ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Uncertainty Analysis, Example for Propulsion Test

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Uncertainty Analysis, Example for Propulsion Test

1. PURPOSE OF PROCEDURE

The purpose of the procedure is to provide an example for the uncertainty analysis of a model scale towing tank propulsion test following the ITTC Procedures 7.5-02-01-01 Rev 00, 'Uncertainty Analysis in EFD, Uncertainty Assessment Methodology' and 7.5-02-01-02 Rev 00, 'Uncertainty Analysis in EFD, Guideline for Towing Tank Tests.'

2. EXAMPLE FOR PROPUL-SION TEST

This procedure provides an example showing an uncertainty assessment for a model scale towing tank propulsion test. The bias and precision limits and total uncertainties for single and multiple runs have been estimated for the thrust deduction factor (t), wake fraction (w_T) and relative rotative efficiency (η_R) at model scale at one Froude number.

In order to achieve reliable precision limits, it is recommended that 5 sets of tests with 3 speed measurements in each set are performed giving in total 15 test points. In this example the recommended sequence was followed.

This example utilises the bias and precision limits derived in ITTC Procedures 7.5-02-02-02 Rev 01 'Uncertainty Analysis, Example for Resistance Test,' and 7.5-02-03-02.2 Rev 00 'Uncertainty Analysis, Example for Open Water Test.'

Extrapolation to full scale has not been considered in this example. Although it might lead to significant sources of error and uncertainty, it is not essential for the present purpose of demonstrating the methodology.

When performing an uncertainty analysis for a real case, the details need to be adapted according to the equipment used and procedures followed in each respective facility.

2.1 Test Design

By measuring the thrust (T), torque (Q), external tow force (F_D), model speed (V), propeller rate of revolution (n) and water temperature (t °), and by measuring or using reference values from the resistance and propeller open water tests, the thrust deduction factor (t), wake fraction (w_T) and relative rotative efficiency (η_R) can be calculated.

Thrust and torque measured in the propulsion tests are expressed in the non-dimensional form as

$$K_{TM} = \frac{T_M}{\rho_M n_M^2 D_M^4} \tag{2-1}$$

$$K_{QM} = \frac{Q_M}{\rho_M n_M^2 D_M^5} \tag{2-2}$$

where D is the nominal diameter of the propeller, n is the propeller rate of revolution, and ρ is the nominal mass density of fresh water in the tank set to ρ=1000 kg/m³ according to ITTC-1978 extrapolation method. T and Q are the measured thrust and torque of the propeller corrected for measured trust and torque of hub and shaft.

With K_T as input, J_T and K_{QT} are read off the model propeller characteristics, and the wake fraction is derived as:

$$w_{TM} = 1 - \frac{J_{TM} n_M D_M}{V_M} \tag{2-3}$$

and the relative rotative efficiency as:

$$\eta_R = \frac{K_{QTM}}{K_{QM}} \tag{2-4}$$

The thrust deduction factor is obtained from

$$t = \frac{T_M + F_D - R_C}{T_M} \tag{2-5}$$

with

$$F_D = \frac{1}{2}\rho_M S_M V_M^2 \left[(1+k)(C_{FMC} - C_{FS}) - \Delta C_F\right] \tag{2-6}$$

where R_C is the resistance corrected for differences in temperature between resistance and propulsion tests:

$$R_C = \frac{(1+k)C_{FMC} + C_R}{(1+k)C_{FM} + C_R} R_{TM} \tag{2-7}$$

where:

$(1+k)$ is the form factor

C_{FC} is the frictional resistance coefficient at the temperature of the propulsion test.

C_F is the frictional resistance coefficient at the temperature of the resistance test.

C_{FS} is the frictional resistance coefficient at 15 degrees of the ship scale.

C_R is the residuary resistance coefficient of the resistance test.

R_T is the total resistance measured in the resistance test

ΔC_F is the roughness allowance, defined as:

$$\Delta C_F = 0.044\left[\left(\frac{k_S}{L_{WL}}\right)^{\frac{1}{3}} - 10Re^{\frac{1}{3}}\right] + 0.000125 \tag{2-8}$$

k_S is the roughness of hull surface

C_F is the frictional resistance coefficient

The water density in the towing tank (ρ) is set to 1000 kg/m^3 according to the ITTC-78 procedure. The viscosity is calculated for the measured tank temperature using ITTC Procedure 7.5-02-01-03 Rev 00 'Density and Viscosity of Water.'

The propulsion test requires both resistance test and open water propeller test results. These tests have bias and precision errors, which may affect not only the uncertainty in scaling to ship powering prediction but also uncertainty in the propulsion test. As these three tests were accepted as a minimum set of required tests for the determination of the ship powering, the effects of bias and precision errors of both resistance and open water tests on propulsion test bias errors were taken into account to predict the uncertainty in wake fraction, thrust deduction factor and relative rotative efficiency.

The total uncertainty for resistance measurement, total resistance coefficient etc was embedded into the propulsion bias error.

2.2 Measurement Systems and Procedure

Figure 2.1 shows a block diagram for the propulsion test including individual measurement systems, measurement of individual variables, data reduction and experimental results.

In Section 2.3.1 the bias limits contributing to the total uncertainty will be estimated for individual measurement systems: hull geometry, propeller geometry, model speed, propeller rate of revolution, thrust, torque, tow force and temperature/density/viscosity. The elementary bias limits are for each measurement system estimated for the categories: calibration, data acquisition, data reduction and conceptual bias.

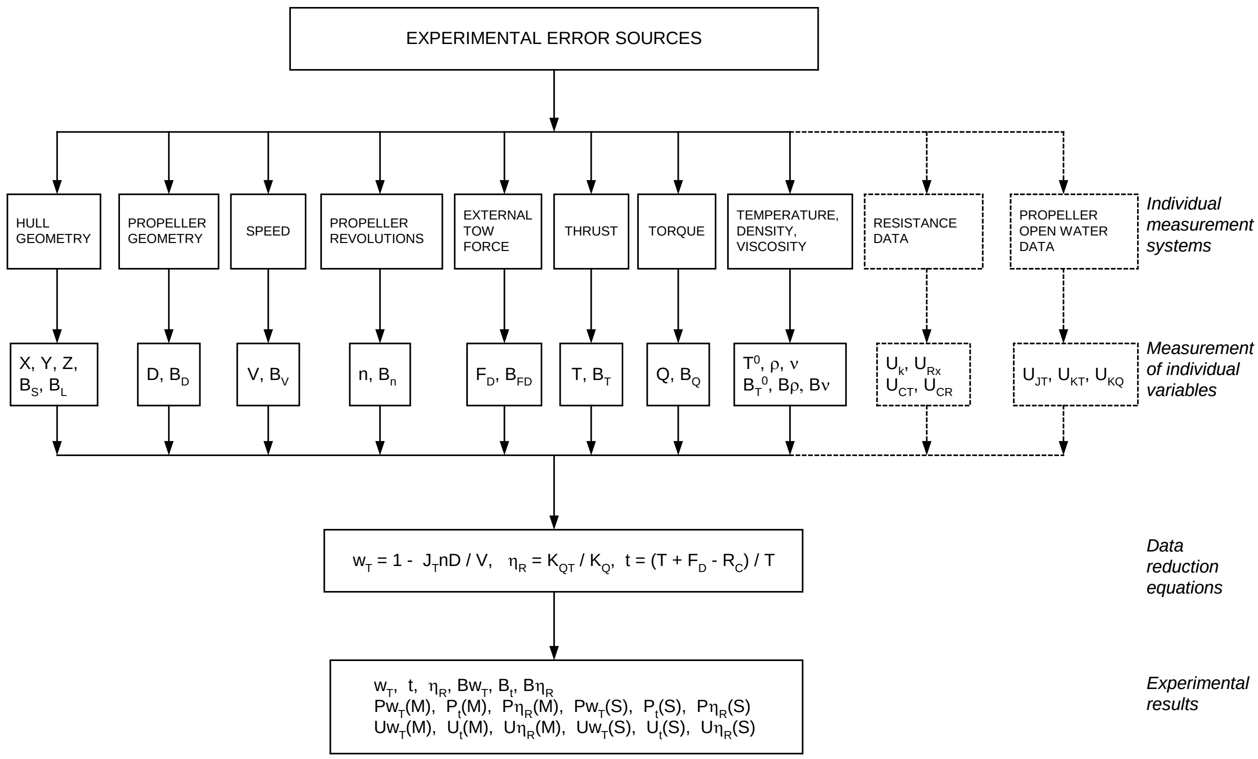

Figure 2.1 Block diagram of test procedure.

Table 2.1 Ship particulars.

