ITTC, 2014

Copenhagen, Denmark

Modelling Nonlinear Seas

Challenges and achievements

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Background: wave generation

Increasingly complex requirements :

Regular (Stokes) waves

Long-crested random waves

Short-crested random waves

Fully incorporating:
- ***unsteadiness***
- ***nonlinearity***
- ***directionality***

For some applications

Nonlinear transient wave groups

(representing the most-probable shape of the largest waves

- in a fully nonlinear sense)

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Background: industrial trends

- Increasing adoption of reliability criteria
- Based upon crest height / wave load statistics
- Often requiring 10^{-4} exceedence probabilities

Implies long random simulations

- Concern over abnormally large (*Freak / Rogue*) waves
- Occur in the tail of the distribution
- Is there some new physics governing these events?

Implies even longer random simulations

- Incorporation of wave breaking
  - Kinematics
  - Loads (possible step change: slamming)

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Background: Scientific understanding

Nonlinear wave-wave interactions:

- Free waves / bound waves, $O(a^2)$, well established
- Resonant interactions, $O(a^3)$, allow spectral change
  - Slow modulation (wave growth)
  - Rapid evolution (characteristic of extreme events?)

Consequences:

- Spectral shape may vary (rapidly) in space & time
- Design spectra:
  - May apply on average
  - But not necessarily local to an extreme wave event
- Spectral broadening → larger maximum crests
- Nonlinear evolution → change in directional spread
- Directionality is key to wave breaking

  → breaking is fundamental to design

Wave Basin at Imperial College London

Calibration of test facilities

(a) Basin calibration:

- Iterative approach to achieve:
  - Desired frequency spectrum
  - Desired directional spread
- What happens to the spectral evolution?
- Have important nonlinear effects been calibrated out?

(b) Paddle calibration

- Generation of underlying linear wave components
- Sea state will evolve as required
- Ideally based upon a theoretical transfer function
- Effective absorption essential (beach and paddles)

Methodology adopted at Imperial College

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Methods of sea state generation

(a) *Double summation method (DSM)*

- All frequencies in all directions
- Sum over both frequency and direction
- Non-ergodic

(b) *Single summation method (SSM)*

- Any one frequency component generated in one direction
- Spectrum sub-divided into narrow (but finite) bands
- Within each band:
  - Sequential components generated in sequential directions
  - OK, but requires high resolution (calibration more difficult)

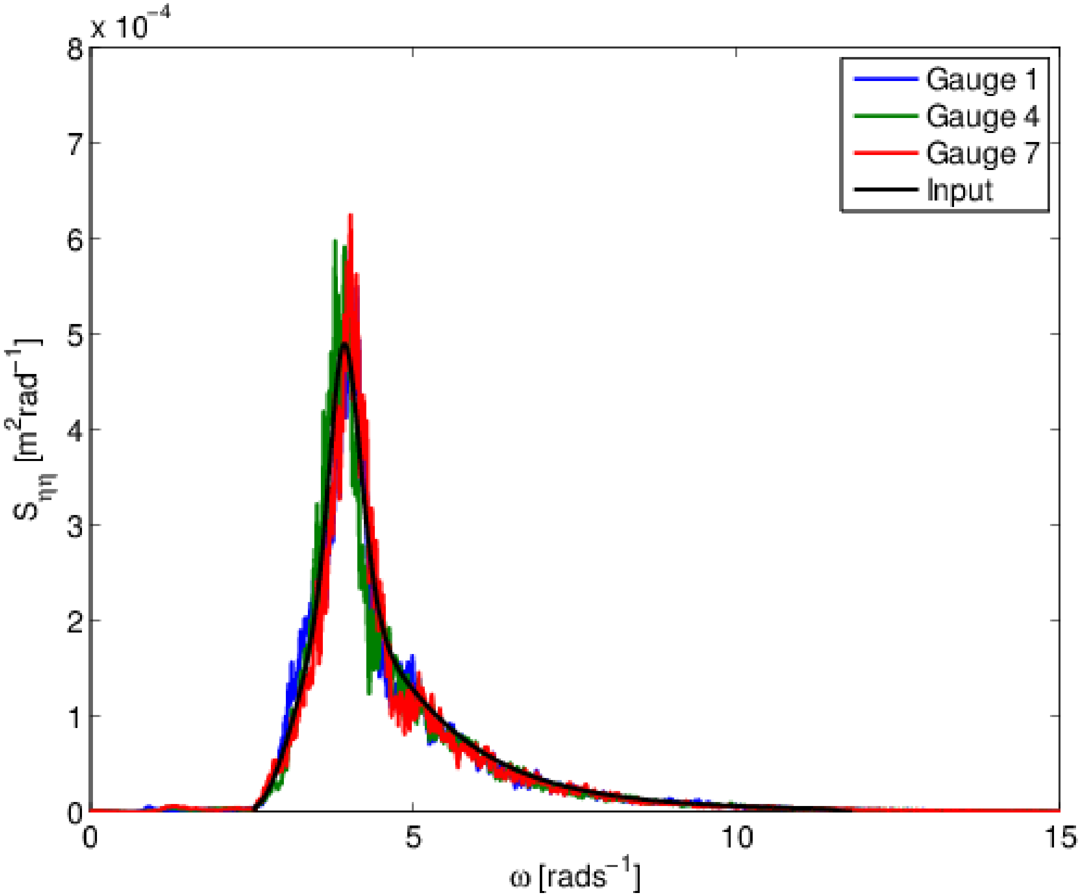
(c) *Random directional method (RDM)*

- Any one component in one direction
- Direction of propagation chosen randomly
- Based on normal distribution weighted by DSF
- Easy to incorporate random amplitudes

Generated data: frequency spectra

JONSWAP

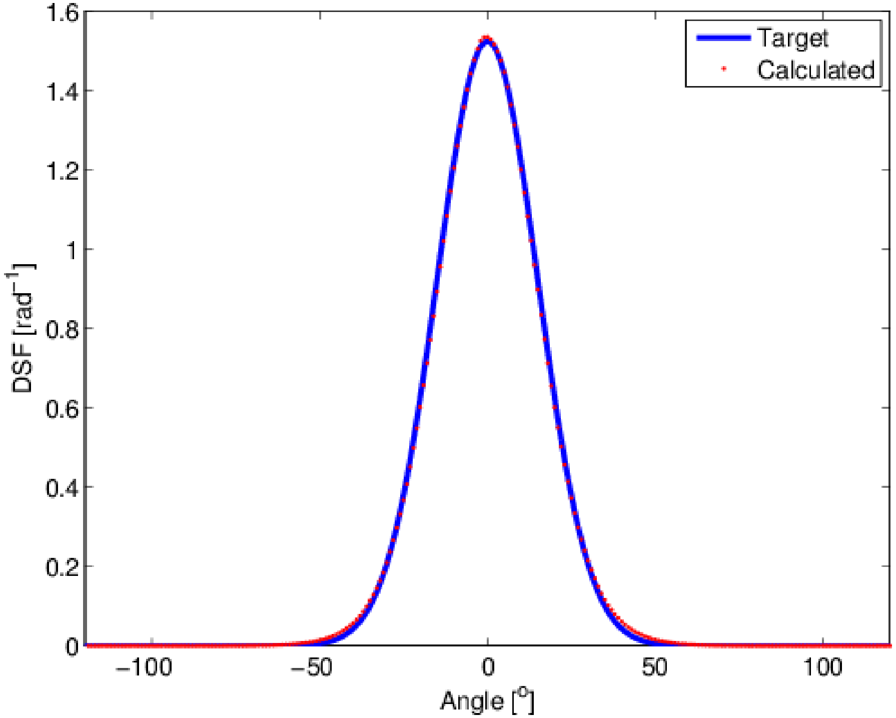
- match to target & spatial uniformity



***H_s*=10m, *T_p*=16s, *σ_θ*=15°**

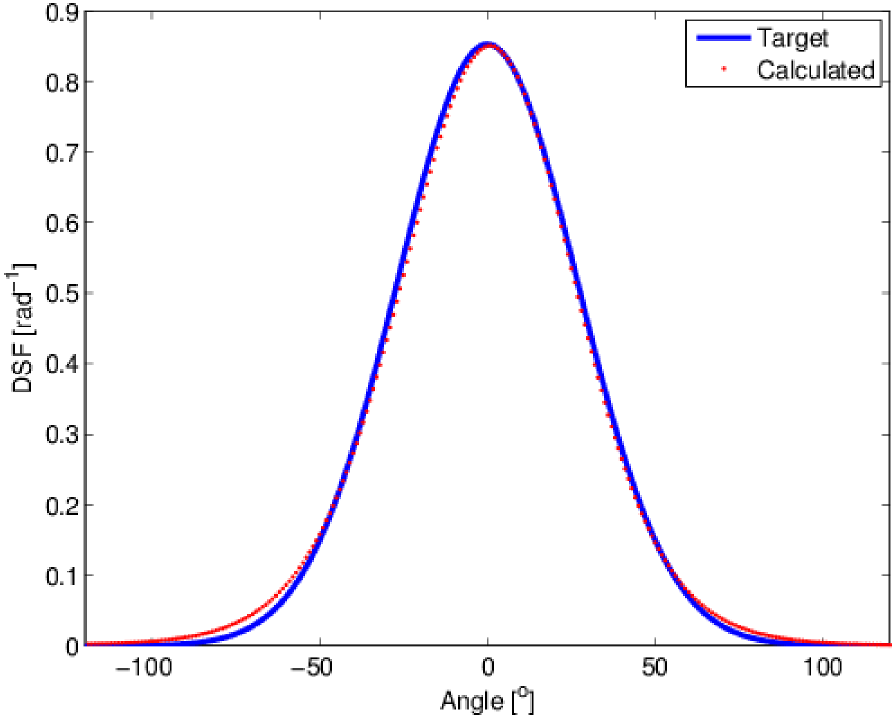
Generated data: directional spreading

- match to target
- linear sea state



σ_θ=15°

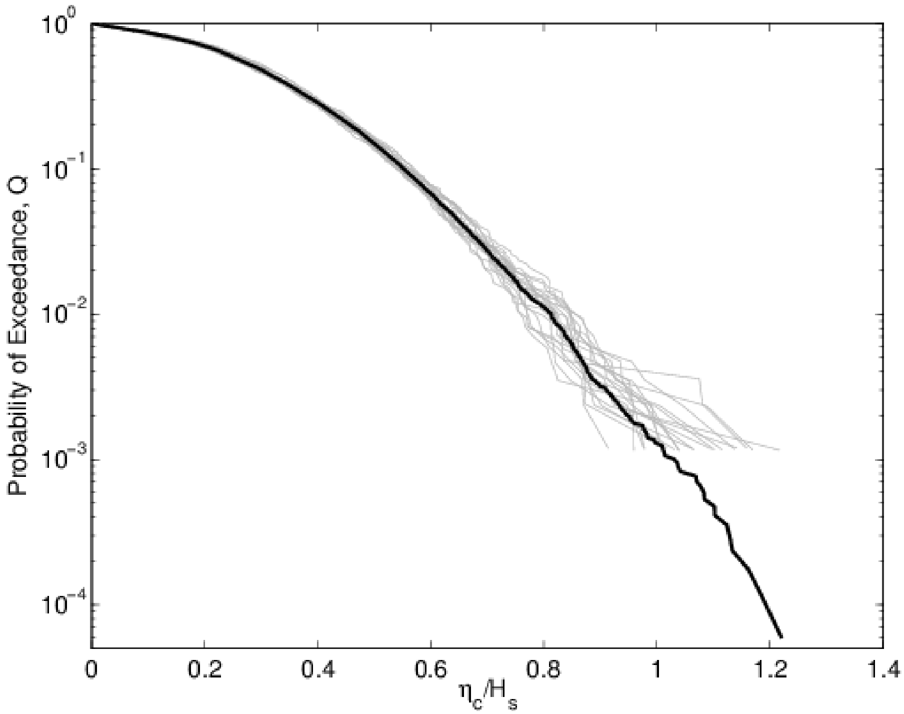
σ_θ=30°



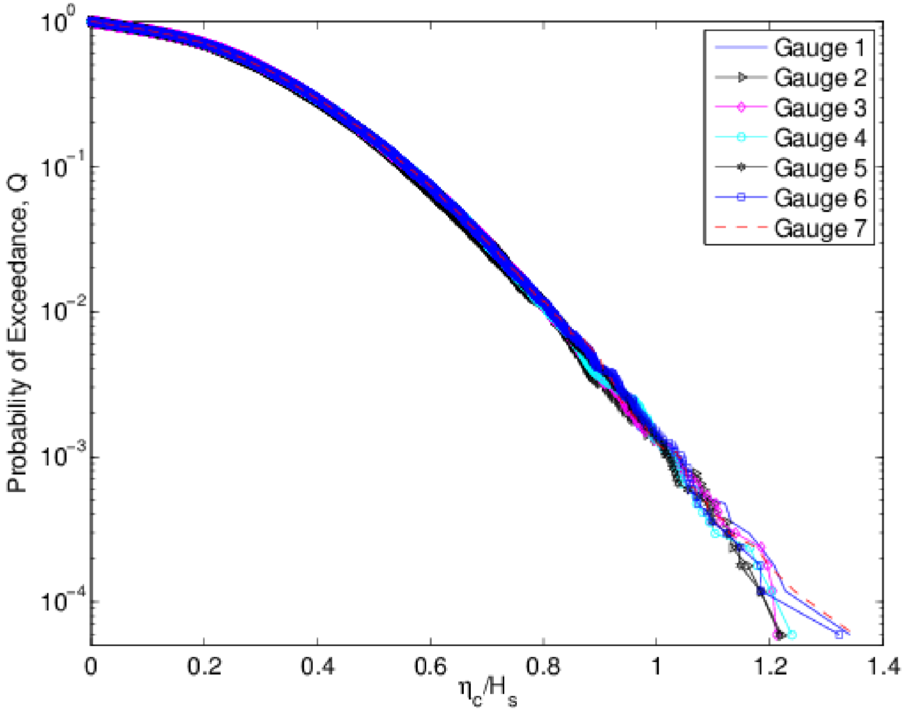
Crest height distributions

Individual wave components:

- random phases (0 → 2π)
- random amplitudes (Rayleigh distributed)
- direction of propagation, random with weighting based upon DSF



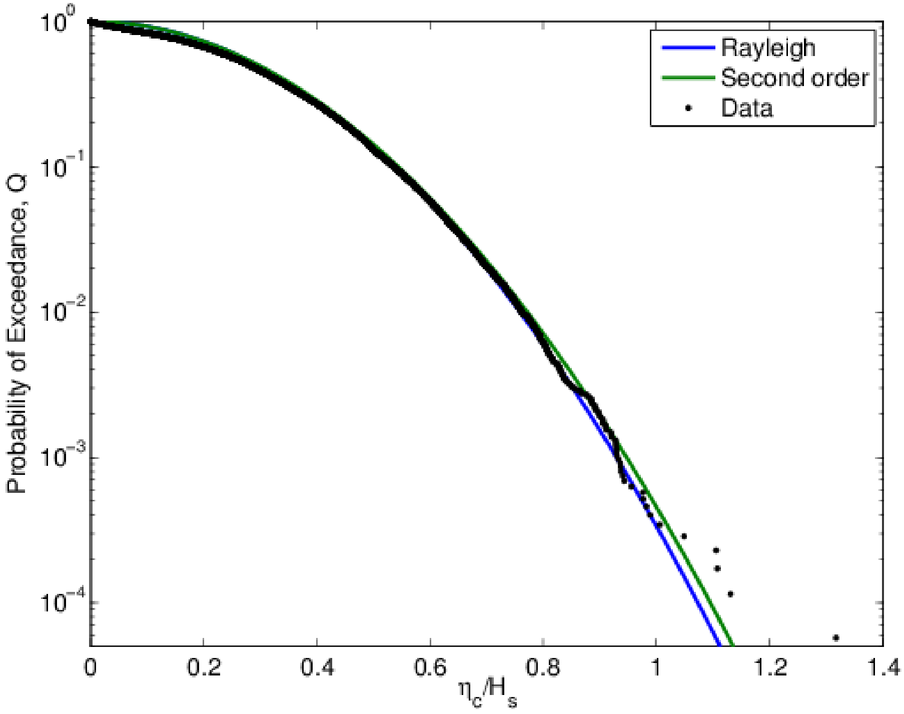
20 x 3-hour seeds



Spatial uniformity

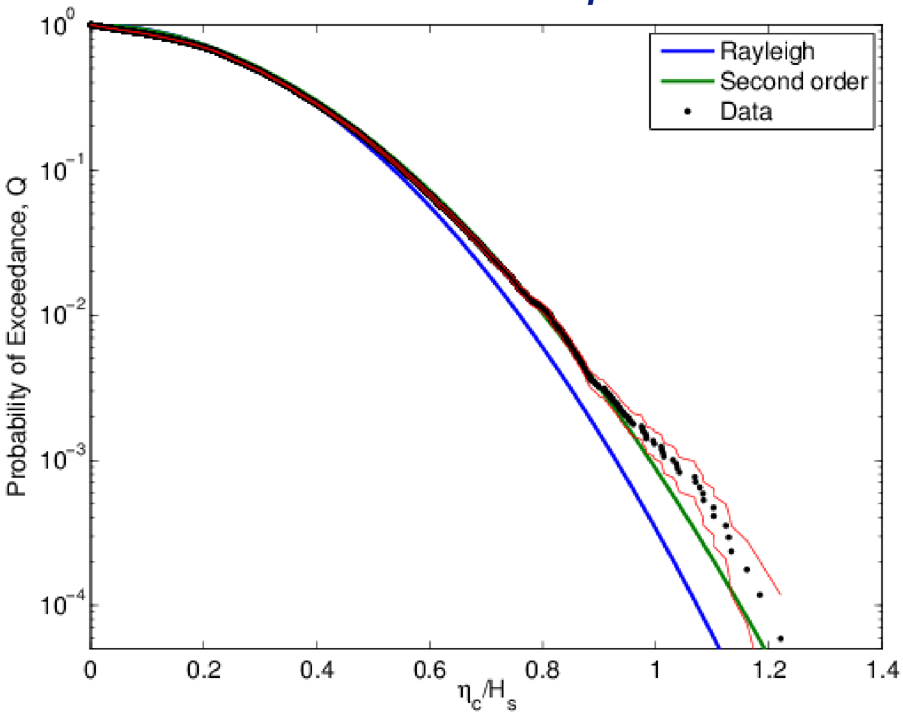
Crest height distributions

Laboratory data (T_p=16s, σ_θ=15°)



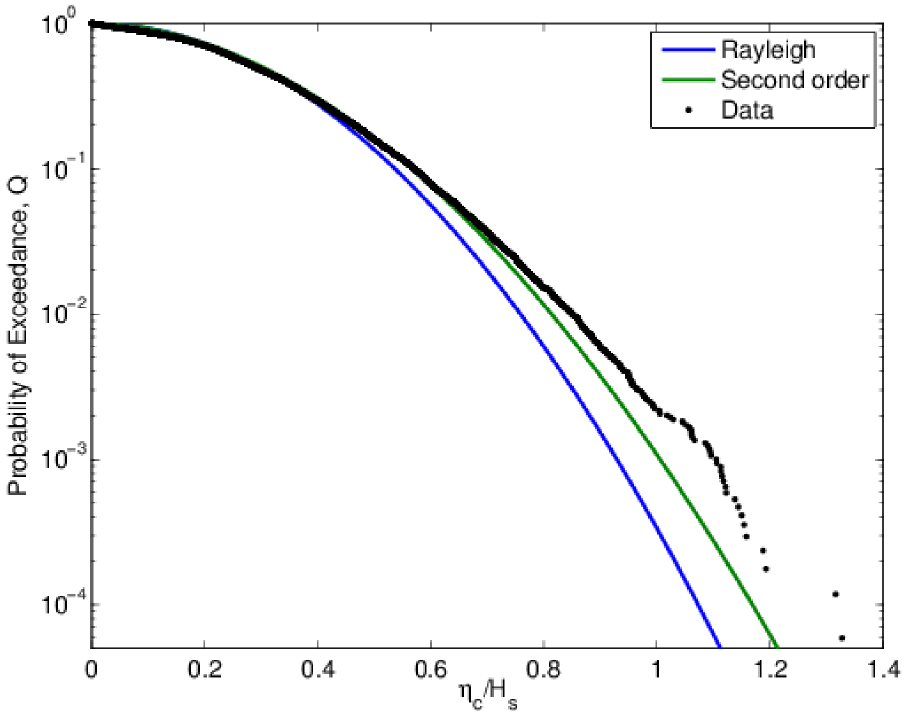
H_s=3m, ½$H_s k_p$=0.024

H_s=10m, ½$H_s k_p$=0.081



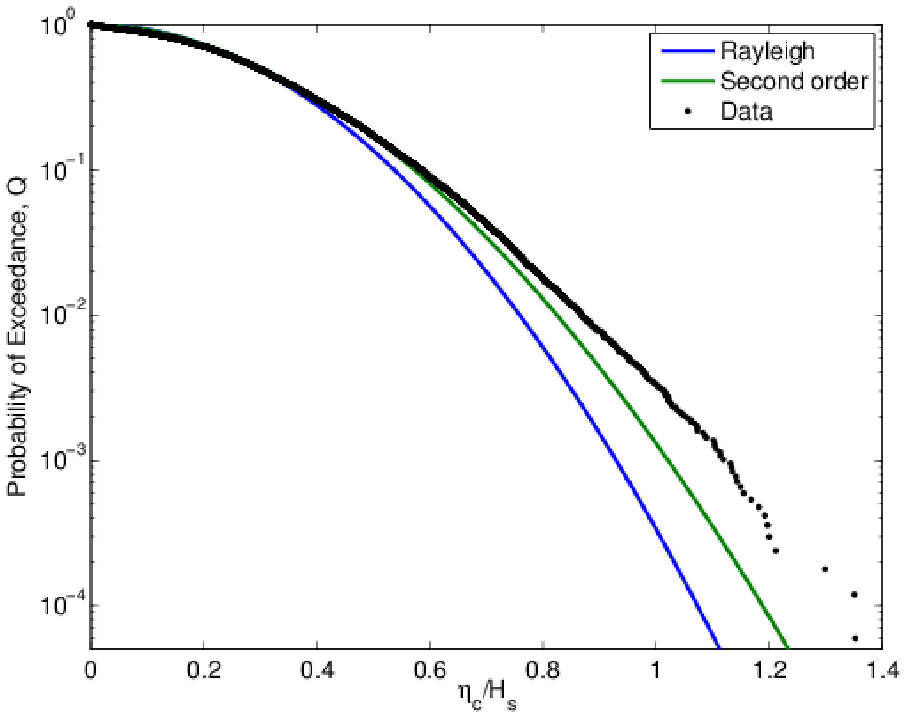
Crest height distributions

Laboratory data (T_p=16s, σ_θ=15°)



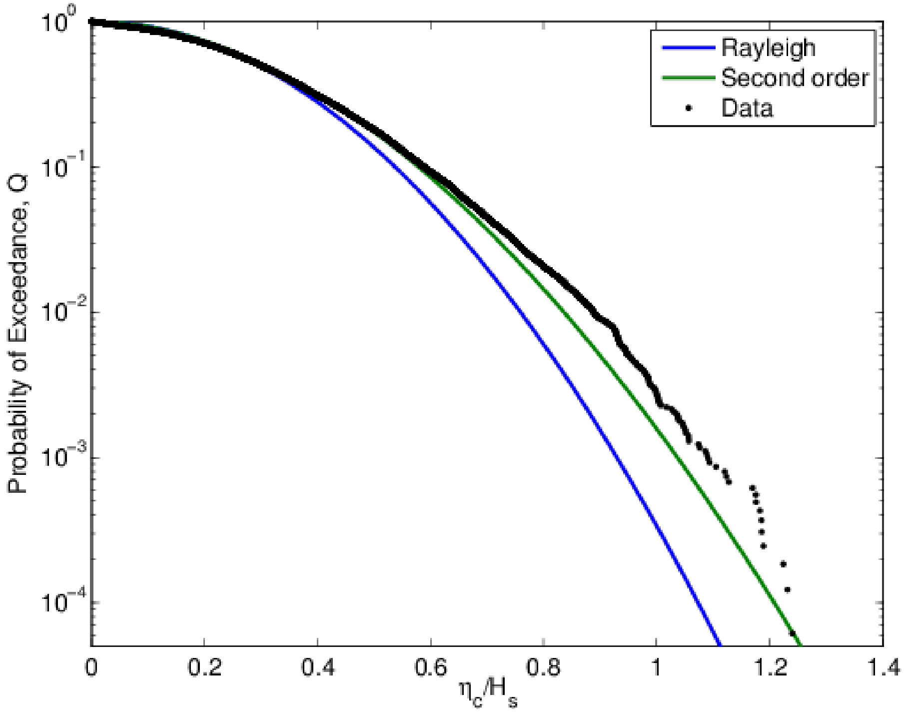
H_s=12.5m, $\frac{1}{2}H_sk_p$=0.102

H_s=15.0m, $\frac{1}{2}H_sk_p$=0.122



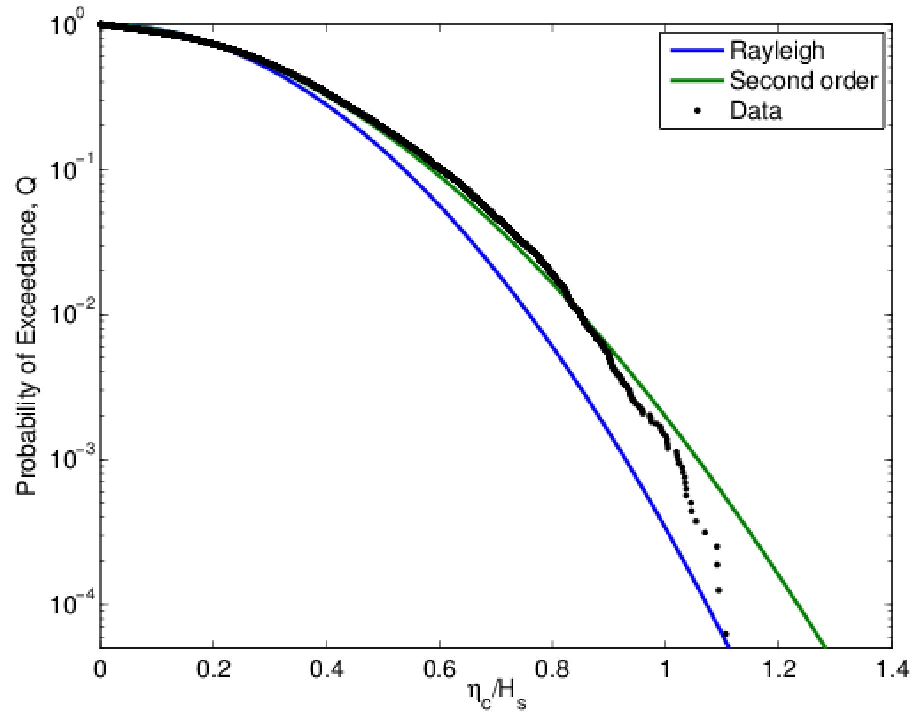
Crest height distributions

Laboratory data (T_p=16s, σ_θ=15°)



