ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

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Experimental Uncertainty Analysis for Ship Resistance in Ice Tank Testing

1. PURPOSE OF THE PROCEDURE

Develop a methodology to calculate uncertainties in the results of ship resistance in ice tank tests.

2. INTRODUCTION

Experimental Uncertainty Analysis (EUA) is an analytical process for estimating uncertainties in the results of a given experimental program. Fundamentally, through the EUA process, experimentalists in the laboratory can quantify the agreement (the closeness or the difference) between the measured results and their "true" values.

Historically, until late 1980's, only marginal work on EUA was reported by ocean and marine test facilities. During the 1990's, the International Towing Tank Conference (ITTC) and the International Ship and Offshore Structure Congress (ISSC) have recommended and supported the application of Uncertainty Analysis (UA) in both experimental and numerical/computational fields.

For clarity, in computational and numerical fields, uncertainty analysis is known as Verification and Validation analysis (V&V analysis). The AIAA (1998) gave very useful definitions for the terminology used in V&V analyses. Among these are the definitions for terms such as verification, validation, modelling, simulation, prediction, uncertainty, error, ...etc. The main objective of V&V analysis is to quantify the uncertainty in the results of a numerical model (or computer simulations). Sources for numerical uncertainties include grid convergence, time step convergence, iterative solution, constitutive model, ...etc. The main objective of EUA, however, is to quantify the uncertainty in the experimental results obtained in a given test program.

This procedure deals exclusively with Experimental Uncertainties (EU) in the results obtained from resistance tests of model ships in a typical ice tank. Up to now, in the literature, there are no standards to quantify and/or minimize uncertainties in ice tank ship resistance testing.

Mathematically, the total uncertainty is the geometric sum of two components. They are the systematic component (also, known as the bias uncertainty) and the precision component (also, known as the repeatability uncertainty). The bias component deals with uncertainties in the instrumentation and equipment calibrations (such as load cells, RVDT's[1], yoyo potentiometers and Data Acquisition System (DAS)). However, the precision component deals with environmental and human factors that affect the repeatability of the test results (such as small temperature fluctuations in the ice tank during testing, small misalignments of the ship model in the test set-up, …etc).

The main objective of this document is to provide ice tank experimentalists with a method of analysis to estimate uncertainties in typical ship resistance in ice experiments. To achieve this objective, experiments for ship resistance in

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[1] RVTD = Rotary Variable Differential Transformer

ice were conducted using a model for a Canadian Icebreaker. The results from these tests were used to develop a procedure for EUA in ice tank ship resistance tests.

3. EXPERIMENTAL DATA

Experiments for ship model resistance in ice were conducted at the Institute for Ocean Technology of the National Research Council of Canada (www.iot-ito.nrc-cnrc.gc.ca/) using a model scale of the Canadian Icebreaker, "Terry Fox". The model is 3.79 m long (at water line), and it has a maximum beam section of 0.79 m. The model is 1/21.8 scale of the actual icebreaker.

The tests were conducted in three phases (as shown Table 1). A brief description of the test program is given as follows:

Phase I tests, test results, and the development of a preliminary method for EUA for ship resistance in ice were documented in two IOT reports (Derradji-Aouat et al., 2002, and Derradji-Aouat, 2002).

The documentation for Phase II test program is also presented in two IOT reports (Derradji-Aouat and Coëffé, 2003, and Derradji-Aouat, 2003). The test matrix in Phase II is the same as that in Phase I (see Table 1). The only difference is the target thickness of the ice. In Phase I, all tests were conducted for only one target ice thickness (40 mm), while Phase II tests were conducted for two additional ice thicknesses (25 mm and 55 mm). Together, the two phases provided information for three different ice thicknesses.

In Phase III, the same test matrix as in Phase I was completed. The difference between Phase I and Phase III test programs is that in Phase I,

the ship model was attached to the carriage using the tow post while in Phase III, the model was attached to the carriage using the PMM (Planar Motion Mechanism). The details were provided by Derradji-Aouat and van Thiel (2004).