Definitions	Symbol	Value (unit)
Length between perp.	L_{PP}	6.500 (m)
Length on waterline	L_{WL}	6.636 (m)
Length overall submerged	L_{OS}	6.822 (m)
Breadth	B	1.100 (m)
Draught even keel	T	0.300 (m)
Wetted surface incl. rudder	S	7.600 (m^2)
Area water plane	A_{WP}	4.862 (m^2)
Displacement	∇	1.223 (m^3)
Block coefficient	$C_B = \nabla / L_{PP}BT$	0.5702 (-)
Water plane coefficient	$C_{WP} = A_{WP}/L_{PP}B$	0.680 (-)
Wetted surface coefficient	$C_S = S/\sqrt{(\nabla L_{PP})}$	2.695 (-)
Surface roughness	ks	$150 \cdot 10^{-6}$ (m)

The bias limits are then, using the data reduction Eq. (2-3), (2-4) and (2-5), reduced into $B(w_T)$, $B(\eta_R)$, $B(t)$ respectively.

The precision limits for the thrust deduction factor $P(t)$, wake fraction $P(w_T)$ and relative rotative efficiency $P(\eta_R)$ in model scale are estimated by an end-to-end method for multiple tests (M) and a single run (S).

In the Tables 2.1, 2.2 and 2.3, the ship, propeller particulars and constants used in the example are tabulated. The example uses the same hull and speed as in the ITTC Procedure 7.5-02-02-02 Rev 01 ‘Uncertainty Analysis, Example for Resistance Test' and the propeller is chosen as the same as in the ITTC Procedure 7.5-02-03-02.2 Rev 00 'Uncertainty Analysis Example for Open Water Test'.

Table 2.2 Propeller particulars.

Definitions	Symbol	Value (unit)
No of Prop	*Z*	1 (-)
Propeller diameter	*D*	0.2275 (m)
Propeller Pitch ratio	*P/D*	0.94 (-)
Propeller Blade Area Ratio	*BAR*	0.60 (-)

Table 2.3 Constants.

Definitions	Symbol	Value (unit)
Gravity	g	9.81 (m/s^2)
Density, model basin	ρ	1000 (kg/m^3)
Water temperature (resistance test average)	$t°$	15.0 °
Water temperature (propulsion test average)	$t°$	15.8 °

2.3 Uncertainty Analysis

The total uncertainty for the propulsive coefficients are given by the root sum square of the uncertainties of the total bias and precision limits

$$U^2 = B^2 + P^2 \tag{2-9}$$

The bias limits for equations (2-3), (2-4) and (2-5) are

$$B(w_T)^2 = \left(\frac{\partial w_{\mathrm{T}}}{\partial J_T}B(J_T)\right)^2 + \left(\frac{\partial w_T}{\partial D}B(D)\right)^2 + \left(\frac{\partial w_T}{\partial n}B(n)\right)^2 + \left(\frac{\partial w_T}{\partial V}B(V)\right)^2 \tag{2-10}$$

$$B(\eta_{\mathrm{R}})^2 = \left(\frac{\partial \eta_{\mathrm{R}}}{\partial K_{QT}}B(K_{QT})\right)^2 + \left(\frac{\partial \eta_{\mathrm{R}}}{\partial K_Q}B(K_Q)\right)^2 \tag{2-11}$$

$$B(t)^2 = \left(\frac{\partial t}{\partial T}B(T)\right)^2 + \left(\frac{\partial t}{\partial F_{\mathrm{D}}}B(F_{\mathrm{D}})\right)^2 + \left(\frac{\partial t}{\partial R_{\mathrm{C}}}B(R_{\mathrm{C}})\right)^2 \tag{2-12}$$

The precision limits will be determined for the propulsive coefficients by an end-to-end method where all the precision errors for speed, thrust, torque, towing force, rate of revolutions and temperature/density/viscosity are included. The precision limits for a single run (*S*) and for the mean value of multiple tests (*M*) are determined. Regardless as to whether the precision limit is to be determined for single or multiple runs the standard deviation must be determined from multiple tests in order to include random errors such as model misalignment, heel, trim etc. If it is not possible to perform repeated tests the experimenter must estimate a value for the precision error using the best information available at that time. The precision limit for multiple tests is calculated according to

$$P(M) = \frac{K\ SDev}{\sqrt{M}} \tag{2-13}$$

where M = number of runs for which the precision limit is to be established, *SDev* is the standard deviation established by multiple runs and K=2 according to the methodology.

The precision limit for a single run can be calculated according to

$$P(S) = K\ SDev \tag{2-14}$$

2.3.1 Bias Limits

Under each group of bias errors (hull geometry, propeller geometry, temperature /density/viscosity, model speed, propeller rate of revolution, resistance, external tow force, thrust and torque) the elementary error sources have been divided into the following categories: calibration; data acquisition; data reduction; and conceptual bias. The categories not applicable for each respective section have been left out.

2.3.1.1 Hull Geometry (Model Length and Wetted Surface Area)

The model is manufactured to be geometrical similar to the drawings or mathematical model describing the hull form. Even though great effort is given to the task of building a model no model manufacturing process is perfect and therefore each model has an error in form and wetted surface. The influence of an error in hull form affects not only the wetted surface but also the measured values by an error in resistance, thrust and torque. For example, two hull forms, with the same wetted surface and displacement, give different resistance, wake fraction, thrust deduction and relative rotative efficiency when towed in water if the geometry is not identical. This error in hull form geometry is very difficult to estimate, and will not be considered here. Only the bias errors in model length and wetted surface area due to model manufacture error are taken into account.

<u>Model Length</u>

Data acquisition:

The bias error in model length (both waterline and overall submerged) due to manufacturing error in model geometry can be adopted from model accuracy of ±1 mm in all co-ordinates as given in ITTC Procedure 7.5-01-01-01 Rev 01 'Ship Models', hence the bias error in model length will be $B(L)$=2 mm.

The bias error in ship length must be estimated for propulsion test uncertainty analysis. As scaling to ship was not considered, ship length bias error is taken as $B(L)$=0 mm.

<u>Wetted surface</u>

Bias errors in wetted surface area of the model due to errors in model manufacture can be calculated as given in ITTC Procedure 7.5-02-02-02 Rev 01 'Uncertainty Analysis, Example for Resistance Test.'

Data acquisition:

The error in wetted surface due to manufacturing error in model geometry can be carried out as given in ITTC Procedure 7.5-02-02-02 Rev 01 'Uncertainty Analysis, Example for Resistance Test', where wetted surface area bias error due to model manufacture error is $B(S_1)$=0.0037 m^2.

The model weight (including equipment) is measured with a balance and the model is loaded to the nominal weight displacement. The balance used when measuring the model weight is calibrated to ± 1.0 kg. The errors in model- and ballast weights are seen in Table 2.4.

The total uncertainty in weight is given by the root sum square of the accuracy of the group of weights, i.e.

$$\sqrt{1^2+0.75^2+1.41^2+1.30^2+0.12^2+0.09^2}$$
$$=2.295\text{kg}$$

which will result in a wetted surface area error of $B(S_2)$=0.0063 m^2 as shown in ITTC Procedure 7.5-02-02-02 Rev 01 'Resistance Uncertainty Analysis, Example for Resistance Test'.

Finally the bias error limit in wetted surface is obtained by the root sum square of the two bias components as

$$B(S)=\sqrt{0.037^2+0.0063^2}=0.0073\ \text{m}^2$$

corresponding to 0.10 % of the nominal wetted surface area of 7.6 m^2.

Table 2.4 Error in displacement

Item	Weights	Weights	
		Individual weights	Group weights
Ship model	260 kg	± 1.0 kg	± 1.00 kg
Equipment	50 kg	± 0.75 kg	± 0.75 kg
Ballast weights	2x200 kg	± 1.0 kg	$\sqrt{2(1.0)^2}$=± 1.414 kg
	3x150 kg	± 0.75 kg	$\sqrt{3(0.75)^2}$=± 1.299 kg
	6x10 kg	± 0.05 kg	$\sqrt{6(0.05)^2}$=± 0.12 kg
	3x1 kg	± 0.005 kg	$\sqrt{3(0.005)^2}$=± 0.09 kg
Total weight displ.	1223 kg		±2.295 kg

2.3.1.2 Propeller Geometry

The model is manufactured to be geometrically similar to the actual propeller geometry. Although efforts are made to produce an accurate propeller model, including the use of NC milling machines, errors in dimensions and offsets can occur leading, for example, to errors in diameter, chord length, pitch and blade section shape. The influence of these errors in dimensions and shape can strongly affect the flow characteristics around the propeller blades and hence the measured thrust and torque and subsequently the wake fraction, thrust deduction and relative rotative efficiency. These errors can only be estimated through systematic variations in propeller geometry and offsets. For example, the sensitivity of pitch setting can be estimated by performing multiple propulsion tests, where the pitch has been re-set between each set of tests. Errors arising from blade shape inaccuracies can only be estimated by performing multiple tests with different propeller models manufactured from the same surface description.