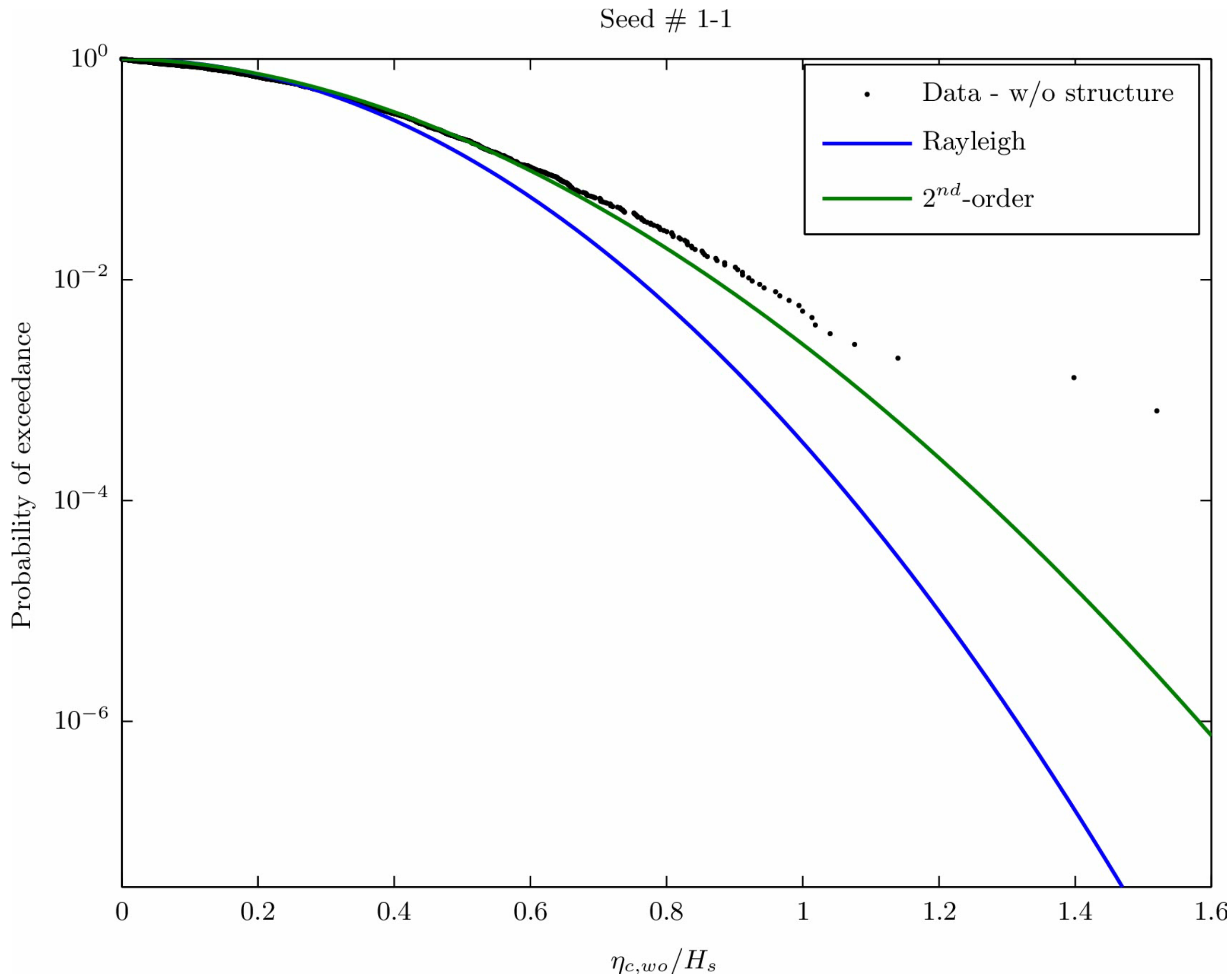
H**$_s$=17.5m, ½H$_s$k$_p$=0.142***

H**$_s$=20.0m, ½H$_s$k$_p$=0.163***



Crest height statistics

Evolution of distribution with progressively more seeds:



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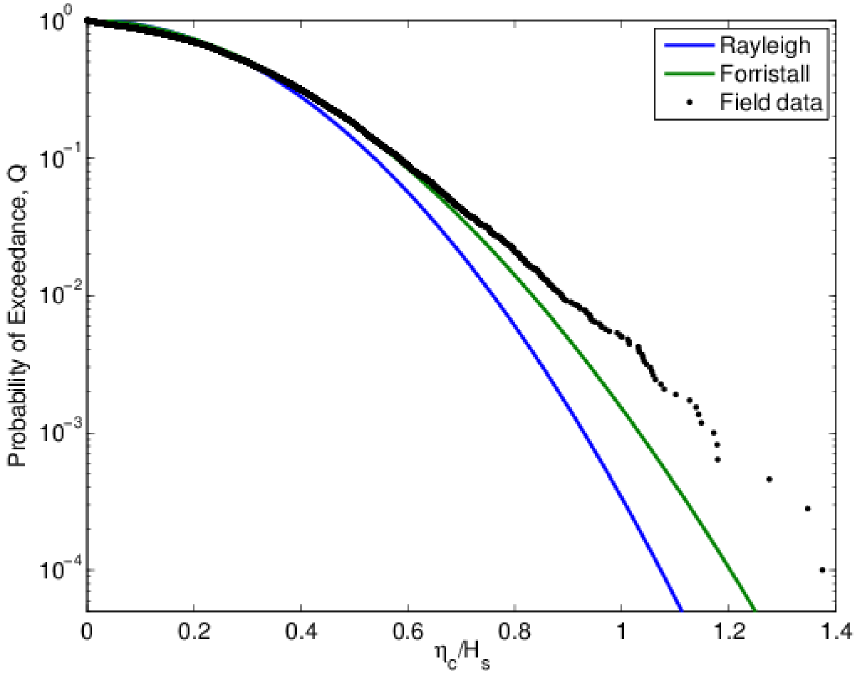
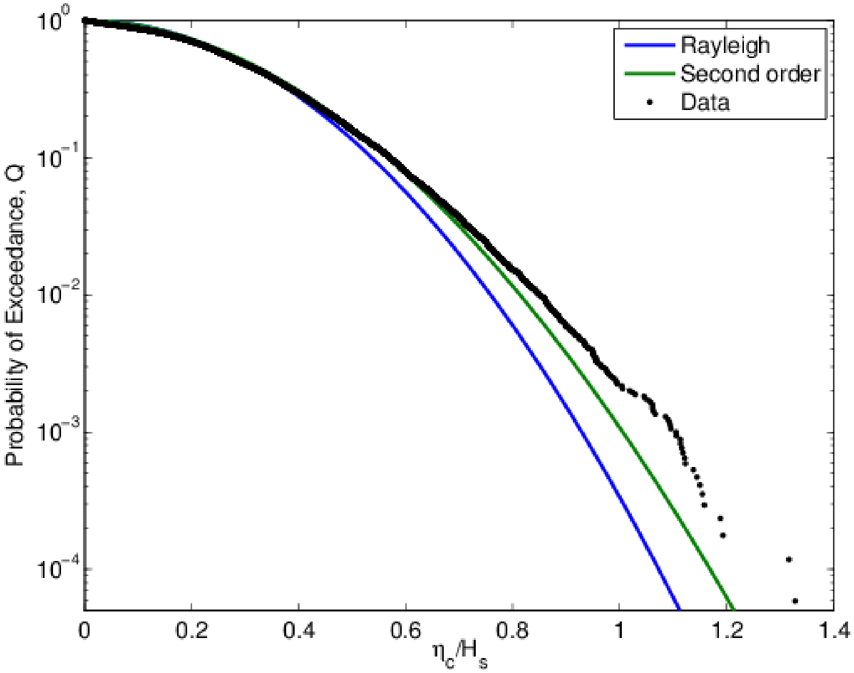
Comparisons to field data

- **Analysis of available field data (>5x10^5 20min records)**
- **Undertaken within the *CresT* JIP**



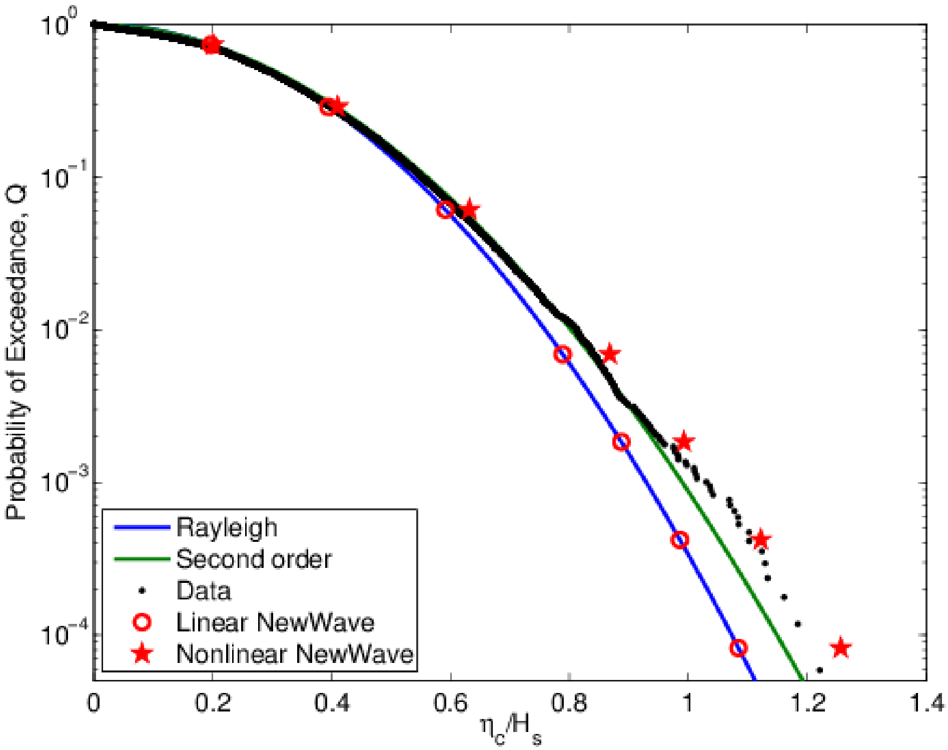
Field data: H_s>12m



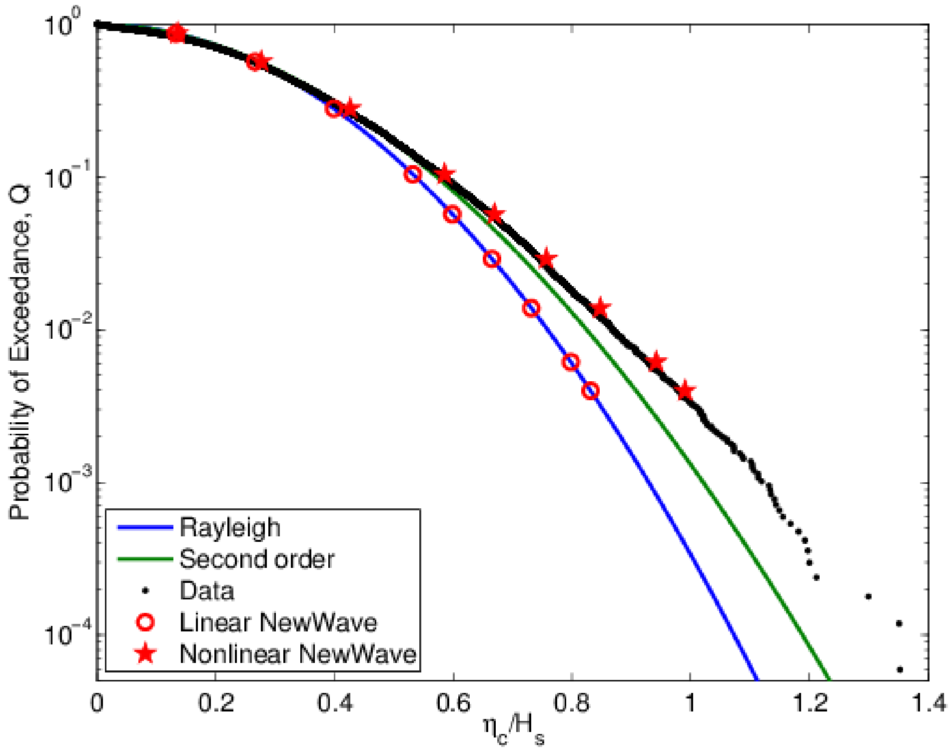
Laboratory data H_s=12.5m

Comparisons with numerical calculations

- **Applied to non-breaking wave events**
- **Numerical calculations based upon <u>focused wave groups</u>**
- **Undertaken using a fully nonlinear BEM solution**