Table 1: Test matrix

	Ice Sheet #	Model Speed, m/s	Ice Thickness, mm	Ice Strength, kPa
Phase I	1	0.1	40	35
	2	0.2	40	35
	3	0.4	40	35
	4	0.6	40	35
Phase II	5	0.1	25	35
	6	0.2	25	35
	7	0.4	25	35
	8	0.6	25	35
	9	0.1	55	35
	10	0.2	55	35
	11	0.4	55	35
	12	0.6	55	35
Phase III	13	0.1	40	35
	14	0.2	40	35
	15	0.4	40	35
	16	0.6	40	35

All three phases involved experiments in ice and in open water. A total of sixteen (16) different ice sheets were tested. All experiments in ice were very long test runs. The model was towed at constant speeds throughout the useable length of the ice tank (76 m).

4. EXPERIMENTAL UNCERTAINTY ANALYSIS – BASIC EQUATIONS

The "total uncertainty, U" is the geometric sum of a "bias uncertainty, B" and a "random uncertainty, P". Bias uncertainties (also called systematic uncertainties) are due to uncertainty sources such as load cell calibrations, accuracy of   and DAS. Random uncertainties (also

called precision or repeatability uncertainties) are a measure of the degree of repeatability in the test results (i.e. if a test was to be repeated several times, would the same results be obtained each time?). Examples for random uncertainty sources are the changing test environment (such as fluctuations in room temperature during testing), small misalignments in the initial test set-up, human factors, …etc.

Mathematically, the total uncertainty is:

$$U = \pm \sqrt{\left(B^2 + P^2 \right)} \qquad (1a)$$

For a single test population (where only one test is performed, and for that one test, n data readings are obtained), random uncertainty "P" from a source "X" is Px:

$$P_X = t * S_X \qquad (1b)$$

The coefficient "t" is obtained from the standard table for a normal Gaussian distribution (Coleman and Steele, 1998). Its value depends on the desired level of confidence (usually, 95%) and the number of the Degree of Freedom (DOF) in the sample population. The DOF = n –1, where n is the numbers of data readings.

In a multi-test population (where the same test is repeated N times, and each test is represented by only one data point in the population of N data points), the random uncertainty from a source "X" is P_{NX}:

$$P_{NX} = \frac{t * S_{NX}}{\sqrt{N}} \qquad (1c)$$

Derradji-Aouat (2002) showed that in a typical ice tank ship resistance test, the bias uncertainty component (B) is much smaller than the

random one (P), He concluded, therefore, that; in routine ship resistance ice tank testing, the total uncertainty (U) can be taken as equal to the random one. Simply, without a loss of accuracy, the bias uncertainty component can be neglected. It follows that:

$$U = \pm P \qquad (1d)$$

The above equations are valid for direct measurements (directly measured variables, such as load, deformation, motion, pitch, roll, …etc.). In most cases, the measured variables are used to compute engineering parameters (such as stress, strain, resistance, …etc.) using Data Reduction Equations (DRE). Additional uncertainties due to the use of DRE need to be considered (as will be discussed later).

The mathematics of this EUA procedure is based on the equations provided by Coleman and Steel (1998). The latter is in harmony with the guidelines of ISO (1995), ASME (PTC-19.1, 1998), and GUM (2003).

5. SHIP RESISTANCE IN ICE

Since the objective of this procedure is to present a methodology to calculate EUA in the results of ship resistance tests in ice tanks, a summary for the standard calculations of ship resistance in ice is given as follows:

The standards for ship resistance in ice (ITTC-4.9-03-03-04.2.1) give the equation for the total resistance in ice, R_t, as the sum of 4 individual components:

$$R_t = R_{br} + R_c + R_b + R_{ow} \qquad (2a)$$

where R_{br} is the resistance component due to breaking the ice, R_c is the component due to

clearing the ice, R_b is the component due to buoyancy of the ice, and R_{ow} is the resistance component in open water.

In order to quantify each component, the test plan should include tests in level ice, tests in pre-sawn ice, creeping speed tests, and tests in open water (as per ITTC-4.9-03-03-04.2.1). The open water tests provide values for R_{ow}, while the creeping speed tests give R_b. In the pre-sawn ice tests, $R_{br} = 0$, and therefore:

$$R_t = R_c + R_b + R_{ow} \qquad (2b)$$

Since R_{ow} and R_b are known (from the open water and the creeping speed tests), thus:

$$R_c = R_t - R_b - R_{ow} \qquad (2c)$$

where R_t, in Eq. 2c, is the measured resistance in the pre-sawn ice test runs.