It is seen that many of the errors in propeller geometry are difficult to estimate and are therefore not considered in this example. Only the bias error on propeller diameter will be considered.

Data acquisition

In this example the error in model propeller diameter due to manufacturing error is estimated from the limits given in ITTC Procedure 7.5-01-01-01 Rev 01 'Ship Models.' By assuming the error in model diameter to be within ±0.1mm the bias error limit is estimated to be $B(D)$=0.0001 m corresponding to 0.044% of the nominal propeller diameter of 0.2275 m.

2.3.1.3 Temperature/Density/Kinematic Viscosity

Density and kinematic viscosity of fresh/sea water are affected by bias errors in temperature measurement and density/viscosity calculation. Bias errors in temperature measurement and density calculation can be derived following the methodology given in ITTC Procedure 7.5-02-02-02 Rev 01 'Uncertainty Analysis, Example for Resistance Test.'

<u>Temperature</u>

Calibration:

The thermometer used is calibrated by the manufacturer with a guaranteed accuracy of ±0.30°C within the interval -5 to +50°C. The bias limit associated with temperature measurement is $B(t°)$ = 0.°C corresponding to 1.899 % of the nominal temperature in propulsion tests of 15.8 °C.

As no scaling effects was considered in this example temperature measurement bias error limit in ship scale was neglected as $B(t°) = 0$.

Density

Calibration:

The density-temperature relationship (table) according to the ITTC Procedure 7.5-02-01-03 Rev 00 'Density and Viscosity of Water' for g=9.81m/s² can be expressed as:

$$\rho = 1000.1 + 0.0552 t° - 0.0077 t°^2 + 0.00004 t°^3 \tag{2-15}$$

$$\left|\frac{\partial \rho}{\partial t°}\right| = \left|0.0552 - 0.0154 t° + 0.000120 t°^2\right| \tag{2-16}$$

Using Eq. (2-16) with $t°$=15.8°C and $B(t°)$=0.3°C the bias $B(\rho_1)$ can be calculated according to:

$$B(\rho_1) = \left|\frac{\partial \rho}{\partial t°}\right| B(t°) = 0.1582 \cdot 0.3 = 0.0474\,\mathrm{kg/m^3} \tag{2-17}$$

Data reduction:

The error introduced when converting the temperature to a density (table lookup) can be calculated as two times the *SEE* of the curve fit to the density/temperature values for the whole temperature range. Comparing the tabulated values with the calculated values (Eq. 2-15) the bias error B(ρ_2) can be calculated as $B(\rho_2)$=0.070 kg/m³.

Conceptual:

The nominal density according to the ITTC-78 method is ρ = 1000 kg/m³. Using this method introduces a bias limit as the difference between ρ(15.8°C) = 999.27kg/m³ and ρ = 1000 kg/m³ such that B(ρ_3)=1000.0-999.27 = 0.7338 kg/m³.

The bias for ρ can than be calculated according to:

$$B(\rho) = \sqrt{B(\rho_1)^2 + B(\rho_2)^2 + B(\rho_3)^2} = \sqrt{(0.1582 \cdot 0.3)^2 + 0.070^2 + 0.7338^2} = 0.7387\,\mathrm{kg/m^3} \tag{2-18}$$

The bias limit for density is thus $B(\rho)$=0.7387 kg/m³ corresponding to 0.074 % of ρ = 1000 kg/m³. If using the density value, determined by the temperature, the bias limit $B(\rho_3)$ will be eliminated.

Fresh Water Kinematic Viscosity

Calibration:

The viscosity-temperature relationship for fresh water adopted by ITTC Procedure 7.5-02-01-03 Rev 00 'Density and Viscosity of Water' can be calculated as

$$\nu = ((0.000585(t° - 12.0) - 0.03361)(t° - 12.0) + 1.2350)10^{-6} = (0.000585 t°^2 - 0.04765 t° + 1.72256)10^{-6} \tag{2-19}$$

Partial derivative of Eq. (2-19) is

$$\frac{\partial \nu}{\partial t°} = (0.00117 t° - 0.04765)10^{-6} \tag{2-20}$$

Using Eq. (2-20) with $t°$ =15.8°C and $B(t°)$=0.3°C the bias $B(\nu_1)$ can be calculated according to:

$$B(\nu_1)=\left|\frac{\partial\nu}{\partial t^\circ}\right|B(t^\circ)=0.029164\ 10^{-6}\cdot 0.3=$$

$$= 0.008749\ 10^{-6}\ \text{m}^2/\text{s} \qquad (2\text{-}21)$$

Data reduction:

For a nominal temperature of 15.8°C this formula results in ν=1.1157 10^{-6} m^2/s. Meanwhile the fresh water kinematic viscosity according to the table in ITTC Procedure 7.5-02-01-03 Rev 00 for 15.8°C is equal to ν=1.1154 10^{-6} m^2/s. Using this method introduces a bias error due to the difference between ν(15.8)= 1.1157 10^{-6} m^2/s and ν=1.1154 10^{-6} m^2/s such as $B(\nu_2)$= -3.294 10^{-10} m^2/s.

Hence the total bias limit associated with fresh water viscosity

$$B(\nu)=\sqrt{B(\nu_1)^2+B(\nu_2)^2} \qquad (2\text{-}22)$$

The bias limit associated with fresh water viscosity due to temperature measurement and viscosity calculation method is thus $B(\nu)$ = 8.7554 10^{-9} m^2/s corresponding to 0.785 % of the kinematic viscosity.

Seawater Kinematic Viscosity

Calibration:

The viscosity-temperature relationship for sea water adopted by ITTC Procedure 7.5-02-01-03 Rev 00 'Density and Viscosity of Water' can be calculated as

$$\nu = ((0.000659(t^\circ-1.0)$$
$$-0.05076)(t^\circ-1.0)+1.7688)10^{-6}$$
$$=(0.000659t^{\circ 2}-0.052074t^\circ+1.820215)10^{-6} \qquad (2\text{-}23)$$

Partial derivative of Eq. (2-23) is

$$\frac{\partial\nu}{\partial t^\circ}=(0.001318t^\circ-0.052074)10^{-6} \qquad (2\text{-}24)$$

Using Eq. (2-24) with t°=15.0°C and $B(t^\circ)$ = 0.0 °C in ship scale the bias $B(\nu_1)$ can be calculated according to:

$$B(\nu_1)=\left|\frac{\partial\nu}{\partial t^\circ}\right|B(t^\circ)=0.032304\ 10^{-6}\cdot 0.0=0.0\,\text{m}^2/\text{s} \qquad (2\text{-}25)$$

Data reduction:

For a nominal temperature of 15.0°C this formula results in ν=1.1873 10^{-6} m^2/s. Meanwhile the sea water kinematic viscosity according to the table in ITTC Procedure 7.5-02-01-03 Rev 00 for 15.0°C is equal to ν=1.1843 10^{-6} m^2/s. Using this method introduces a bias error due to the difference between ν(15.0)= 1.1873 10^{-6} m^2/s and ν=1.1843 10^{-6} m^2/s such that $B(\nu_2)$= -3.014 10^{-9} m^2/s.

Hence the total bias limit associated with sea water viscosity

$$B(\nu)=\sqrt{B(\nu_1)^2+B(\nu_2)^2} \qquad (2\text{-}26)$$

The bias limit associated with sea water viscosity due to temperature measurement and viscosity calculation method is thus $B(\nu)$ = 3.014 10^{-9} m^2/s corresponding to 0.254 % of the kinematic viscosity.

2.3.1.4 Model Speed

The carriage speed measurement system consists of individual measurement systems for pulse count (*c*), wheel diameter (*D*) and 12 bit DA and AD card time base (Δt). The speed is determined by tracking the rotations of one of the wheels with an optical encoder. The encoder is perforated around its circumference with 8000 equally spaced and sized windows. As the wheel rotates, the windows are counted with a pulse counter. The speed circuit has a 100 ms time base which enables update of the pulse every 10th of a second. A 12-bit DA conversion in the pulse count limits the maximum number of pulses in 100 ms to 4096. The output of the speed circuit is 0-10 V so that 4096 counted in 100 ms corresponds to 10 V output. The output from the encoder is calculated with the equation

$$V = \frac{c\pi D}{8000\Delta t} \qquad (2\text{-}27)$$

where c is the number of counted pulses in Δt=100 ms and D is the diameter of the carriage wheel (0.381 m).