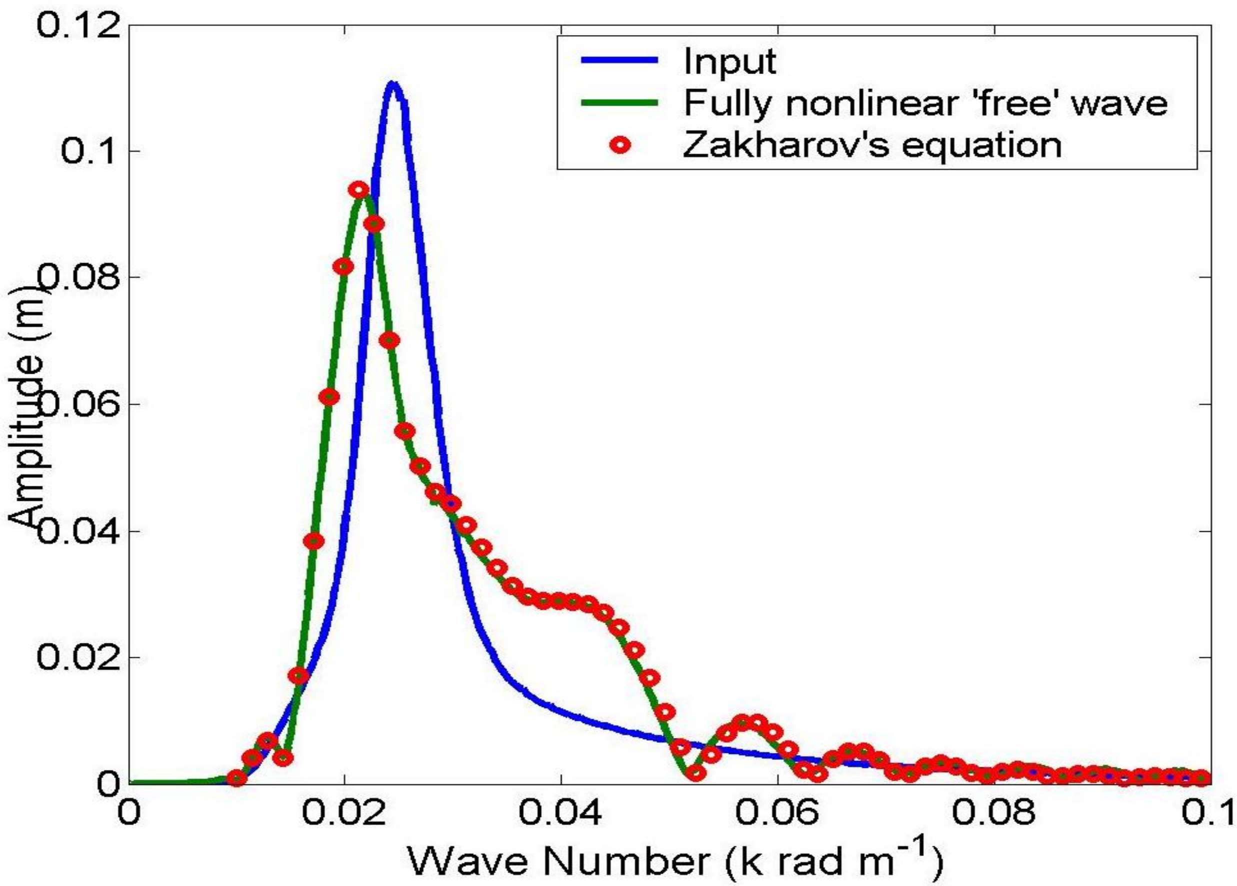
H_s=10m, ½H_sk_p=0.081



H_s=15m, ½H_sk_p=0.122

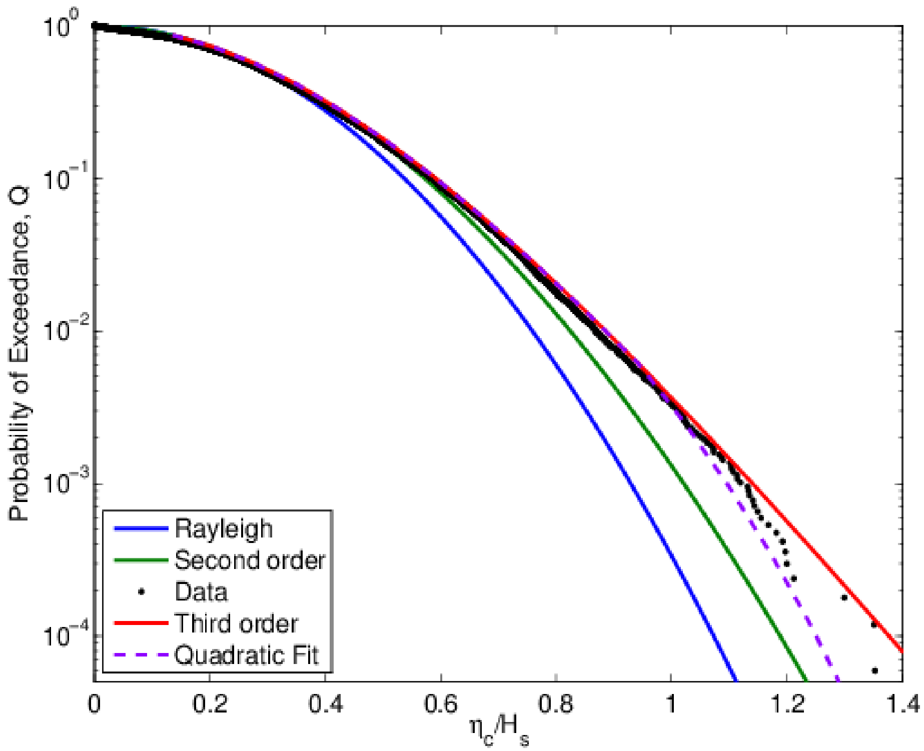
Crest amplifications: Physical explanation

- **Local and rapid spectral change**
- **Movement of energy to the higher frequencies**
- **Due to third-order resonant interactions**



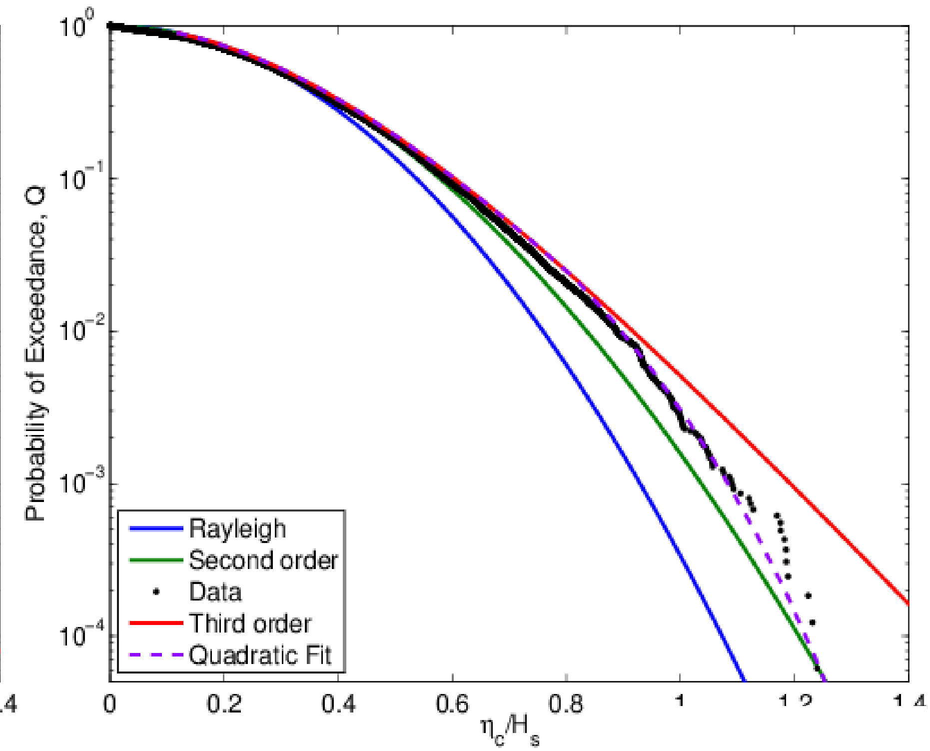
Long-term goal

- **An empirical crest height distribution**
- **Incorporating: - nonlinear amplification**
  - wave breaking
- **Based upon experimental & theoretical input**



***H_s*=15m, *½$H_s k_p$*=0.122, *σ_θ*=15°**

- limited breaking

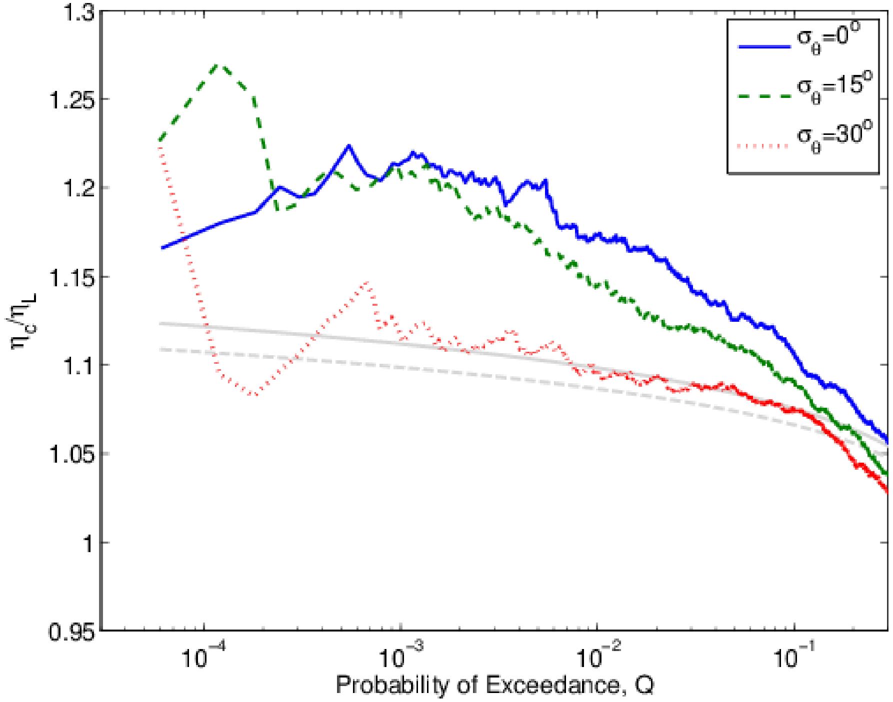


***H_s*=17.5m, *½$H_s k_p$*=0.142, *σ_θ*=15°**

- significant breaking

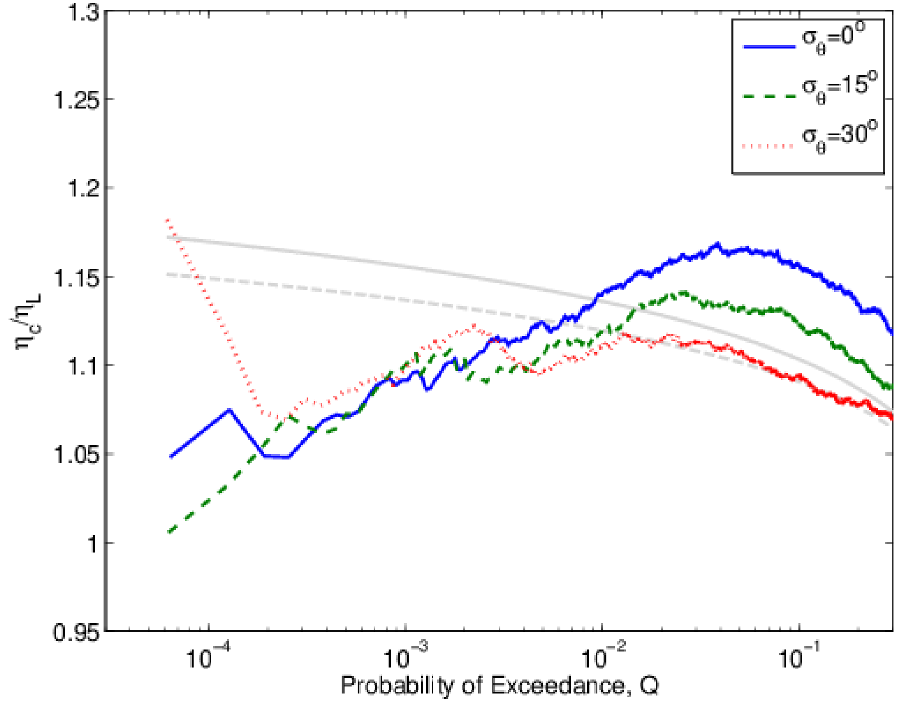
Nonlinear amplification and breaking

Critically dependent on steepness & directional spread



***T_p*=16s, *H_s*=15.0m, ½*$H_s k_p$*=0.122**

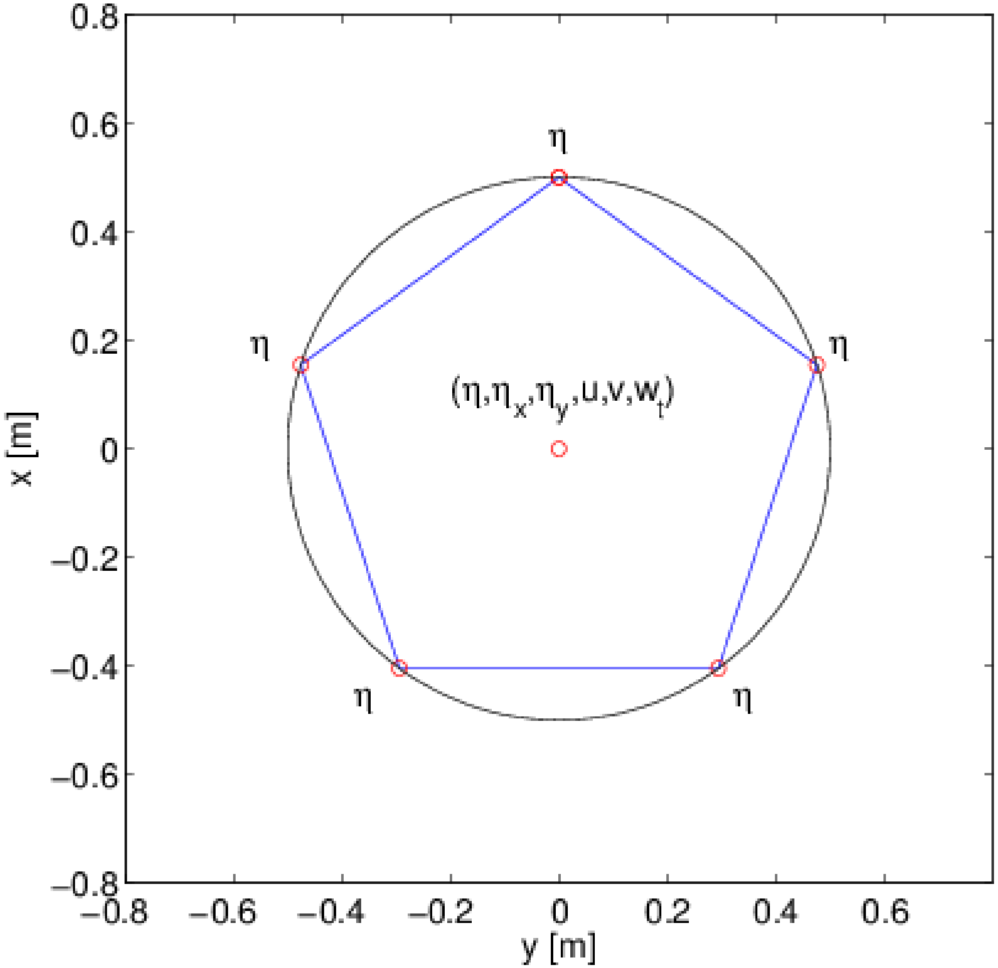
***T_p*=16s, *H_s*=20.0m, ½*$H_s k_p$*=0.163**