From tests in level ice, the total resistance R_t is measured, and the ice breaking component, R_{br}, is calculated as (from Eq. 2a):

$$R_{br} = R_t - R_c - R_b - R_{ow} \qquad (2d)$$

6.   EUA – A PROCEDURE FOR ICE TANK TESTING

This procedure was developed on the basis of one hypothesis and one requirement:

- Segmentation hypothesis, and
- Steady state requirement.

6.1   Segmentation Hypothesis

To conduct the test program (indicated in Section 3), several reasons have contributed to the decision for keeping the speed of the ship model constant throughout most of the useable length of the ice tank (76 m). The main one is the hypothesis that the time history from one long test run can be divided into segments, and each segment can be analysed as a statistically independent test. The hypothesis states that:

"The history for a measured parameter (such as tow force versus time) can be divided into 10 (or more) segments, and each segment is analyzed as a statistically independent test. Therefore, the 10 segments in one long test run are regarded as 10 individual (independent but identical) tests."

Coleman and Steel (1998) reported that, in statistical uncertainty analysis, a population of at least 10 measurements (10 data points) is needed. However, in ice tank testing, conducting the same test 10 times is very costly and very time consuming. Therefore, the principle of segmenting a time history of a measured parameter over a long test run into 10 segments, results in significant savings in costs and efforts. In this case, uncertainties are calculated from the means and standard deviations of the individual segments.

Basically, the hypothesis calls for dividing the long time history into at least 10 equal (more or less equal) segments, calculate the mean and standard deviation for each segment, and then calculate the mean of the means and the standard deviation of the means. An example for segmentation calculations is shown in Table 2.

It should be cautioned that the segmentation hypothesis is valid only if the following 3 conditions are satisfied.

- Each segment should span over 1.5 to 2.5 times the length of the ship model,

- Each segment should include at least 10 events for ice breaking (10 ice load peaks),
- General trends (of a measured parameter such as tow force versus time) are repeated in each segment.

Condition # 1 is based on the fact that the ITTC procedure for resistance tests in level ice (ITTC-4.9-03-03-04.2.1) requires that a test run should span over at least 1.5 times the model length. For high model speeds (> 1 m/s), however, the ITTC procedure requires test spans of 2.5 times the model length.

Condition # 2 is based on the fact that in EUA, for an independent test, a population of at least 10 data points is needed to achieve the minimum value for the factor t (in Eq. 1). The gain in any further reduction in the value of t, by having more than 10 segments, is minimum (Derradji-Aouat, 2004a).

Condition # 3 is introduced to ensure that the overall trends in a measurement are repeated in each segment. This condition serves to provide further assurance into the main hypothesis ("…the 10 segments in one long test run are regarded as 10 individual, independent but identical, tests"). Fundamentally, if the trends are not, reasonably, repeated, then the segments could not be analyzed as "independent but identical" tests.

The time histories measured in creeping speed tests are not subjected to the segmentation hypothesis. Furthermore, it is recognized that the division of the results of a test run into segments is valid only for the steady state portion of the measured data (only the steady state portion of the measured time history is to be used for the segmentation). This is required to eliminate the effects of the initial ship penetration into the ice (transient stage) and the effects of

the slowdown and full stop of the carriage during the final stages of the test run (also transient stage).

6.2 Steady State Requirement

In ice tank testing, for any given ice sheet, the ice properties are not completely (100%) uniform (same thickness) and homogeneous (same mechanical material properties) all over the ice sheet. This is attributed, mainly, to the ice growing processes and refrigeration system in the ice tank (Derradji-Aouat, 2004b).

In addition to the spatial variability of the material properties of ice, during an ice test run, the carriage speed may (or may not) be maintained at exactly the required nominal constant speed. The control system maintains the carriage speed constant. However, when ice breaks, small fluctuations in carriage speed may take place.

Because of this inherent non-uniformity of ice sheets, the non-homogeneity of ice properties and the small fluctuations in the carriage speed, steady state in the time history of a measurement may not be achieved.

Theoretically, if the time history of a measured parameter is changing, then the segments could not be analysed as "identical" tests. The steady state requirement, therefore, calls for a corrective action to account for the effects of non-uniform ice thickness, non-homogenous ice mechanical properties and small fluctuations in carriage speed on the test measurements.