Bias errors in model speed can be calculated as given in ITTC Procedure 7.5-02-02-02 Rev 01 'Uncertainty Analysis, Example for Resistance Test'.

Using nominal values of V=1.7033, c=1138, D=0.381, Δt=0.1, the total bias limit associated with the speed is $B(V)$=0.00357 m/s corresponding to 0.21% of the nominal speed of 1.7033 m/s.

The bias error associated with ship speed is assumed to be $B(V)$ =0.0 m/s as scaling to ship is not considered.

2.3.1.5 Propeller Rate of Revolution

The propeller rate of revolution measurement system consists of individual measurement systems for pulse count (c) and 12 bit AD conversion at a time base (Δt). The rate of rotation is sensed by tracking the rotation of propeller shaft with an optical encoder. The encoder has a pulse capacity of 600 due to equally spaced and sized windows at one revolution. As the wheel rotates, the windows are counted with the encoder. The speed circuit has a 100 ms time base which enables update of the signal every 10th of a second. A 12-bit DA conversion in the pulse count limits the maximum number of pulses in 100 ms to 4096. The output of the speed circuit is 0-10 V so that 4096 counted in 100 ms corresponds to 10 V output. The output from the encoder is calculated with the equation

$$n = \frac{c}{NoPulse * \Delta t} \qquad (2\text{-}28)$$

where c is the number of counted pulses in Δt=100 ms and $NoPulse$ is the number of pulses at one revolution (600).

<u>Pulse count</u>

Calibration:

The optical encoder is factory calibrated with a rated accuracy of ±1 pulse on every update. This value is a bias limit and represents the minimum resolution of the 12-bit AD data acquisition card. Therefore, the bias limit associated with the calibration error will be $B(C_1)$=1 pulse ($10V/2^{12}$=0.00244 V).

Data acquisition:

In the given data acquisition cycle, the rate of revolution pulse data is converted to dc signal and acquired by the PC through one 12-bit conversion. The resolution is $resol$=10 V/ 2^{12} = 0.00244V / bit. The conversion is accurate to 1.5 pulse and AD boards are accurate to 1.5 bits or pulses, which was determined by calibrating the boards against a precision voltage source. Therefore, the bias associated with the conversions is $B(C_2)$=$B(C_3)$= 1.5 pulses (0.00366 V).

The total bias limit for pulse count will then be

$$B(C) = \left(B(C_1)^2 + B(C_2)^2 + B(C_3)^2\right)^{\frac{1}{2}} = \left(1^2 + 1.5^2 + 1.5^2\right)^{\frac{1}{2}}$$
$$= 2.345\, pulse \qquad (2\text{-}29)$$

Time base (Δt)

The time base of the speed circuitry is related to the clock speed of its oscillator module.

Calibration:

The oscillator module is factory calibrated and its rated accuracy is 1.025 10^{-5} seconds on every update giving $B(\Delta t)$ = 1.025 10^{-5} seconds.

The data reduction equation is derived from Eq. (2-28) and can be written

$$B(n)=\left(\left(\frac{\partial n}{\partial c}B(c)\right)^2+\left(\frac{\partial n}{\partial \Delta t}B(\Delta t)\right)^2\right)^{\frac{1}{2}} \qquad (2\text{-}30)$$

Using the nominal values of c=500.4, and Δt=0.1 s for the mean speed of n=8.34 rps the partial derivatives can be calculated as

$$\frac{\partial n}{\partial c}=\frac{1}{600\Delta t}=0.01667 \qquad (2\text{-}31)$$

$$\frac{\partial n}{\partial \Delta t}=\frac{c}{600}\left(-\frac{1}{\Delta t^2}\right)=-83.4 \qquad (2\text{-}32)$$

The total bias limit can then be calculated from Eq. (2-30) as

$$B(n)=\left((0.01667\cdot 2.345)^2+(83.4\cdot 1.025\cdot 10^{-5})^2\right)^{\frac{1}{2}}$$
$$=0.0391 \qquad (2\text{-}33)$$

The total bias limit associated with the speed is $B(n)$=0.0391 rps corresponding to % 0.469 of the nominal rate of revolution of 8.34 rps.

2.3.1.6 Total Resistance Measurement and Total Resistance Coefficient

Bias Errors in model resistance measurement can be calculated as given in ITTC Procedure 7.5-02-02-02 Rev 01 'Uncertainty Analysis, Example for Resistance Test.'

The total bias limit associated with the resistance measurement is $B(Rx)$=0.1814 N corresponding to 0.434 % of the mean resistance of 41.791 N, and the precision limit for total resistance is $P(Rx(\mathrm{S}))$=0.3692 N corresponding to 0.883 % of the mean resistance of 41.791 N. Hence total uncertainty in total resistance is $U(Rx(\mathrm{S}))$=0.4113 N. This total uncertainty will in the propulsion test be used as a bias error. Hence for the propulsion test purposes $B(R_T)$ =0.4113 N corresponding to 0.984 % of the mean resistance of 41.791 N at 15 degrees water temperature.

Similarly, the resistance coefficient bias limit can be used as $B(C_\mathrm{T})$ =2.3296 10^{-5} corresponding to 0.615 % of the mean resistance coefficient of C_T=3.791 10^{-3} and the precision limit for total resistance coefficient is $P(C_\mathrm{T}(\mathrm{S}))$ =3.8289 10^{-5} corresponding to 1.01 % of the mean resistance coefficient of C_T =3.791 10^{3}. Hence the total uncertainty in total resistance coefficient is $U(C_\mathrm{T}(\mathrm{S}))$=4.4819 10^{-5}. This total uncertainty will in the propulsion test be used as a bias error. Hence for the propulsion test purposes $B(C_\mathrm{T})$ =4.4819 10^{-5} corresponding to 1.182 % of the mean resistance coefficient of C_T=3.791 10^{-3}

2.3.1.7 Frictional Resistance Coefficients

Frictional resistance coefficient is calculated according to the ITTC-1957 line as:

$$C_\mathrm{F}=\frac{0.075}{(Log\frac{VL}{\nu}-2)^2} \qquad (2\text{-}34)$$

Bias errors in skin friction calculation may be traced back to errors in model length, speed and viscosity.

Bias limit associated with C_F can be a found as

$$B(C_F)^2 = \left(\frac{\partial C_F}{\partial V} B(V)\right)^2 + \left(\frac{\partial C_F}{\partial L} B(L)\right)^2 + \left(\frac{\partial C_F}{\partial \upsilon} B(\nu)\right)^2 \quad (2\text{-}35)$$

Partial derivatives of Eq. (2-34) by model speed, model length and viscosity are

$$\frac{\partial C_F}{\partial V} = 0.075\left(-\frac{2}{(\log\frac{VL}{\nu}-2)^3}\right)\left(\frac{1}{V\ln 10}\right) \quad (2\text{-}36)$$

$$\frac{\partial C_F}{\partial L} = 0.075\left(-\frac{2}{(\log\frac{VL}{\nu}-2)^3}\right)\left(\frac{1}{L\ln 10}\right) \quad (2\text{-}37)$$

$$\frac{\partial C_F}{\partial \upsilon} = 0.075\left(-\frac{2}{(\log\frac{VL}{\nu}-2)^3}\right)\left(-\frac{1}{\nu\ln 10}\right) \quad (2\text{-}38)$$

By substituting $B(V)$ =0.00357 m/s, $B(L)$=0.002 mm, $B(\nu)$ =9.0405 10^{-9} m^2/s into Eq. (2-36), (2-37), (2-38), Bias limits associated with C_F in model scale (resistance test temperature, 15 degrees) is $B(C_F) = 4.2579\ 10^{-6}$ corresponding to 0.142 % nominal value of C_F= 2.990 $10^{-3.}$

By substituting $B(V)$=0.00357 m/s, $B(L)$=0.002 mm, $B(\nu)$ =8.7554 10^{-9} m^2/s into Eq. (2-36), (2-37), (2-38), Bias limits associated with C_F in model scale (propulsion test temperature) is $B(C_F) = 4.2216\ 10^{-6}$ corresponding to 0.142 % nominal value of C_F= 2.979 $10^{-3.}$

By substituting $B(V)$=0.0 m/s, $B(L)$=0.000 mm, $B(\nu)$=3.014 10^{-9} m^2/s into Eq. (2-36), (2-37), (2-38), Bias limits associated with C_F in ship scale is $B(C_F) = 4.6003\ 10^{-7}$ corresponding to 0.031 % nominal value of C_F= 1.479 $10^{-3.}$

2.3.1.8 Roughness Allowance

Roughness allowance is affected by surface roughness and ship length.

$$\Delta C_F = \left[105\left(\frac{k_s}{L_{WL}}\right)^{1/3} - 0.64\right]10^{-3} \quad (2\text{-}39)$$

According to the ITTC-1978 method the value of k_s can be assumed as 150 µm. Both k_s and ship length (L_{WL}) are generally accepted as true values resulting in $B(k_S) = 0$ and $B(L_{WL}) = 0$ and hence the bias limit associated with ship roughness is $B(\Delta C_F) = 0$

2.3.1.9 Form Factor

The recommended method for the experimental evaluation of the form-factor is that proposed by Prohaska. If the wave-resistance component in a low speed region (say 0.1 < Fr <0.2) is assumed to be a function of Fr^4, the straight-line plot of C_T/C_F versus Fr^4/C_F will intersect the ordinate (Fr =0) at (1+k), enabling the form factor to be determined.