Directional analysis: input data

Input data:

η	**surface elevation**
η_x, η_y	**surface gradients**
u, v, w	***velocity components***



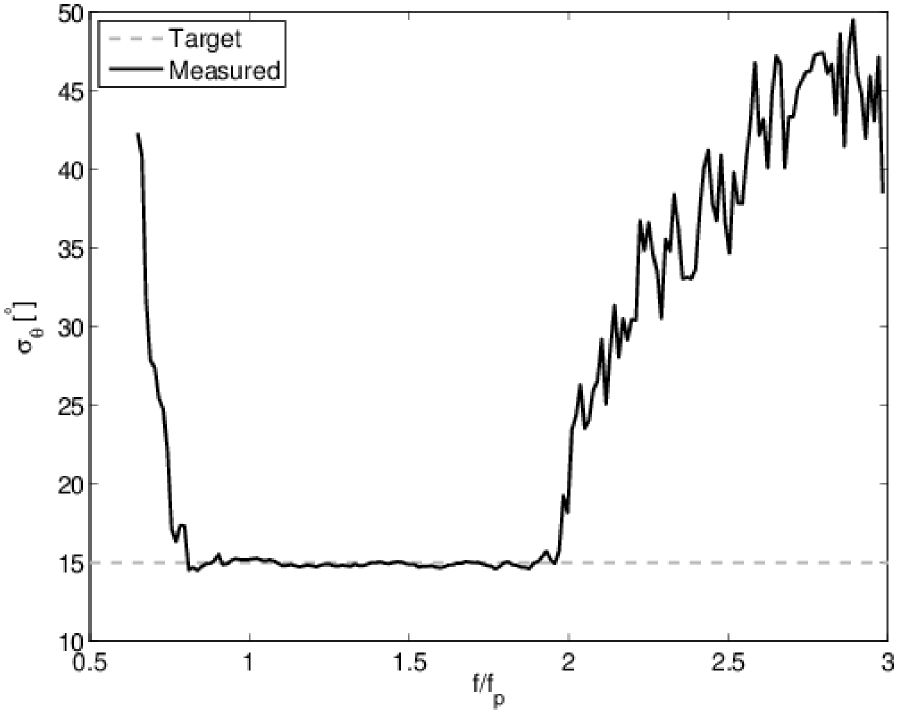
Preferred approach: **Input data:** ***η, u, v***

Methodology: **EMEP**

Directionality: nonlinear changes

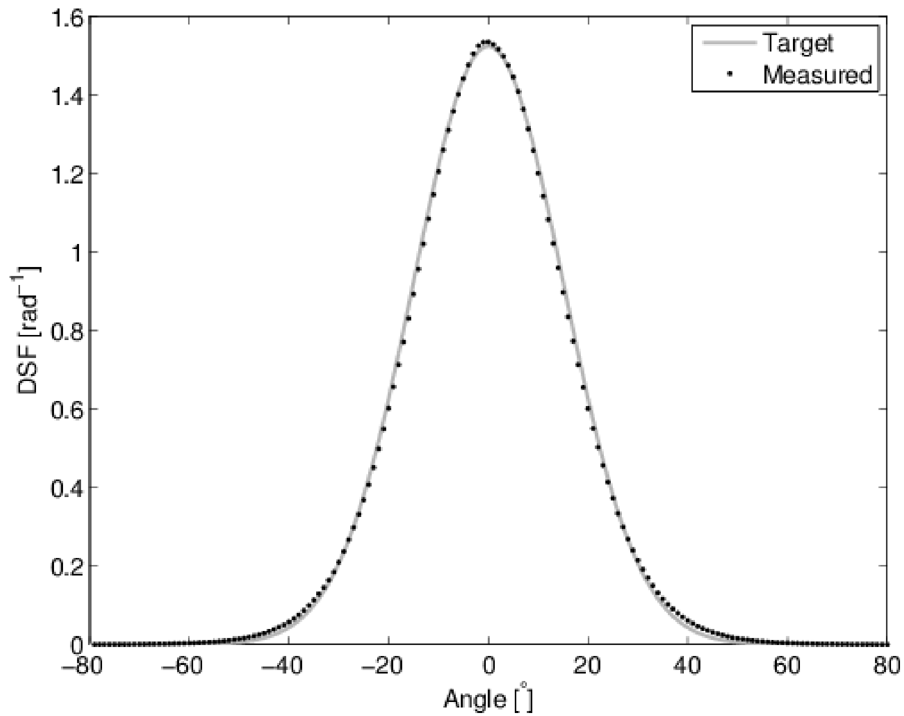
Comparisons to laboratory data (H_s=10m, $\frac{1}{2}H_s k_p$=0.081)

- σ_θ=15°
- input data: η, u, v
- calculated using the EMEP
- sea state generated using RDM



σ_θ vs. f/f_p

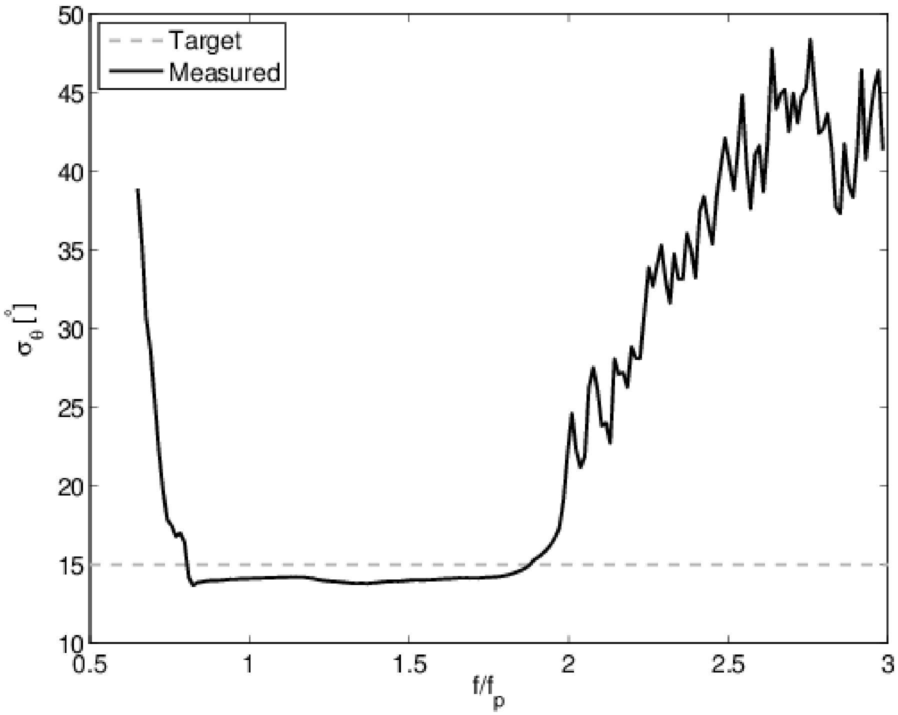
Directional spreading function, DSF



Directionality: nonlinear changes

Comparisons to laboratory data (H_s=15.0m, ½$H_s k_p$=0.122)

- σ_θ=15°
- input data: η,u,v
- calculated using the EMEP
- sea state generated using RDM



σ_θ vs. f/f_p

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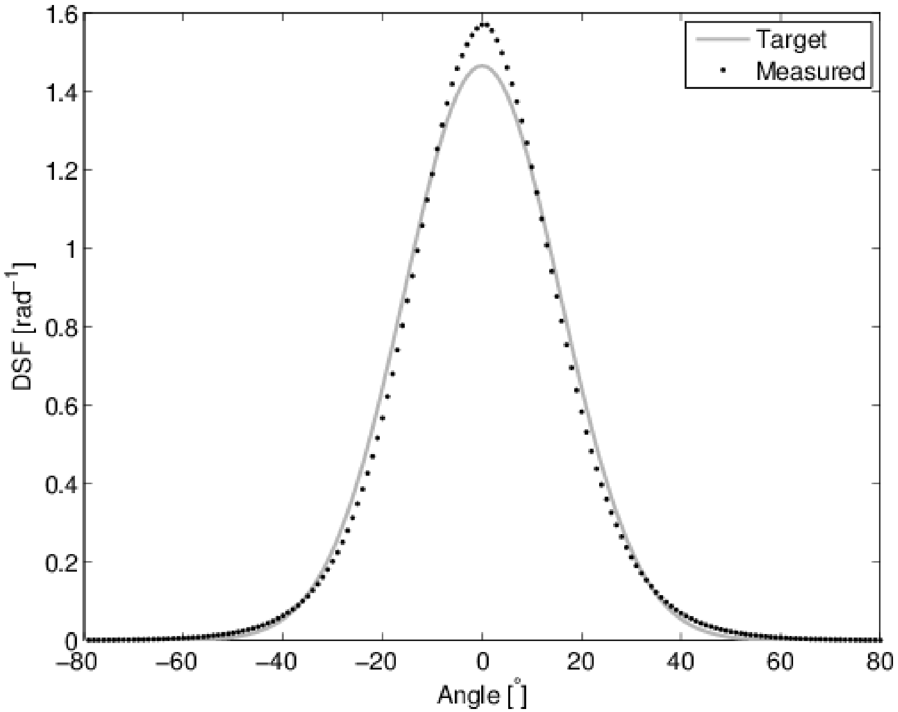
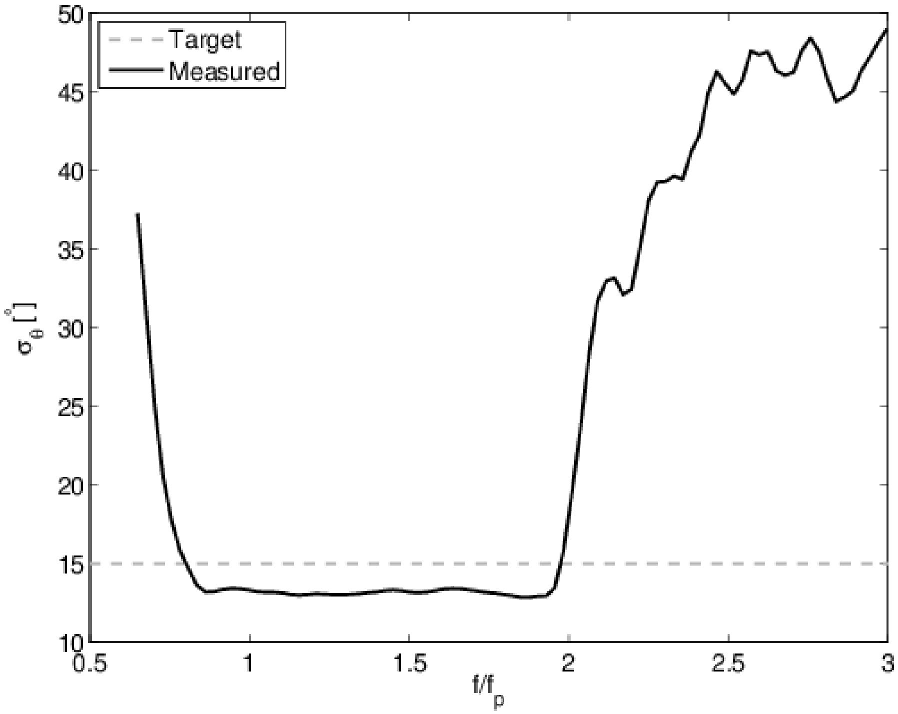