To identify whether or not the time history for a measured parameter has reached its steady state, the following procedure was applied. The time histories for the measured parameters were

plotted along with their linear trend lines (Derradji-Aouat and van Thiel, 2004). A linear trend line with a slope of about zero indicates that a steady state in a measured parameter is achieved.

The non-steady state condition may be attributed to one (or all) of the following 3 factors:

- A changing carriage speed (or small fluctuations in carriage speed) during testing,
- Non-uniform ice thickness,
- Non-uniform mechanical properties of the ice (flexural/compressive strengths, elastic modulus, density of ice, …etc.).

The contribution of each factor was investigated by Derradji Aouat and van Thiel (2004), and they concluded that the effect of changing carriage speed can be ignored (that is factor # 1). The effects of the other two factors are given as:

6.2.1 Non-Uniform Ice Thickness

Mean ice thickness profiles were calculated, each mean profile is the average of 3 measured ice thickness profiles. Each profile is a series of ice thickness measurements (every 2 m) along the length of the ice tank.

The linear trend lines, through the mean thickness profiles, indicate that the ice thickness varied within a range of 0.69% to 2.64%.

To correct for the effects of non-uniform ice thickness on the resistance measurements, the following rational was followed.

The ice thickness corrections are applied only to the resistance due to the ice. Therefore, the total ice resistance ($R_{Total\ Ice}$) is equal to the measured resistance in the ice tests ($R_{Measured}$) minus the resistance measured in the open water tests ($R_{Open\ Water}$).

$$\left(R_{\text{Total}-\text{ice}}\right) = \left(R_{\text{Measured}}\right) - \left(R_{\text{Open}-\text{Water}}\right) \quad (3a)$$

To correct for the ice thickness, the following equation is used:

$$\left(R_{\text{Total}-\text{ice}}\right)_{\text{correct}} = \left(R_{\text{Total}-\text{ice}}\right) * \frac{h_0}{h_m} \quad (3b)$$

where $(R_{\text{Total Ice}})_{\text{correct}}$ is the corrected total ice resistance, $(R_{\text{Total Ice}})$ is the measured total ice resistance (Eq. 3a), h_0 is the nominal ice thickness, and h_m is the measured ice thickness.

The time histories measured in the creeping speed test runs are not subjected to corrections for ice thickness variation. The length of each creeping speed test run is small (only one ship length $\approx$ 3.8 m), the variation of ice thickness over this small length can be ignored.

6.2.2 Non-Homogeneous Ice Properties

Mean flexural strength profiles along the length of the ice tank were given by Derradji-Aouat and van Thiel (2004). Typically, the flexural strength profiles are obtained using in-situ cantilever beam tests. The beam dimensions have the proportions of 1:2:5 (thickness, h_f: width, w: length, L). The flexural strength σ_f is calculated as:

$$\sigma_f = \frac{6PL}{wh_f^2} \quad (4a)$$

where P is the applied point load.

The uncertainty in the flexural strength is $U_{\sigma f}$:

$$U\sigma_f = \sqrt{U_P^2 + U_L^2 + U_W^2 + 2U_{hf}^2} \quad (4b)$$

where U_L, U_W, and U_{hf} are the uncertainties in the measured dimensions (L, w and h_f). U_p is the uncertainty in the measured point load.

Derradji-Aouat (2002) reported that any data correction for ice thickness includes, implicitly, the correction for the flexural strength of the ice. This is due to the fact that ice thickness is a fundamental measurement while the flexural strength is a calculated material property (flexural strength is calculated from measurements of applied point load and dimensions of the ice cantilever beam). Since this work deals with EUA of actual "fundamental" measurements, it is recognized that if corrections were to be made for both ice thickness and flexural strength, double correction (double counting) would take place, and the final uncertainty values would be overestimated. The same argument is valid for corrections for the comprehensive strength of ice (the latter is calculated from applied axial load and measurements of actual dimensions of the ice sample).