Hence at low Froude numbers

$$(1+k) = \frac{C_T}{C_F} \quad (2\text{-}40)$$

In the case of a bulbous bow near the water surface these assumptions may not be valid and care should be taken in the interpretation of the results.

The bias limit $B(1+k)$ can be determined from the data reduction Eq. (2-40). The determination of the precision limit requires about 15 set of tests (5 sets with 3 repeat measurements for each speed) for several speeds. As there was no example data available, the uncertainty in form factor has for the time being and for indicative purposes been assumed to be 0.02, equal to 10% of k or 1.66% of $1+k$.

2.3.1.10 Residuary Resistance Coefficient

Bias Errors in residuary resistance coefficient can be calculated as given in ITTC Procedure 7.5-02-02-02 Rev 01 ‘Uncertainty Analysis, Example for Resistance Test.’

The residuary resistance coefficient bias limit is $B(C_R)$ =6.4377 10^{-5} corresponding to 31.719 % of the mean residuary resistance coefficient of C_R=2.0296 10^{-4} and the precision limit for residuary resistance coefficient is $P(C_R(S))$=3.832 10^{-5} corresponding to 18.89 % of the mean resistance coefficient of C_R=2.0296 10^{-4}. Hence total uncertainty in residuary resistance coefficient is $U(C_R(S))$=7.4921 10^{-5}. This total uncertainty will in the propulsion test be used as a bias error. Hence for the propulsion test purposes $B(C_R)$ =7.4921 10^{-5} corresponding to 36.91 % of the mean resistance coefficient of C_R=2.0296 10^{-4}.

2.3.1.11 Corrected Resistance

R_C is the corrected resistance coefficient at the temperature of the propulsion test i.e.

$$R_C = \frac{(1+k)C_{FC} + C_R}{(1+k)C_F + C_R} R_T \tag{2-41}$$

By defining function F

$$R_C = F R_T \tag{2-42}$$

$$F = \frac{(1+k)C_{FC} + C_R}{(1+k)C_F + C_R} \tag{2-43}$$

The bias error in F is written as

$$B(F)^2 = \left(\frac{\partial F}{\partial k} B(k)\right)^2 + \left(\frac{\partial F}{\partial C_F} B(C_F)\right)^2 + \left(\frac{\partial F}{\partial C_{FC}} B(C_{FC})\right)^2 + \left(\frac{\partial F}{\partial C_R} B(C_R)\right)^2 \tag{2-44}$$

$$\frac{\partial F}{\partial k} = \frac{C_{FC}\left[(1+k)C_F + C_R\right] - C_F\left[(1+k)C_{FC} + C_R\right]}{\left[(1+k)C_{FC} + C_R\right]^2} = 2.979\ 10^{-3} \tag{2-45}$$

$$\frac{\partial F}{\partial C_{FC}} = \frac{1+k}{(1+k)C_F + C_R} \tag{2-46}$$

$$\frac{\partial F}{\partial C_F} = \frac{\left[(1+k)C_{FC} + C_R\right](1+k)}{\left[(1+k)C_F + C_R\right]^2} \tag{2-47}$$

$$\frac{\partial F}{\partial C_R} = \frac{\left[(1+k)C_F + C_R\right] - \left[(1+k)C_{FC} + C_R\right]}{\left[(1+k)C_F + C_R\right]^2} \tag{2-48}$$

$$B(F) = \sqrt{\left(2.979\ 10^{-3} \cdot 0.02\right)^2 + \left(1.200 \cdot 4.222\ 10^{-6}\right)^2 + \left(1 \cdot 7.492\ 10^{-5}\right)^2} = 9.586\ 10^{-5} \tag{2-49}$$

The bias limit at corrected resistance Eq. (2-42) is calculated from

$$B(R_C)^2 = \left(\frac{\partial R_C}{\partial F} B(F)\right)^2 + \left(\frac{\partial R_C}{\partial R_T} B(R_T)\right)^2 \quad (2\text{-}50)$$

$$\frac{\partial R_C}{\partial F} = R_T = 41.791 \quad (2\text{-}51)$$

$$\frac{\partial R_C}{\partial R_T} = F = 0.9966 \quad (2\text{-}52)$$

$$B(R_C) = \sqrt{(41.791 \cdot 0.001896)^2 + (0.9966 \cdot 0.4099)^2} = 0.186 \quad (2\text{-}53)$$

The total bias limit associated with corrected resistance is $B(R_C)$=0.186 N corresponding to 0.447 % of nominal value of R_C=41.647 N.

2.3.1.12 External Tow Force

The horizontal external tow force is measured for the model when towed through the water. The bias errors associated with the measurement of external tow force are similar to the resistance measurement, hence the bias limit for the measurement is $B(F_{D1})$ =0.1814 N. Additionally, a bias error is added due to a curve fit of measurement forces.

Additional bias errors will arise due to bias errors for the determination of external tow force

$$F_D = \frac{1}{2}\rho S V^2 \left[(1+k)(C_{FC} - C_{FS}) - \Delta C_F\right] \quad (2\text{-}54)$$

$$B(F_D)^2 = \left(\frac{\partial F_D}{\partial \rho} B(\rho)\right)^2 + \left(\frac{\partial F_D}{\partial S} B(S)\right)^2 + \left(\frac{\partial F_D}{\partial V} B(V)\right)^2 + \left(\frac{\partial F_D}{\partial C_{FC}} B(C_{FC})\right)^2 + \left(\frac{\partial F_D}{\partial C_{F_S}} B(C_{FS})\right)^2 + \left(\frac{\partial F_D}{\partial \Delta C_F} B(\Delta C_F)\right)^2 \quad (2\text{-}55)$$

Partial derivatives can be calculated as

$$\frac{\partial F_D}{\partial \rho} = \frac{1}{2} S V^2 \left[(1+k)(C_{FC} - C_{FS}) - \Delta C_F\right] = 0.0159 \quad (2\text{-}56)$$

$$\frac{\partial F_D}{\partial S} = \frac{1}{2}\rho V^2 \left[(1+k)\cdot(C_{FC} - C_{FS}) - \Delta C_F\right] = 2.0927 \quad (2\text{-}57)$$

$$\frac{\partial F_D}{\partial V} = \rho S V \left[(1+k)\cdot(C_{FC} - C_{FS}) - \Delta C_F\right] = 18.6751 \quad (2\text{-}58)$$

$$\frac{\partial F_D}{\partial C_{FC}} = \frac{1}{2}\rho S V^2 (1+k) = 13229.3 \quad (2\text{-}59)$$

$$\frac{\partial F_D}{\partial C_{FS}} = -\frac{1}{2}\rho S V^2 (1+k) = -13229.3 \quad (2\text{-}60)$$

$$\frac{\partial F_D}{\partial \Delta C_F} = -\frac{1}{2}\rho S V^2 = -11024.2 \quad (2\text{-}61)$$

$$B(F_{D2}) = 0.0893 \quad (2\text{-}62)$$

$$B(F_D) = \sqrt{0.1814^2 + 0.0893^2} = 0.2022\,\text{N} \quad (2\text{-}63)$$

The total bias limit in external tow force is $B(F_D)$ =0.2022 corresponding to 1.605 % of nominal value of F_D=12.594 N.

2.3.1.13 Thrust Measurement

Thrust measurements are made with a propeller dynamometer.

Calibration:

The thrust transducer is calibrated with weights. The weights are the standard for the calibration and are a source of error, which depends on the quality of the standard. The weights have a certificate that certifies their calibration to a certain class. The tolerance for the individual weights used is certified to be ± 0.005%. The bias error arising from the tolerance of the calibration weights, $B(T_{x1})$, is calculated as the accuracy of the weights, times the resistance measured according to Eq. (2-64).

$$B(T_{x1}) = 0.00005 \cdot 35.657 = 0.0018 \text{ N} \quad (2\text{-}64)$$

Data acquisition:

The data from the calibration tabulated below shows the mass/volt relation. It is proposed by Coleman and Steele (1999) that a ±2(*SEE*) band about the regression curve will contain approximately 95% of the data points and this band is a confidence interval on the curve fit.

$$SEE = \sqrt{\frac{\sum_{i=1}^{N}\left(Y_i - (aX_i + b)\right)^2}{N-2}} \quad (2\text{-}65)$$

From these values the *SEE* can be calculated with Eq. (2-65) to *SEE*=0.0941 resulting in a bias for the curve fit to be $B(T_{x2})$=0.1883 N.