Directionality: nonlinear changes

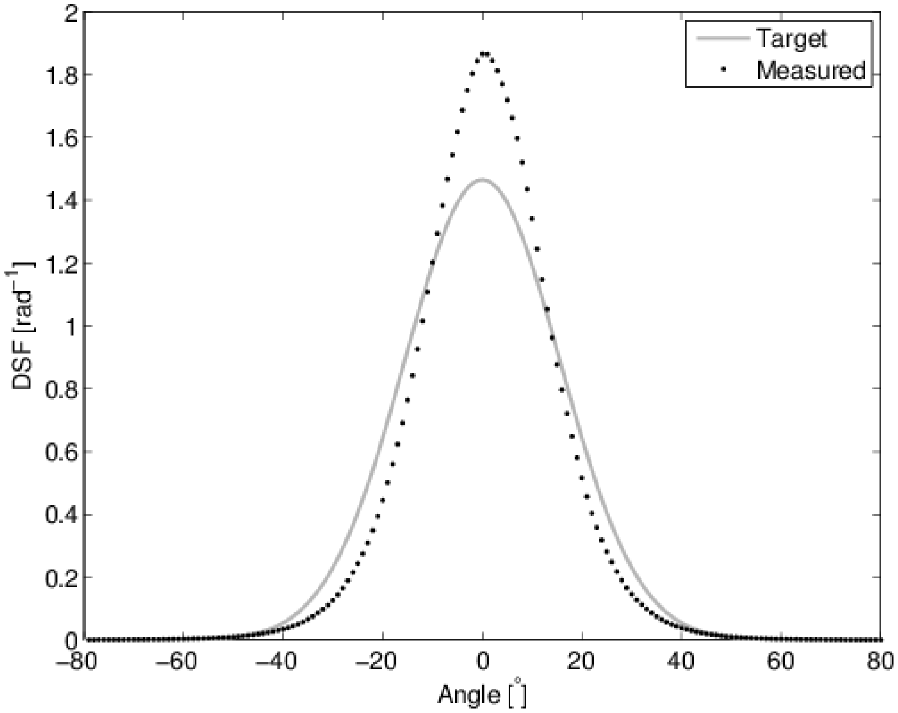
Comparisons to laboratory data (H_s=20.0m, ½$H_s k_p$=0.163)

- σ_θ=15°
- input data: η, u, v
- calculated using the EMEP
- sea state generated using RDM



σ_θ vs. f/f_p

Directional spreading function, DSF

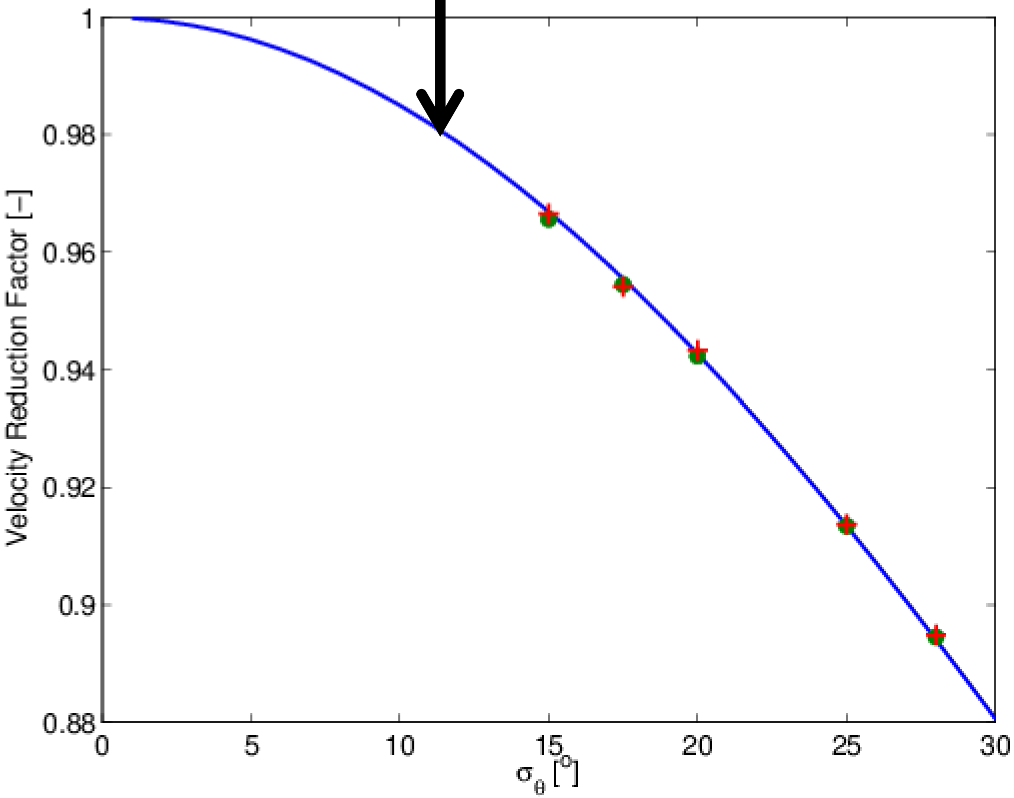


Directionality, alternative quantification

Based upon the velocity reduction factor (VRF)

VRF= $\left[\frac{1}{2}+\frac{1}{2}\,exp\left(-2\sigma^{2}\right)\right]^{\frac{1}{2}}$ or **VRF**= $\frac{(rms\ u_x\ in\ directional\ sea)}{(rms\ u_x\ in\ uni\text{-}directional\ sea)}$

(Tucker & Pitt, 2001)

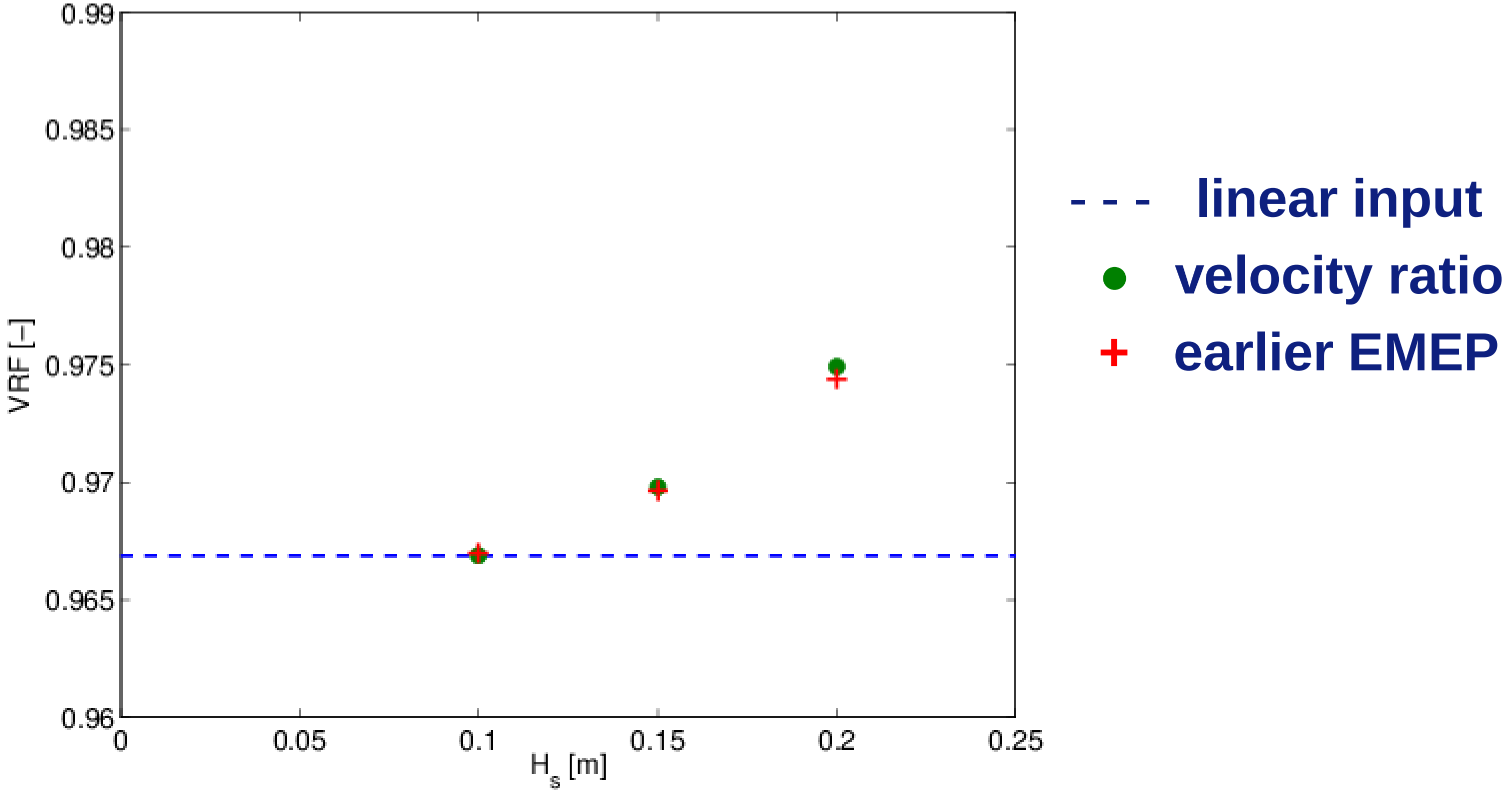


Based upon:

- ● **the analysis of an entire wave record**
- **+** **the average of a wave-by-wave analysis**

Velocity Reduction Factor (VRF)

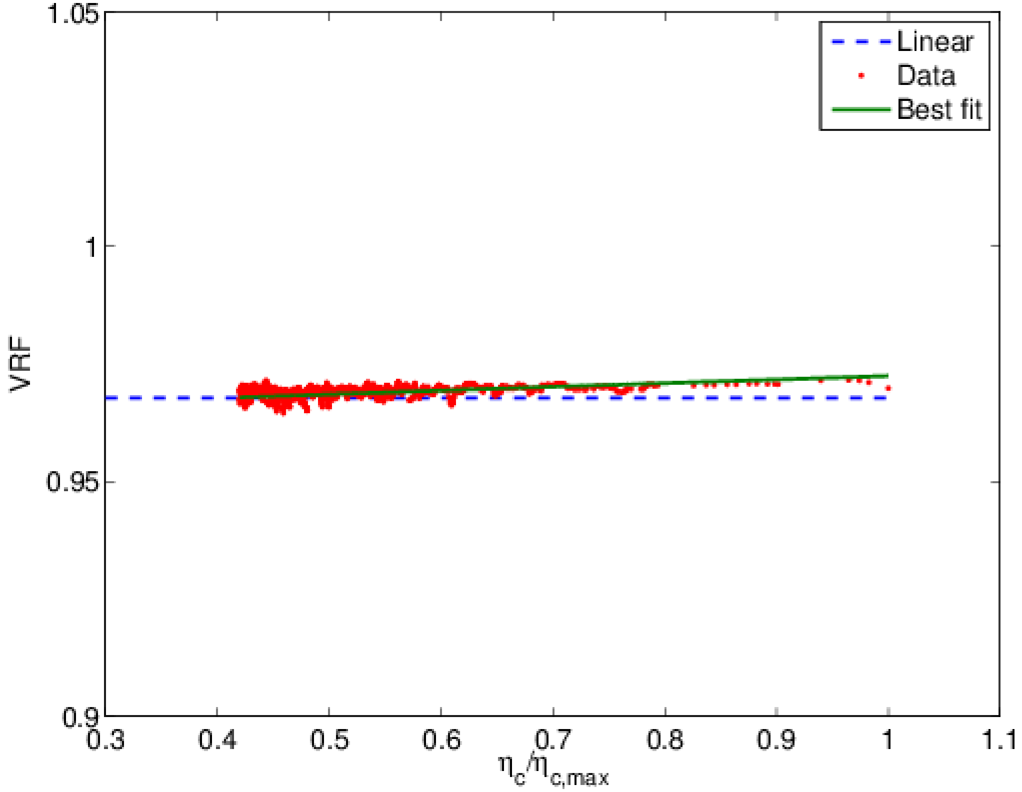
- **comparisons to laboratory data**
- **VRF averaged over 20 x 3-hour seeds for each sea state**
- **changes with H_s**



Conclusion: steepest seas are more uni-directional

Velocity Reduction Factor (VRF)

- **Comparisons to laboratory data (H_s=10m, σ_θ=15°, ½$H_s k_p$=0.081)**
- **VRF calculated for individual waves**
- **Plotted in terms of the normalised crest elevation, η_c/η_{cmax}**



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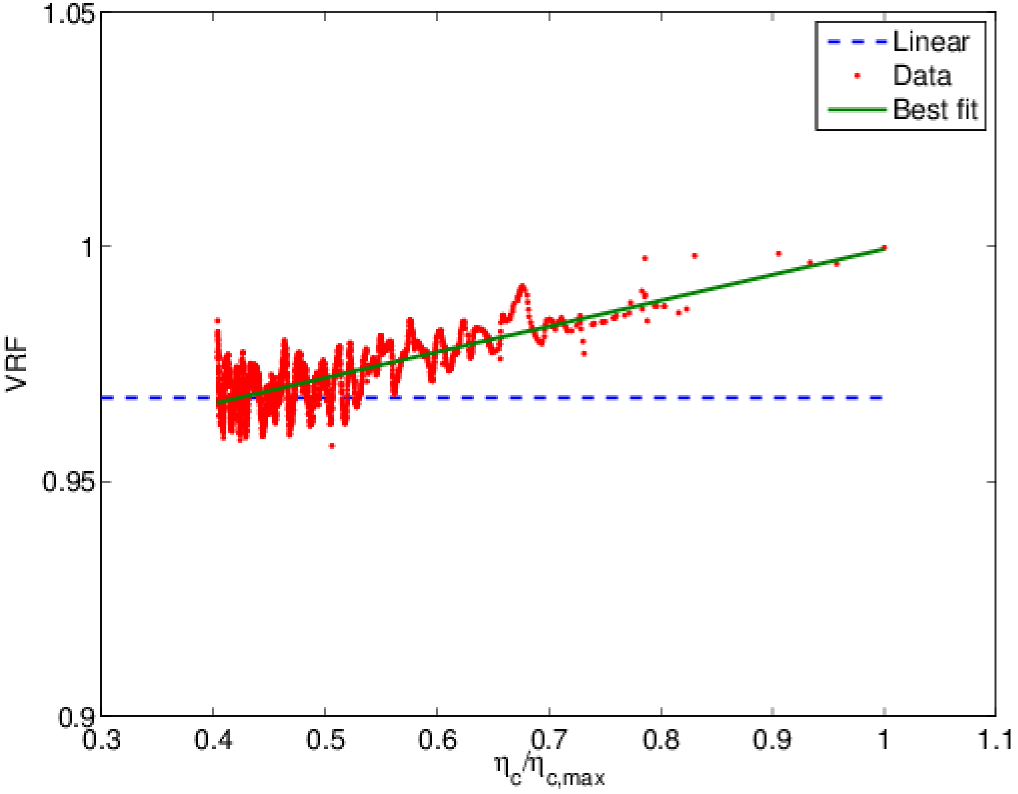