Measured ice density profiles along the length of the ice tank were also given by Derradji-Aouat and van Thiel (2004). The density of ice, ρ_i, is given as:

$$\rho_i = \rho_w - \frac{M}{V} \qquad (4c)$$

where ρ_w is the density of water. M is the mass of the ice sample. The volume, V, is calculated from the sample dimensions (length, L, width, W, and thickness, H): The uncertainty in the ice density is:

$$U_{\rho_i} = \sqrt{U_H^2 + U_L^2 + U_W^2 + U_M^2} \qquad 4d)$$

During testing, it was noted that the variation of density along the centre line of the tank varied between 4.58% and 8.60%.

6.3 Calculation of Random Uncertainties

In the following example, the discussion will be focused on the results given in Figure 1. Other examples were given by Derradji-Aouat et al. (2004). Figure 1a is the measured tow force time history a resistance test in level ice at model speed of 0.1 m/s. Figure 1b shows examples for the segments, in this particular test, the time history was divided into 15 segments. Table 2 shows the segments for the mean tow force history; all ice sheets in Phase I are presented. The tow force history in each test is divided into > 10 segments. Mean tow force (F_{T_mean}) is obtained for each segment.

For each time history, the mean of the > 10 means (Mean_F_{T_mean}) and the standard deviation of the 10 means (STD_ F_{T_mean})) were calculated (as shown in Table 2).

Random uncertainties in the tow forces $U(F_{T\_mean})$) are calculated in three (3) steps:

Step # 1: In Table 2, after the calculations of the mean of means and standard deviation of means, the Chauvenet's criterion is applied to identify outliers (outliers are discarded data points). The Chauvenet number for mean tow forces is (Chauv #)$_{Mean}$:

$$(Chauv\,\#)_{\text{Mean}} = \left| \frac{F_{\text{T\_mean}} - \left(Mean\_F_{\text{T\_mean}} \right)}{\left(STD\_F_{\text{T\_mean}} \right)} \right| \qquad (5a)$$

For 10 to 15 segments, the Chauv # should not exceed 1.96 to 2.13. In Table 2, data points with Chauv # greater than 1.96 were disregarded. A new mean of means and a new standard deviation of means are calculated from the remaining data points (Table 2).

Step # 2: After calculating the new mean of the means and the new standard deviation of the means (from the remaining segments), random uncertainty in the mean tow force is:

$$\left(U(F_{T\_mean})\right) = \frac{t*\left(STD\_F_{T\_mean}\right)}{\sqrt{N}} \tag{5b}$$

where $t \approx 2$, and N is the number of the remaining data points (valid segments).

Step # 3: Random uncertainties are expressed in terms of uncertainty percentage (U_P):

$$\left(U_P(F_{T\_mean})\right) = \frac{U(F_{T\_mean})}{Mean\_F_{T\_mean}} * 100 \tag{5c}$$

It is important to note that the above procedure (segmentation of the measured time history, checks for the steady state requirement, correction for ice thickness, the use of the three calculation steps) is valid for calculating random uncertainties in all other measured ship motion parameters (such as pitch, heave, yaw and sway).

6.3.1 Effects of Data Reduction Equations

Equation 3b was proposed to correct for the effects of ice thickness variations on the values of random uncertainties in resistance. It should be recognized that the corrected resistance curves are not direct laboratory measurements, but they are calculated from the analytical Eq. 3b. The process of using analytical equations to correct measured parameters is called: "Application of Data Reduction Equations, DRE".

In EUA, there are additional random uncertainties involved in using DRE. The uncertainty involved in using Eq. 3b is:

$$\left(\frac{U_R}{R}\right) = \left(\left(\frac{U_{R_0}}{R_0}\right)^2 + \left(\frac{U_h}{h_0}\right)^2\right)^{\frac{1}{2}} \tag{6}$$

In the above equation, (U_R/R) is the total uncertainty in resistance. Both (U_{R0}/R_0) and (U_h/h_0) are the relative uncertainty in the measured ice resistance and the relative uncertainty in the measured ice thickness, respectively. In Eq. 5, the value of (U_h/h_0) is an additional relative uncertainty that is induced to account for the use of the DRE.

6.4   Calculation of Bias Uncertainties

6.4.1   Sources for Bias Uncertainties

Bias uncertainties are attributed to the DAS and the instrumentation used for measurements (such as load cells, yoyo potentiometers and RVTD's). Table 3 is an example for how bias uncertainties are calculated. The first column of Table 3 is a list of the major bias uncertainty sources involved. Essentially, the list was developed by the DAS system specialists, electronics and instrumentation technologists. The experience and skills of these professionals play a significant and critical role in identifying major sources for uncertainties. Typically, calculations of bias uncertainties are based on the instrument data sheets, load cell calibration curves and DAS manufacturer design and gain specifications (details are given by Derradji-Aouat (2002).