Table 2.5 Thrust transducer calibration.

Output (Volt)	Mass (kg)	Force (N)
0.001	0	0
0.81	1	9.81
1.621	2	19.62
2.421	3	29.43
3.21	4	39.24
4.021	5	49.05
4.811	6	58.86

T=-0.1118+ 12.24·Volt

The third error is due to AD conversion of the thrust values. ADC has 1bit accuracy over 0-10 volts range and 12 bit conversion accuracy, resulting in 0.00245 volts error. This also corresponds to $B(T_{x3})$=0.0299 N with the calibration explained above.

The total bias limit in thrust is obtained by the root sum square of the three bias components considered

$B(T_x) = \sqrt{0.0018^2 + 0.1883^2 + 0.0299^2}$ =0.1906 N corresponding to 0.535 % of the mean thrust of 35.48 N.

2.3.1.14 Torque Measurement

The torque dynamometer is calibrated with weights applied at a nominal distance from the shaft line. The weights have the same tolerance as for the thrust calibration i.e. ± 0.005%. The moment arm is 0.1 m with accuracy of 0.2 mm.

Calibration:

As moment is produced by a reduction equation

$$Q_w = F\ ma \quad (2\text{-}66)$$

$$B(Q_w)^2 = \left(\frac{\partial Q_w}{\partial F} B(F)\right)^2 + \left(\frac{\partial Q_w}{\partial ma} B(ma)\right)^2 \quad (2\text{-}67)$$

$$B(Q_w)^2 = \left(ma \cdot B(F)\right)^2 + \left(F \cdot B(ma)\right)^2 \quad (2\text{-}68)$$

If F=1.235/0.1=12.35N, $B(F)$ = 0.00005·12.35 = 0.0006N, ma=0.1m and $B(ma)$=0.0002m the data reduction can then be written as:

$$B(Q_1) = B(Q_W) = \sqrt{(0.1 \cdot 0.0006)^2 + (12.35 \cdot 0.0002)^2} = 0.0025 \quad (2\text{-}69)$$

Data acquisition:

The data from the calibration tabulated below shows the mass/volt relation. From these values the *SEE* can be calculated with Eq. (2-65) to *SEE*=0.0014 resulting in a bias for the curve fit to be $B(Q_2)$=0.0028 Nm.

Table 2.6 Torque transducer calibration.

Output (Volt)	Mass (kg)	Moment (Nn)
0.003	0	0
-1.01	1	0.981
-2.025	2	1.962
-3.04	3	2.943
-4.053	4	3.924
-5.07	5	4.905
-6.08	6	5.886

Q=0.0031-0.96728·Volt

The third error is due to AD conversion of the thrust values. ADC has 1bit accuracy over 0-10 volts range and 12 bit conversion accuracy, resulting in 0.00245 volts error. This also corresponds to $B(Q_3)$=0.0024 Nm with the calibration explained above.

The total bias limit in torque measurement is obtained by the root sum square of the three bias components considered:

$$B(Q)=\sqrt{0.0025^2+0.0028^2+\ 0.0024^2}\ =0.0044$$

Nm corresponding to 0.358 % of the mean thrust of 1.2348 Nm.

2.3.1.15 Thrust Coefficient

Thrust coefficient is utilised in the wake fraction and relative rotative efficiency bias limits, and is expressed as

$$K_T=\frac{T}{\rho n^2 D^4} \qquad (2\text{-}70)$$

Bias Limit can be expressed as:

$$B(K_T)^2=\left(\frac{\partial K_T}{\partial T}B(T)\right)^2+\left(\frac{\partial K_T}{\partial \rho}B(\rho)\right)^2+ \\ +\left(\frac{\partial K_T}{\partial n}B(n)\right)^2+\left(\frac{\partial K_T}{\partial D}B(D)\right)^2 \qquad (2\text{-}71)$$

Partial derivatives

$$\frac{\partial K_T}{\partial T}=\frac{1}{\rho n^2 D^4}=\frac{1}{1000\cdot 8.34\cdot 0.2275^4}=0.005367 \qquad (2\text{-}72)$$

$$\frac{\partial K_T}{\partial \rho}=\frac{T}{n^2 D^4}\left(-\frac{1}{\rho^2}\right)= \\ \frac{35.48}{8.34^2\cdot 0.2275^4}\left(-\frac{1}{1000^2}\right)=-0.00019 \qquad (2\text{-}73)$$

$$\frac{\partial K_T}{\partial n}=\frac{T}{\rho D^4}\left(-\frac{2}{n^3}\right)= \\ \frac{35.48}{1000\cdot 0.2275^4}\left(-\frac{2}{8.34^3}\right)=-0.04583 \qquad (2\text{-}74)$$

$$\frac{\partial K_T}{\partial D}=\frac{T}{\rho n^2}\left(-\frac{4}{D^5}\right)= \\ \frac{35.48}{1000\cdot 8.34^2}\left(-\frac{4}{0.2275^2}\right)=-3.3607 \qquad (2\text{-}75)$$

Using Eq. (2-71) and $B(T)$=0.1906 N, $B(\rho)$=0.7387 kg/m^3, $B(n)$=0.0391 rps, $B(D)$=0.0001 m

$$B(K_T)=\sqrt{\begin{aligned}&(0.005367\cdot 0.1906)^2+(-0.00019\cdot 0.7387)^2\\&+((-0.04583)\cdot 0.0391)^2+((-3.3607)\cdot 0.0001)^2\end{aligned}} \\ =0.0021 \qquad (2\text{-}76)$$

The total bias limit in thrust coefficient is $B(K_T)$ =0.0021 corresponding to 1.096 % of the mean thrust coefficient of 0.1912.

2.3.1.16 Torque Coefficient

Torque coefficient is utilised in the relative rotative efficiency bias limit, and is expressed as

$$K_Q=\frac{Q}{\rho n^2 D^5} \qquad (2\text{-}77)$$

Bias Limit can be expressed as:

$$B(K_Q)^2 = \left(\frac{\partial K_Q}{\partial Q}B(Q)\right)^2 + \left(\frac{\partial K_Q}{\partial \rho}B(\rho)\right)^2 + \left(\frac{\partial K_Q}{\partial n}B(n)\right)^2 + \left(\frac{\partial K_Q}{\partial D}B(D)\right)^2 \quad (2\text{-}78)$$

By calculating

$$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5} = \frac{1}{1000 \cdot 8.34^2 \cdot 0.2275^5} = 0.02356 \quad (2\text{-}79)$$

$$\frac{\partial K_Q}{\partial \rho} = \frac{Q}{n^2 D^5}\left(-\frac{1}{\rho^2}\right) = \frac{1.2348}{8.34^2 \cdot 0.2275^5}\left(-\frac{1}{1000^2}\right) = -0.00002911 \quad (2\text{-}80)$$

$$\frac{\partial K_Q}{\partial n} = \frac{Q}{\rho D^5}\left(-\frac{2}{n^3}\right) = \frac{1.2348}{1000 \cdot 0.2275^5}\left(-\frac{2}{8.34^3}\right) = -0.006986 \quad (2\text{-}81)$$

$$\frac{\partial K_Q}{\partial D} = \frac{Q}{\rho n^2}\left(-\frac{5}{D^6}\right) = \frac{1.2348}{1000 \cdot 8.34^2}\left(-\frac{5}{0.2275^6}\right) = -0.6399 \quad (2\text{-}82)$$

Using Eq. (2-78) and $B(Q)$=0.0044 N, $B(\rho)$=0.7387 kg/m^3, $B(n)$=0.0391 rps, $B(D)$=0.0001 m

$$B(K_Q) = \sqrt{\begin{array}{l}(0.02356 \cdot 0.0044)^2 + (-0.00002911 \cdot 0.7387)^2 \\ +(-0.00698 \cdot 0.0391)^2 + (-0.6399 \cdot 0.0001)^2\end{array}} = 0.00029985 \quad (2\text{-}83)$$

The total bias limit in torque coefficient is $B(K_Q)$=0.00030 corresponding to 1.030 % of the mean torque coefficient of 0.02912.

2.3.1.17 Open Water Advance Coefficient

The propeller advance coefficient is interpolated for thrust coefficient input in the open water propeller diagram. The advance coefficient includes errors due to input thrust coefficient, open water test J-K_T relation and interpolation errors.