Velocity Reduction Factor (VRF)

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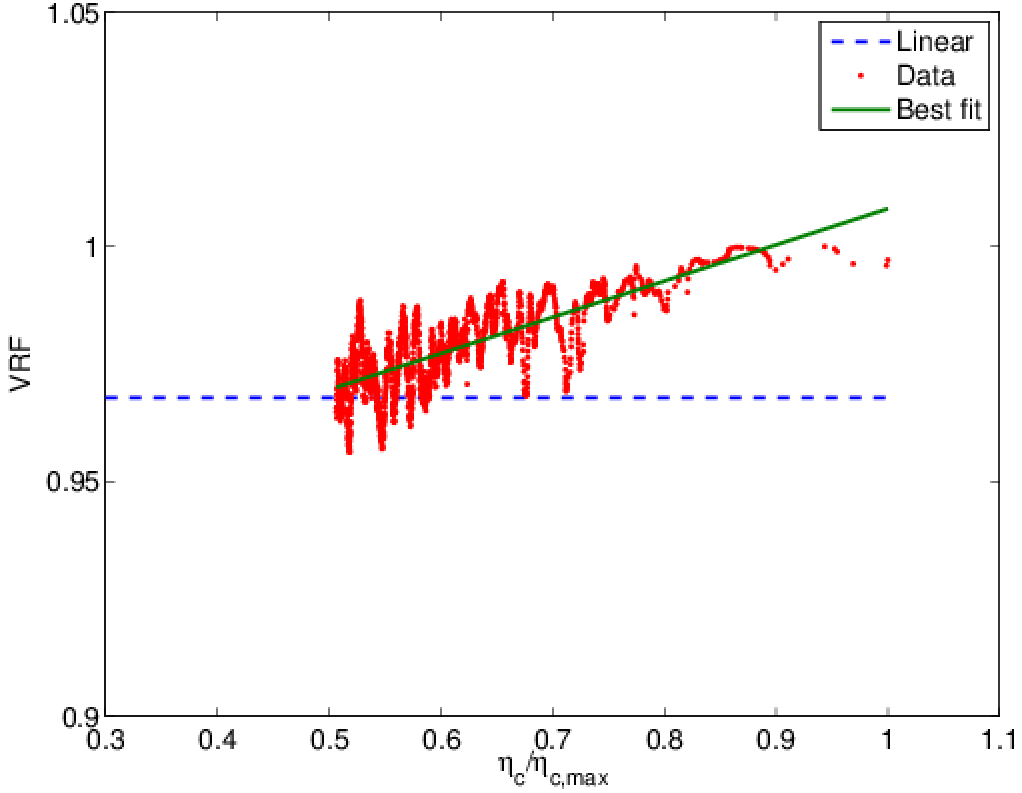
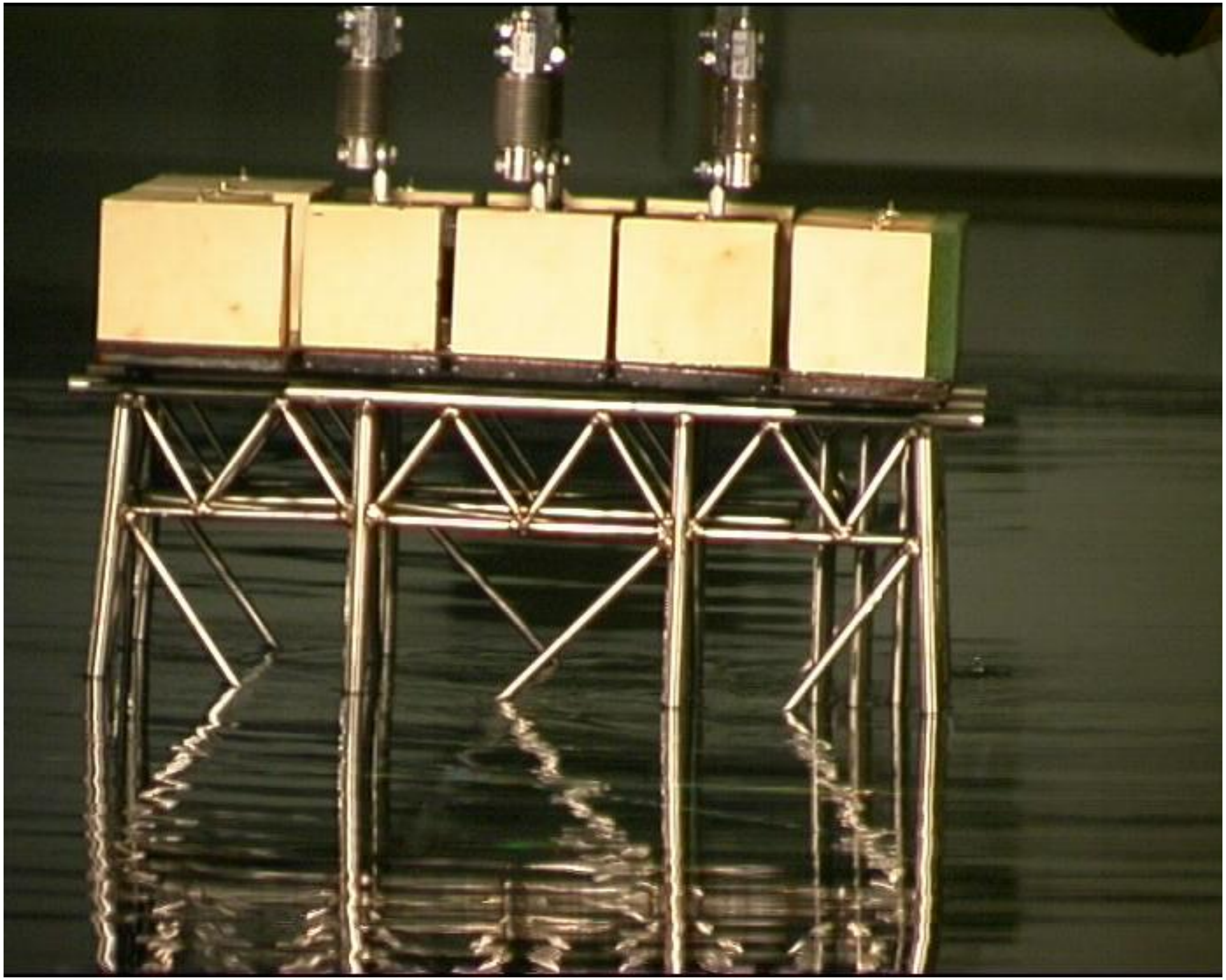


Conclusion: largest individual waves more long-crested

Wave breaking: Is it important in the field?

Laboratory data: 10^{-4} deep water design wave



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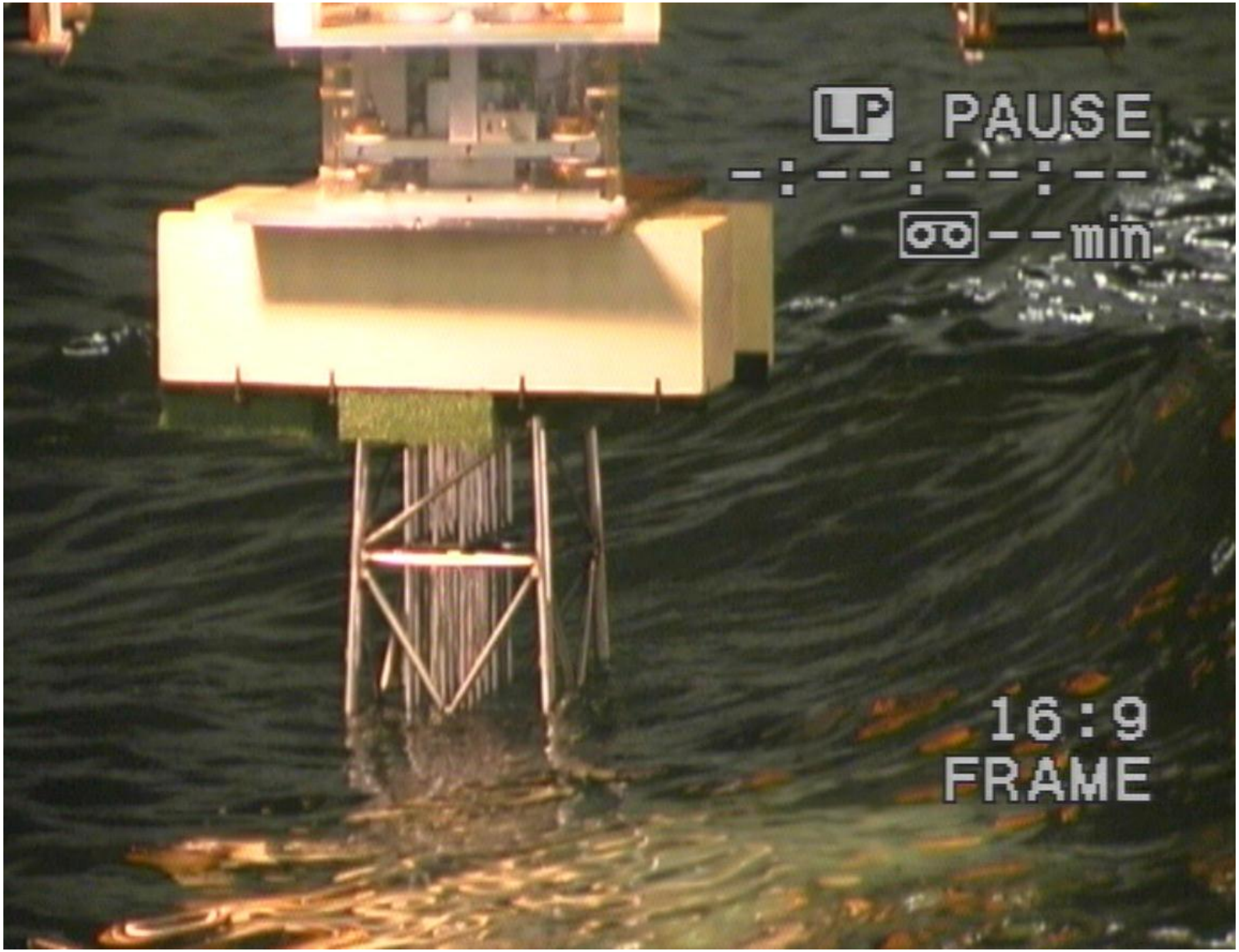
10^{-4} design sea state (North Sea): WID event



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Evidence of an over-turning wave

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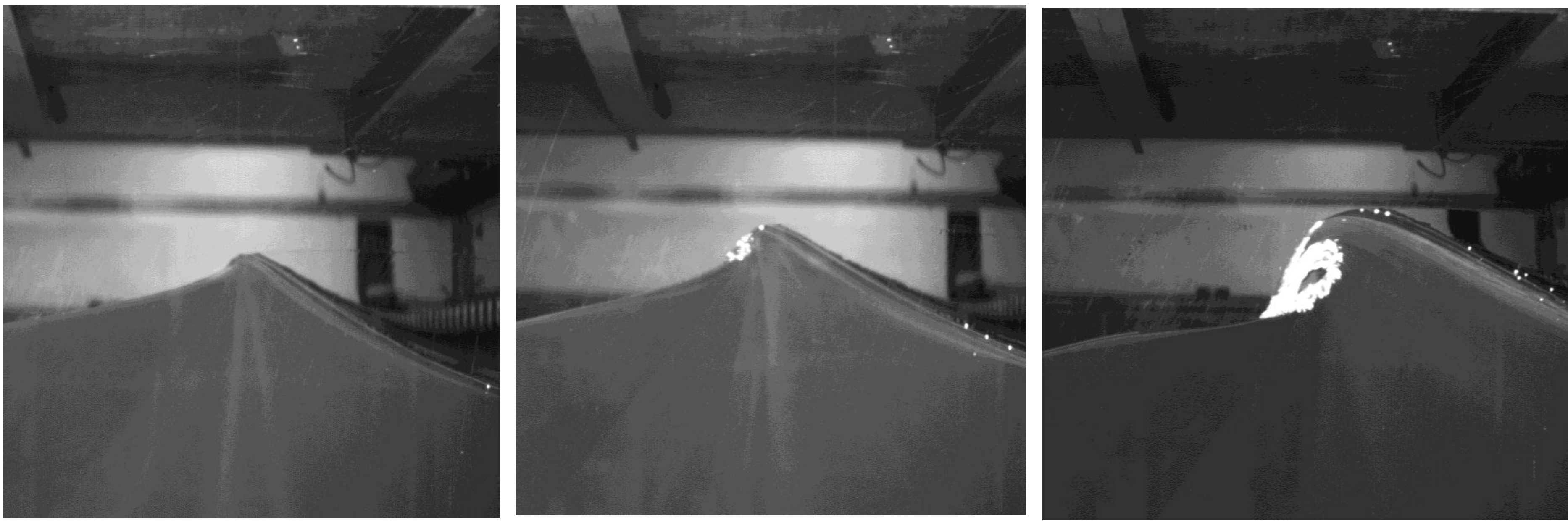
Wave breaking