6.4.2   Determination of Bias Uncertainties in Ice Tank Testing

As shown in Table 3, the DAS on board of the ice tank carriage comprises three main sub-components: The amplifier, the multiplier and the Daq-board. The instrumentation used for measurements included a load cell (to measure tow force), a yoyo potentiometers (to measure heave) and two RVDT's to measure pitch and roll of the model. The carriage speed was measured automatically via a dedicated channel in the carriage control system.

The results (in Table 3) show that the sum of all bias uncertainties for any given instrument is below 0.4%.

6.5   Calculation of Total Uncertainties

In ice tank experiments, bias uncertainties are much smaller than the random once. Subsequently, it is recommended that; in ice tank testing and without a loss of accuracy of the uncertainty analysis, the total uncertainty can be taken as equal to the random one (Eq. 1d). Simply, the bias uncertainty component can be neglected (Derradji-Aouat et al., 2004).

7.   SUMMARY OF PROCEDURE

To compute the uncertainties in the results of a ship resistance in ice test program, the following procedure should be followed:

a) Perform one test for ship resistance in ice. The test run should be long enough so that it can be divided into 10 segments (satisfying the 3 conditions given in section 6.1).
b) Check the measured resistance time history for the steady state requirement (satisfying the requirement in section 6.2).

c) Apply the segmentation (at least 10 segments should be obtained, as shown in Figure 1 and Table 1).
d) Correct the resistance for the variation of ice thickness (using Eq. 3b).
e) Use the three steps to calculate random uncertainties (using Eqs. 5a, 5b and 5c).
f) Estimate bias uncertainties using calibration data and components data sheets, as shown in Table 3.
g) Calculate total uncertainties using Eq. 1a (or 1b if bias uncertainties are neglected).
h) Correct for the application of any DRE (using Eq. 6).

8.   VALIDATION

8.1   Test Results and Comparisons

In the three phases of testing, uncertainty values varied between 3% and 10% (Derradji-Aouat et al., 2004).

The 23rd ITTC Specialist Committee on Ice presented an example for how to estimate random uncertainties in ice testing. In that example, the committee used the results of tests for ship resistance conducted by Kitagawa et. al, (1991 and 1993) in the Japanese NMRI ice tank. Comparisons between the calculations presented by the 23rd ITTC and those reported in the present test program resulted in the following conclusions:

Although the calculations of uncertainties were performed using the results of two different test programs, conducted at two different ice tanks in two different countries (Canada and Japan) and about 10 to 12 years apart, the final calculations converged to about the same range of uncertainties (3% to 10%).

The range of uncertainty (3% to 10%) is not far from the range (10% to 12%) reported by Newberry (1992), using a different ship model (R class icebreaker), 12 years ago, at the IMD ice tank.