The propeller open water characteristics were investigated at three points for the current example and the results in Table 2.7 were obtained. Curve fits in the range of J were defined as

$$K_T = 0.4924 - 0.524J + 0.04J^2 \quad (2\text{-}84)$$

$$K_Q = 0.06407 - 0.0553J - 0.002J^2 \quad (2\text{-}85)$$

Table 2.7 Open water characteristics of propeller.

J	K_T	K_Q
0.55	0.216287	0.033053
0.60	0.192411	0.030169
0.65	0.168742	0.027279

Conceptual:

J_T has bias error due to the open water propeller test uncertainty which is calculated following the ITTC Procedure 7.5-02-03-02.2 Rev 00 ‘Example for Open Water Test.’ $B(J_{T1}) = 0.002032$

Data acquisition:

Interpolation errors will be related to the fit of the regression curve over the J-K_T values in the open water test. If the value of K_T is expressed as Eq. (2-84) then for a small change of propeller operation, the open water propeller performance is:

$$J_T = aK_T + b = 1.0048 - 2.1032K_T \quad (2\text{-}86)$$

The value of K_T is determined from the propulsion test and J_T is derived from the regression equation. In the current example three points are defined for three J values in the open water propeller curve, and maximum error related to J_T can be defined as the difference between a linear regression line through these three points and a polynomial through these three points. For the current example, propeller J_T =1.0048–2.1032 K_T is found. The regression error can be determined from Eq. (2-65). $B(J_{T2}) = 0.000356$.

The thrust coefficient error is found to be 0.0021 in section 2.3.1.15, and the corresponding error in J_T can be found from regression equation as

$$B(J_{T3}) = \left(1.0048 - 2.1032 \cdot \left(K_T + B(K_T)\right)\right) - \left(1.0048 - 2.1032 \cdot K_T\right) \quad (2\text{-}87)$$

$$B(J_{T3}) = (1.0048\text{-}2.1032 \cdot (0.1912 + 0.0021)) - (1.0048\text{-}2.1032\ (0.1912)) = 0.0044$$

The total bias error associated with the advance coefficient will be the summation of these two errors

$$B(J_T) = (0.002032^2 + 0.000356^2 + 0.0044^2)^{0.5}$$

$$= 0.00485$$

corresponding to 0.808 % of advance coefficient 0.6028.

2.3.1.18 Open Water Torque Coefficient

Propeller open water torque coefficient is interpolated for J_T in the open water propeller diagram. Open water torque coefficient bias error will include errors due to input J_T coefficient, open water test J-K_Q relation and interpolation errors.

K_{QT} has bias error due to open water propeller test uncertainty, which is calculated following ITTC Procedure 7.5-02-03-02.2 Rev 00 'Uncertainty Analysis, Example for Open Water Tests.'

$$B(K_{Q1}) = 0.0002628$$

The open water diagram with a small change of J can be expressed as

$$K_{QT} = aJ_T + b \quad (2\text{-}88)$$

The value of J_T is determined and K_{QT} is derived from the regression equation. For the current example propeller K_{QT}=0.06481-0.05774 J_T is found. The regression error (2SEE) can be determined from Eq. (2-65).

$$B(K_{QT2}) = 5.217\ 10^{-6}$$

K_{QT} has also errors due to input J_T for the interpolation

$$B(K_{QT3}) = (0.06481\text{-}0.05774(0.06028 + 0.00905)) - (0.06481\text{-}0.05774(0.6028\text{-}0.00905)) = -0.00105 \quad (2\text{-}89)$$

The total bias error associated with Advance coefficient will be the summation of these three errors $B(K_{QT})$ = (0.0002628^2 + 0.00005217^2 + 0.00028^2)$^{0.5}$ = 0.000384 corresponding to 1.28 % of torque coefficient 0.03001.

2.3.1.19 Total Bias Limit in Wake Fraction

The total bias limit can be calculated according to Eq. (2-3) as

$$B(w_T)^2 = \left(\frac{\partial w_T}{\partial J_T} B(J_T)\right)^2 + \left(\frac{\partial w_T}{\partial D} B(D)\right)^2 + \left(\frac{\partial w_T}{\partial n} B(n)\right)^2 + \left(\frac{\partial w_T}{\partial V} B(V)\right)^2 \quad (2\text{-}90)$$

partial derivative of Eq.(2-3)

$$\frac{\partial w_T}{\partial J_T} = -\frac{Dn}{V} = -\frac{0.2275 \cdot 8.34}{1.703} = -1.1144 \quad (2\text{-}91)$$

$$\frac{\partial w_T}{\partial D} = -\frac{J_T n}{V} = -\frac{0.6027 \cdot 8.34}{1.7033} = -2.952 \quad (2\text{-}92)$$

$$\frac{\partial w_T}{\partial n} = -\frac{J_T D}{V} = -\frac{0.6027 \cdot 0.2275}{1.7033} = = -0.0805 \quad (2\text{-}93)$$

$$\frac{\partial w_T}{\partial V} = -J_T nD\left(-\frac{1}{V^2}\right) = 0.6027 \cdot 8.34 \cdot 0.2275\left(\frac{1}{1.703^2}\right) = 0.3943 \quad (2\text{-}94)$$

By substituting these in to Eq.(2-90)

$$B(w_T) = 0.0064$$

The total bias error associated with wake fraction is 0.0064 corresponding to 1.961 % of wake fraction w_T=0.328.

2.3.1.20 Total Bias Limit in Thrust Deduction

The total bias limit can be calculated according to Eq. (2-5) as

$$B(t)^2 = \left(\frac{\partial t}{\partial T} B(T)\right)^2 + \left(\frac{\partial t}{\partial F_D} B(F_D)\right)^2 + \left(\frac{\partial t}{\partial R_C} B(R_C)\right)^2 \quad (2\text{-}95)$$

Partial derivative of Eq.(2-5)

$$\frac{\partial t}{\partial T} = (F_D - R_C)\left(-\frac{1}{T^2}\right) = (12.7 - 41.781)\left(-\frac{1}{35.48^2}\right) = 0.02285 \quad (2\text{-}96)$$

$$\frac{\partial t}{\partial F_D} = \frac{1}{T} = \frac{1}{35.48} = 0.0280 \quad (2\text{-}97)$$

$$\frac{\partial t}{\partial R_C} = -\frac{1}{T} = -\frac{1}{35.48} = 0.0280 \quad (2\text{-}98)$$

By substituting these into Eq.(2-95)

$$B(t) = 0.0089$$

The total bias error associated with thrust deduction is 0.0089 corresponding to 4.781 % of thrust deduction t=0.1852.

2.3.1.21 Total Bias Limit in Relative Rotative Efficiency

The total bias limit can be calculated according to Eq. (2-4) as

$$B(\eta_R)^2 = \left(\frac{\partial \eta_R}{\partial K_{QT}} B(K_{QT})\right)^2 + \left(\frac{\partial \eta_R}{\partial K_Q} B(K_Q)\right)^2 \quad (2\text{-}99)$$

Partial derivatives of Eq.(2-4) by K_Q and K_{QT}

$$\frac{\partial \eta_R}{\partial K_{QT}} = \frac{1}{K_Q} = \frac{1}{0.02912} = 34.342 \quad (2\text{-}100)$$

$$\frac{\partial \eta_R}{\partial K_Q} = K_{QT}\left(\frac{1}{K_Q{}^2}\right) = 0.02912\frac{1}{0.03001^2} = -35.39 \quad (2\text{-}101)$$

By substituting these into Eq.(2-99)

$$B(\eta_R) = 0.0385$$

The total bias error associated with relative rotative efficiency is $B(\eta_R)$=0.0169 corresponding to 1.643 % of relative rotative efficiency η_R =1.028.

2.3.2 Precision Limit

In order to establish the precision limits, the standard deviation for a number of tests, with the model removed and reinstalled between each set of measurements, must be determined. In this example, 5 sets of testing (A-E) with 3 speed measurements in each set have been performed giving in total 15 test points. This is a suitable way to include random errors in the set-up such as model misalignment, trim, heel etc.

When performing a propulsion test at the balance point of nominal loading (F_D), small deviations will occur. When a run is performed, the thrust, torque and rate of revolutions are normally corrected to the nominal F_D under the assumption that the thrust deduction (t), wake (w_T) and relative rotative efficiency (η_R) are constant. When the measurements are repeated it is likely that the measured quantities will be taken at slightly different speeds for the different runs, and with different water temperatures between the different set of tests.

In this case no corrections have to be made for these deviations. The values used for the extrapolation of model test results are the thrust deduction; wake fraction; and relative rotative efficiency. These values are assumed to be constant for small deviations in loading (as described above) and for small deviations in speed and should therefore not be corrected. A correction for difference in temperature between the different sets of tests is also not carried out. Such a correction would incorrectly change the uncertainty, as the temperature of the propulsion test is not considered in the extrapolation to full scale when using the ITTC-78 method.

In Table 2.8 the thrust deduction (t), wake (w_T) and relative rotative efficiency (η_R) given have been calculated based on measured quantities. The values below are valid for a model speed of 1.7033 m/s.

In Table 2.8 the mean values, standard deviation (*SDev*), precision limits for single runs Eq. (2-14), and precision limits for multiple runs Eq. (2-13) have been calculated. The corresponding percentage values have also been given within brackets.

The mean value over 15 runs for thrust deduction is calculated as $\bar{t} = 0.185$ as shown in Table 2.8. The precision limit for the mean value of 15 runs is calculated as

$$P(t) = \frac{K\ SDev(t)}{\sqrt{M}} = \frac{2 \cdot 0.0064}{\sqrt{15}} = 0.0033 \qquad (2\text{-}102)$$

corresponding to 1.80 % of t. For a single run the precision limit is calculated as

$$P(t) = K\ SDev(t) = 2 \cdot 0.0064 = 0.0129 \qquad (2\text{-}103)$$

corresponding to 6.96 % of t.

The mean value over 15 runs for thrust deduction is calculated as $\overline{w_T} = 0.327$ as shown in Table 2.6. The precision limit for the mean value of 15 runs is calculated as

$$P(w_T) = \frac{K\ SDev(w_T)}{\sqrt{M}} = \frac{2 \cdot 0.0021}{\sqrt{15}} = 0.0011 \qquad (2\text{-}104)$$

corresponding to 0.33 % of w_T. For a single run the precision limit is calculated as

$$P(w_T) = K\ SDev(w_T) = 2 \cdot 0.0021 = 0.0042 \quad (2\text{-}105)$$

corresponding to 1.27 % of w_T

The mean value over 15 runs for thrust deduction is calculated as $\overline{\eta_R} = 1.028$ as shown in Table 2.6. The precision limit for the mean value of 15 runs is calculated as

$$P(\eta_R) = \frac{K\ SDev(\eta_R)}{\sqrt{M}} = \frac{2 \cdot 0.0051}{\sqrt{15}} = 0.00261 \quad (2\text{-}106)$$

corresponding to 0.25 % of η_R.

For a single run the precision limit is calculated as

$$P(\eta_R) = K\ SDev(\eta_R) = 2 \cdot 0.0051 = 0.0101 \quad (2\text{-}107)$$

corresponding to 0.99 % of η_R

Table 2.8 Mean values, standard deviation and precision limits of thrust deduction (t), wake (w_T) and relative rotative efficiency (η_R).

Series /run	**Propulsive factors based on measured values**		
	t	w_{TM}	η_R
A1	0.179	0.327	1.027
A2	0.179	0.329	1.026
A3	0.183	0.330	1.027
B1	0.183	0.326	1.033
B2	0.181	0.326	1.032
B3	0.182	0.327	1.032
C1	0.192	0.325	1.033
C2	0.181	0.327	1.034
C3	0.184	0.329	1.035
D1	0.196	0.323	1.022
D2	0.200	0.329	1.017
D3	0.181	0.329	1.023
E1	0.183	0.324	1.029
E2	0.182	0.326	1.026
E3	0.191	0.325	1.027
MEAN	0.185	0.327	1.028
SDev	0.0064 (3.48%)	0.0021 (0.64%)	0.0051 (0.49%)
P (S)	0.0129 (6.96%)	0.0042 (1.27%)	0.0101 (0.99%)
P (M)	0.0033 (1.80%)	0.0011 (0.33%)	0.0026 (0.25%)

2.3.3 Total Uncertainties

By combining the precision limits for multiple and single tests with the bias limits, the total uncertainty can be calculated according to Eq. (2-9)

Total uncertainty of thrust deduction (t) for the mean value of 15 runs will then be

$$U(t)=\left(B(t)^2+P(t)^2\right)^{\frac{1}{2}}$$
$$=\left(0.00886^2+0.0033^2\right)^{\frac{1}{2}}=0.00946 \quad (2\text{-}108)$$

which is corresponding to 5.1081 % of *t*.

Correspondingly the total uncertainty for a single run can be calculated as

$$U(t)=\left(B(t)^2+P(t)^2\right)^{\frac{1}{2}}$$
$$=\left(0.00886^2+0.01289^2\right)^{\frac{1}{2}}=0.01564 \quad (2\text{-}109)$$

which is 8.4449 % of *t*.

Total uncertainty of wake fraction (w_T) for the mean value of 15 runs will then be

$$U(w_T)=\left(B(w_T)^2+P(w_T)^2\right)^{\frac{1}{2}}$$
$$=\left(0.00644^2+0.00107^2\right)^{\frac{1}{2}}=0.00652 \quad (2\text{-}110)$$

which is corresponding to 1.9876 % of w_T.

Correspondingly, the total uncertainty for a single run can be calculated as

$$U(w_T)=\left(B(w_T)^2+P(w_T)^2\right)^{\frac{1}{2}}$$
$$=\left(0.00644^2+0.00415^2\right)^{\frac{1}{2}}=0.00766 \quad (2\text{-}111)$$

which is 2.3335 % of w_T.

Total uncertainty of relative rotative efficiency (η_R) for the mean value of 15 runs will then be

$$U(\eta_R)=\left(B(\eta_R)^2+P(\eta_R)^2\right)^{\frac{1}{2}}$$
$$=\left(0.01693^2+0.00262^2\right)^{\frac{1}{2}}=0.01713 \quad (2\text{-}112)$$

which is corresponding to 1.6626 % of η_R.

Correspondingly the total uncertainty for a single run can be calculated as

$$U(\eta_R)=\left(B(\eta_R)^2+P(\eta_R)^2\right)^{\frac{1}{2}}$$
$$=\left(0.01693^2+0.01015^2\right)^{\frac{1}{2}}=0.01974 \quad (2\text{-}113)$$

which is 1.9155 % of η_R.

As can be seen from the values above, the uncertainty will decrease if it is calculated for the mean value of 15 tests compared with the single run value.

Table 2.9 Uncertainties in propulsion tests.

Total uncertainties		**Percentage values**
$B(t)(M)$	0.008856	87.62 % of $U(t)(M)$
$P(t)(M)$	0.003329	12.38 % of $U(t)(M)$
$U(t)(M)$	0.009461	5.11 % of t
$B(t)(S)$	0.008856	32.06 % of $U(t)(S)$
$P(t)(S)$	0.012892	67.94 % of $U(t)(S)$
$U(t)(S)$	0.01564	8.44 % of t
$B(w_T)(M)$	0.006436	97.30 % of $U(w_T)(M)$
$P(w_T)(M)$	0.00107	2.70 % of $U(w_T)(M)$
$U(w_T)(M)$	0.006525	1.99 % of w_T
$B(w_T)(S)$	0.006436	70.59 % of $U(w_T)(S)$
$P(w_T)(S)$	0.004154	29.41 % of $U(w_T)(S)$
$U(w_T)(S)$	0.00766	2.33 % of w_T
$B(\eta_R)(M)$	0.016932	97.66 % of $U(\eta_R)(M)$
$P(\eta_R)(M)$	0.00262	2.34 % of $U(\eta_R)(M)$
$U(\eta_R)(M)$	0.017134	1.66 % of η_R
$B(\eta_R)(S)$	0.016932	73.57 % of $U(\eta_R)(S)$
$P(\eta_R)(S)$	0.010147	26.43 % of $U(\eta_R)(S)$
$U(\eta_R)(S)$	0.01974	1.92 % of η_R

Expressed in relative numbers the bias represents 87.6 % and 32.1 % of the total uncertainty for multiple tests and single run respectively for thrust deduction. For wake fraction the bias represents 97.3 % and 70.6 % of the total uncertainty for multiple tests and a single run respectively. Meanwhile, the bias represents 97.7 % and 73.6 % of the total uncertainty for multiple tests and a single run respectively for relative rotative efficiency. A significant part of the bias limits are originating from embedding the uncertainties of resistance test and open water test into the propulsion test bias limits.

By comparing the bias and precision limits and the uncertainties, the relative contribution of each term can be calculated. This makes it possible to determine where an upgrade in the measurement system has the largest effect. Compare ITTC Procedure 7.5-02-02-02 Rev. 01, 'Uncertainty Analysis, Example for Resistance Test.'

The bias and precision limits and the total uncertainties are summarised in Table 2.9

2.4 REFERENCES

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