It should be recognized that more EUA comparisons using data from various ice tanks (various model ice types and test conditions) are very much needed to accurately estimate and compare uncertainties involved in various tanks. At this point in time, the limited number of EUA publications, in the literature of ice tank testing, inhibited the work towards a larger and more comprehensive comparison study in uncertainties among tanks worldwide (only qualitative comparisons are possible).
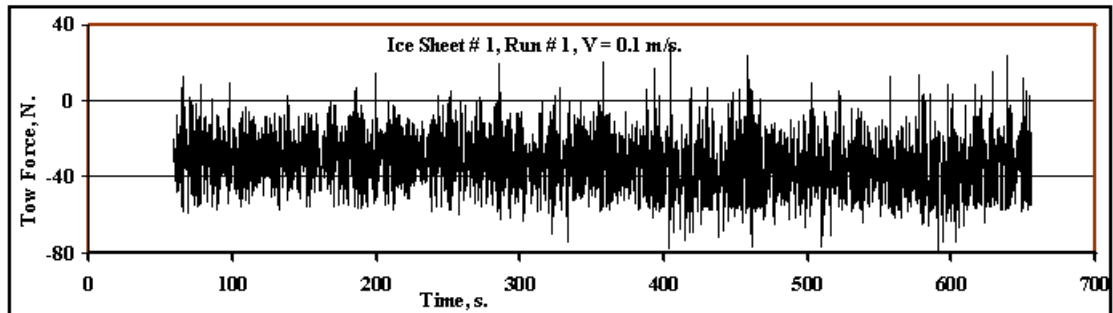
Figure 1a: An example for typical test measurement - Tow force versus time
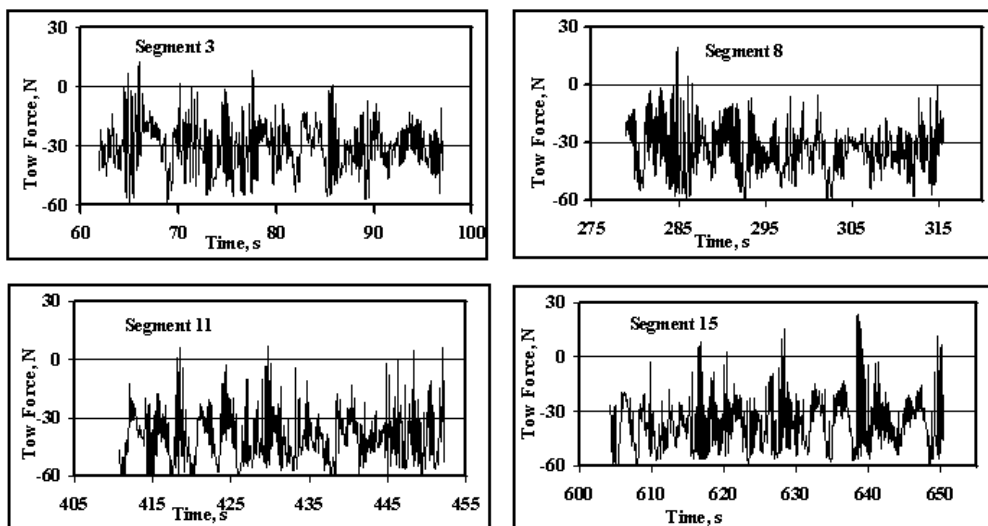(Constant speed, v = 0.1 m/s, level ice, ice thickness = 40 mm, length of run = 65 m).

Figure 1b: Division of measured test results (in Fig. 1a) into segments
(Four segments are shown as examples)

Table 2: Examples for calculations for random uncertainties in mean tow force

Mean Tow Force, Continuous Ice Sheet - RUN # 1									
Ice Sheet #	Segment #	TF-mean (N)	Mean (TF_mean)	STD_mean (N)	(Chauv#)	New Mean TF_mean	New STD_mean	Uncertainty Value (N)	Uncertainty %
1	3	-30.45	34.67	0.88	1.54			0.49	1.41%
	4	-30.49	34.67	0.88	1.28				
	5	-29.88	34.67	0.88	1.03				
	6	-30.63	34.67	0.88	0.77				
	7	-31.69	34.67	0.88	0.51				
	8	-34.03	34.67	0.88	0.26				
	9	-32.91	34.67	0.88	0.00				
	10	-35.80	34.67	0.88	0.26				
	11	-40.16	34.67	0.88	0.51				
	12	-38.19	34.67	0.88	0.77				
	13	-38.23	34.67	0.88	1.03				
	14	-40.24	34.67	0.88	1.28				
	15	-36.26	34.67	0.88	1.54				
2	3	-42.80	36.57	0.27	1.55			0.14	0.39%
	4	-44.29	36.57	0.27	1.31				
	5	-42.40	36.57	0.27	1.08				
	6	-45.13	36.57	0.27	0.84				
	7	-44.51	36.57	0.27	0.60				
	8	-43.25	36.57	0.27	0.36				
	9	-44.30	36.57	0.27	0.12				
	10	-45.43	36.57	0.27	0.12				
	11	-47.42	36.57	0.27	0.36				
	12	-46.87	36.57	0.27	0.60				
	13	-47.57	36.57	0.27	0.84				
	14	-47.82	36.57	0.27	1.08				
	15	-51.21	36.57	0.27	1.31				
	16	-50.96	36.57	0.27	1.55				
3	3	-54.79	48.74	0.47	1.53			0.27	0.56%
	4	-56.09	48.74	0.47	1.25				
	5	-53.44	48.74	0.47	0.97				
	6	-53.90	48.74	0.47	0.69				
	7	-55.66	48.74	0.47	0.42				
	8	-49.25	48.74	0.47	0.14				
	9	-56.16	48.74	0.47	0.14				
	10	-54.26	48.74	0.47	0.42				
	11	-62.24	48.74	0.47	0.69				
	12	-60.64	48.74	0.47	0.97				
	13	-60.51	48.74	0.47	1.25				
	14	-60.96	48.74	0.47	1.53				
4	3	-60.84	61.77	0.89	1.51			0.54	0.87%
	4	-60.17	61.77	0.89	1.21				
	5	-62.57	61.77	0.89	0.90				
	6	-61.22	61.77	0.89	0.60				
	7	-60.53	61.77	0.89	0.30				
	8	-62.17	61.77	0.89	0.00				
	9	-65.25	61.77	0.89	0.30				
	10	-67.83	61.77	0.89	0.60				
	11	-66.28	61.77	0.89	0.90				
	12	-67.63	61.77	0.89	1.21				
	13	-69.09	61.77	0.89	1.51				

Note: Calculations for all other test runs were given by Derradji-Aouat (2004b). Note that the segment # starts always as # 3. During testing and data acquisition, segment # 1 was designated for the raw data and segment # 2 was designated for the tarred data.

Table 3: Ice tank bias uncertainty calculations

Transducer Type	Load Cell	YoYo Pot	RVDT	RVDT	Carriage
Parameter	Tow Force (N)	Sinkage (m)	Pitch (degrees)	Roll (degrees)	Speed (m/s)
DAS Channel	CH. 8	CH. 21	CH. 27	CH. 28	CH. 33
Transducer	% F.S.	% F.S.	% F.S.	% F.S.	% F.S.
Non Linearity	0.0200		0.1870	0.1930	
Hysterisis	0.0200				
Non Repeatability	0.0100		0.0200	0.0200	
Zero Offset Drift	0.0216	0.0432			
Span Temp. Coefficient		0.0432	0.2160	0.2160	
Accuracy		0.1500			
DAS - NEFF Amplifier	% F.S.	% F.S.	% F.S.	% F.S.	% F.S.
Gain Stability	0.0350	0.0350	0.0350	0.0350	0.0350
Non Linearity	0.0200	0.0200	0.0200	0.0200	0.0200
Zero Stability	0.0160	0.0030	0.0030	0.0030	0.0030
Zero Drift	0.0220	0.0130	0.0130	0.0130	0.0130
Common-Mode Rejection	0.0500				
DAS-Iotech DBK12 Multiplexer	% F.S.	% F.S.	% F.S.	% F.S.	% F.S.
Gain Accuracy	0.2500	0.2500	0.2500	0.2500	0.2500
Non Linearity	0.0150	0.0150	0.0150	0.0150	0.0150
Offset Drift	0.0250	0.0250	0.0250	0.0250	0.0250
Common-Mode Voltage	0.0020	0.0020	0.0020	0.0020	0.0020
DAS Iotech Daqboard /200	% F.S.	% F.S.	% F.S.	% F.S.	% F.S.
A/D Linearity	0.0031	0.0031	0.0031	0.0031	0.0031
A/D Zero Drift	0.0050	0.0050	0.0050	0.0050	0.0050
A/D Gain Drift	0.0150	0.0150	0.0150	0.0150	0.0150
Analog i/p Resolution	0.0015	0.0015	0.0015	0.0015	0.0015
Analog i/p Accuracy	0.0250	0.0250	0.0250	0.0250	0.0250
Analog i/p Gain Temp. Coefficient	0.0015	0.0015	0.0015	0.0015	0.0015
Analog i/p Offset Temp. Coefficient	0.0012	0.0012	0.0012	0.0012	0.0012
TOTAL BIAS (SYSTEM)	% F.S.	% F.S.	% F.S.	% F.S.	% F.S.
UNCERTAINTY % Fs = Full Scale	0.2655	0.3038	0.3848	0.3878	0.2570

Note: Total bias uncertainty values are the same for all test runs since the same DAS and same transducers are in all test runs (an all test types: in ice or in open water).

8.2 Benchmark Tests

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