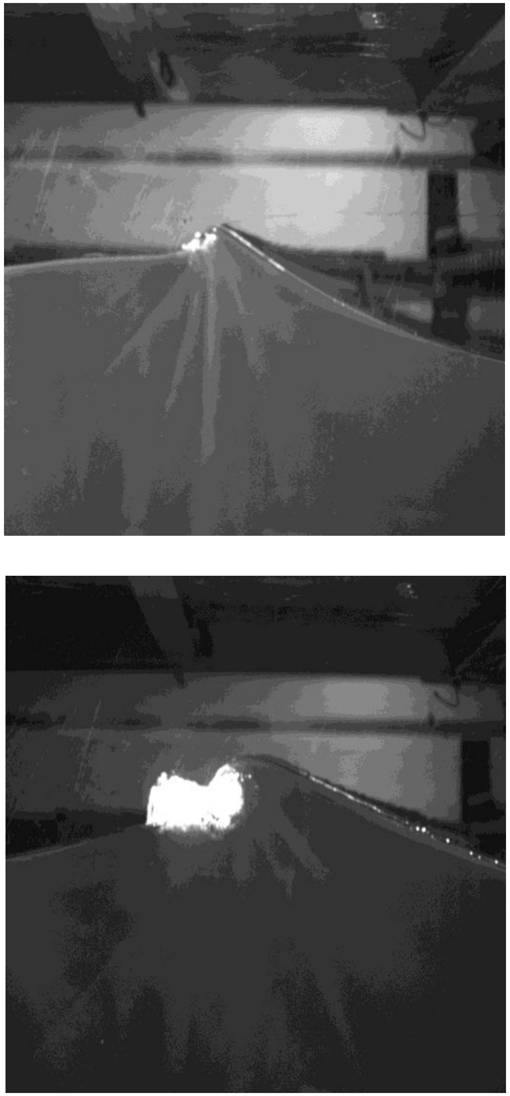
Should be viewed as a process, not a single deterministic event

- **constant spectral shape**
- **constant directional spread**
- **increased energy levels**

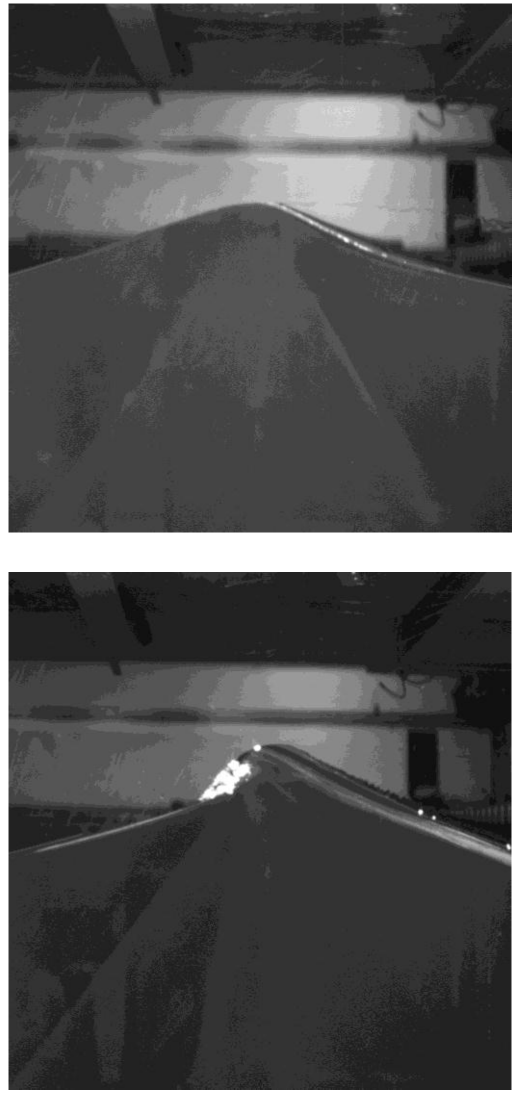


Wave breaking: the role of directionality

σ_θ=0°



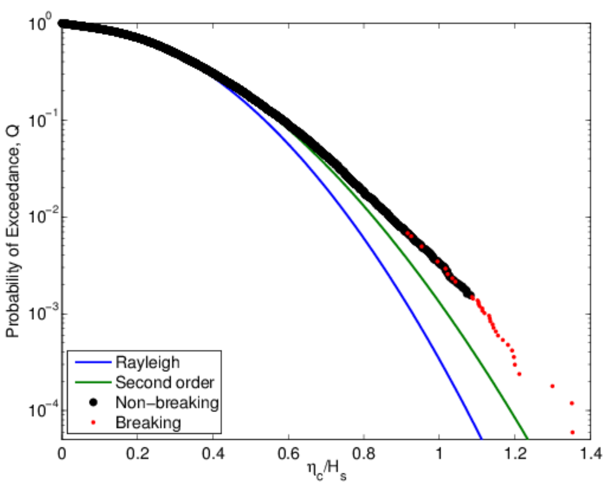
σ_θ=15°



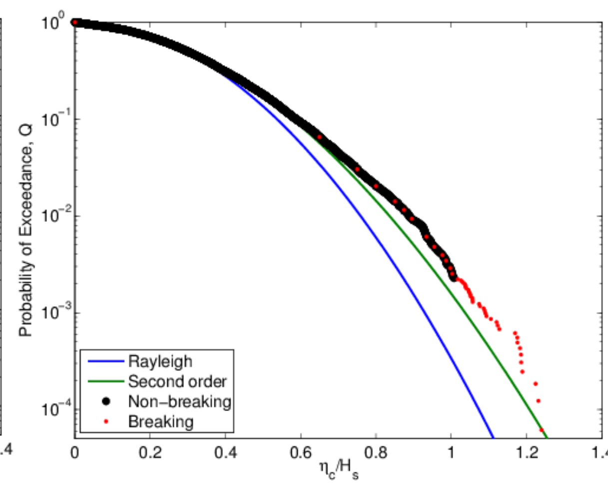
σ_θ=30°

The occurrence of wave breaking

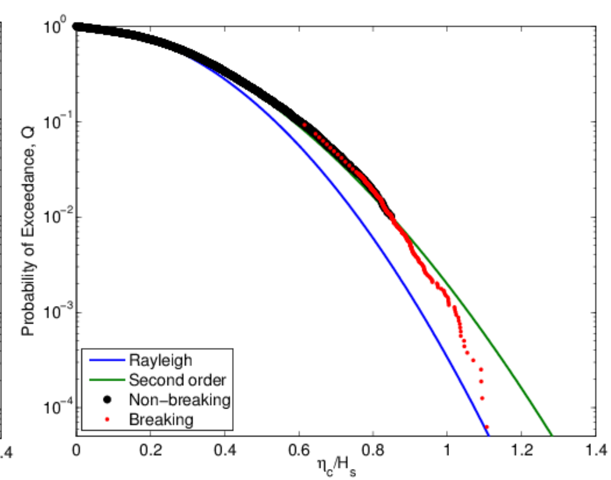
- Visual observations allow breaking waves to be identified
- Where breaking is dominant (on average) data is given in red
- With increasing steepness, the tail of the distribution is controlled by breaking, hence the reduction in crest heights
- Laboratory data relates to σ_θ=15°



H_s=15m, $\frac{1}{2}H_s k_p$=0.122, σ_θ=15°



H_s=17.5m, $\frac{1}{2}H_s k_p$=0.142, σ_θ=15°



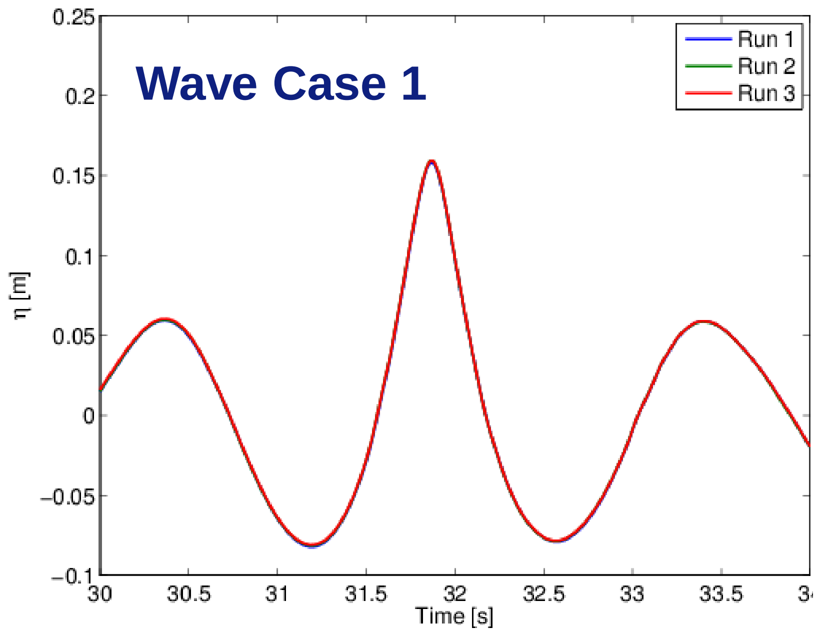
H_s=20m, $\frac{1}{2}H_s k_p$=0.163, σ_θ=15°

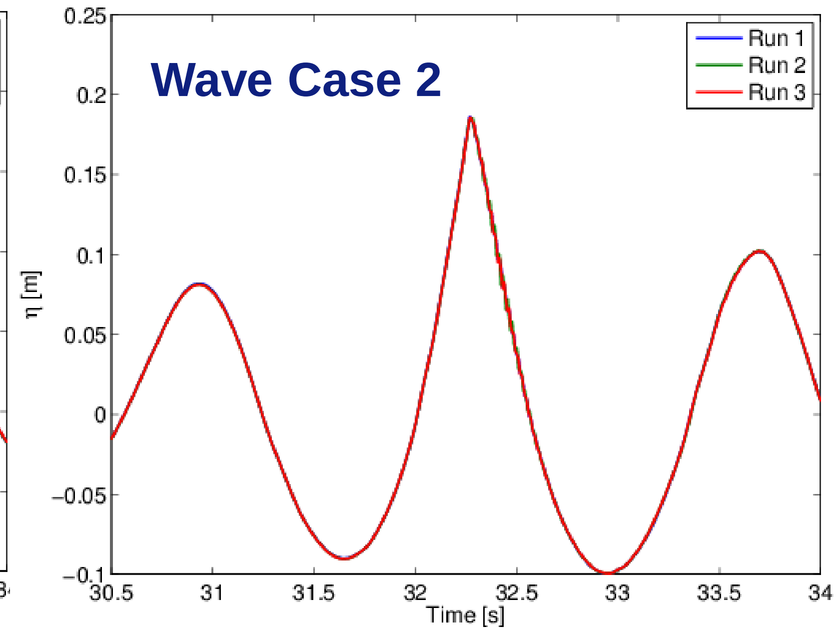
Kinematics measurements

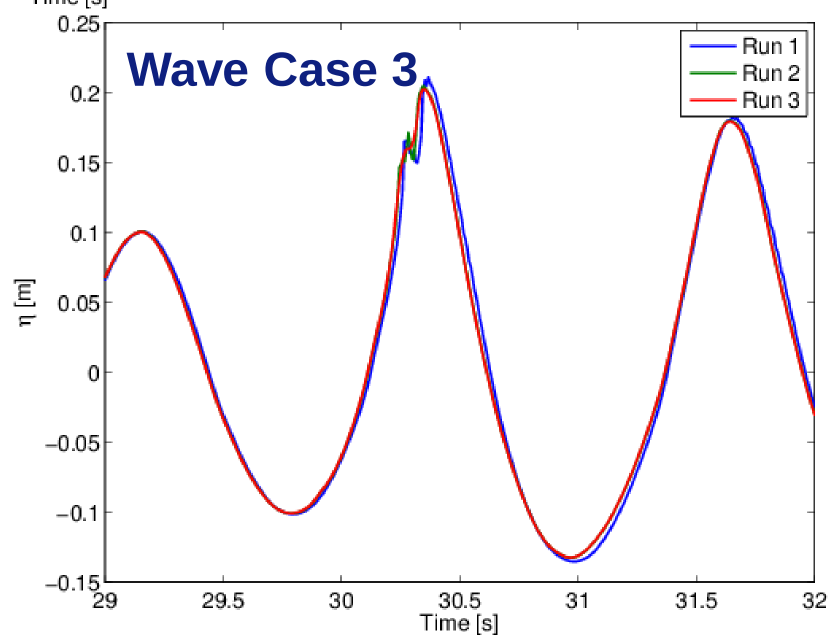
- **Laser Doppler Anemometry (LDA).**
    - **Provides time-history at a single point**
    - $u(t)$, $v(t)$ & $w(t)$
    - **Multiple runs to build spatial profiles**
    - **Highest accuracy (better than ±1%)**
    - **Largest data rate (kHz)**
    - **Required seeding density more easily achieved**

    - **Very time consuming**

Repeatability of wave records:







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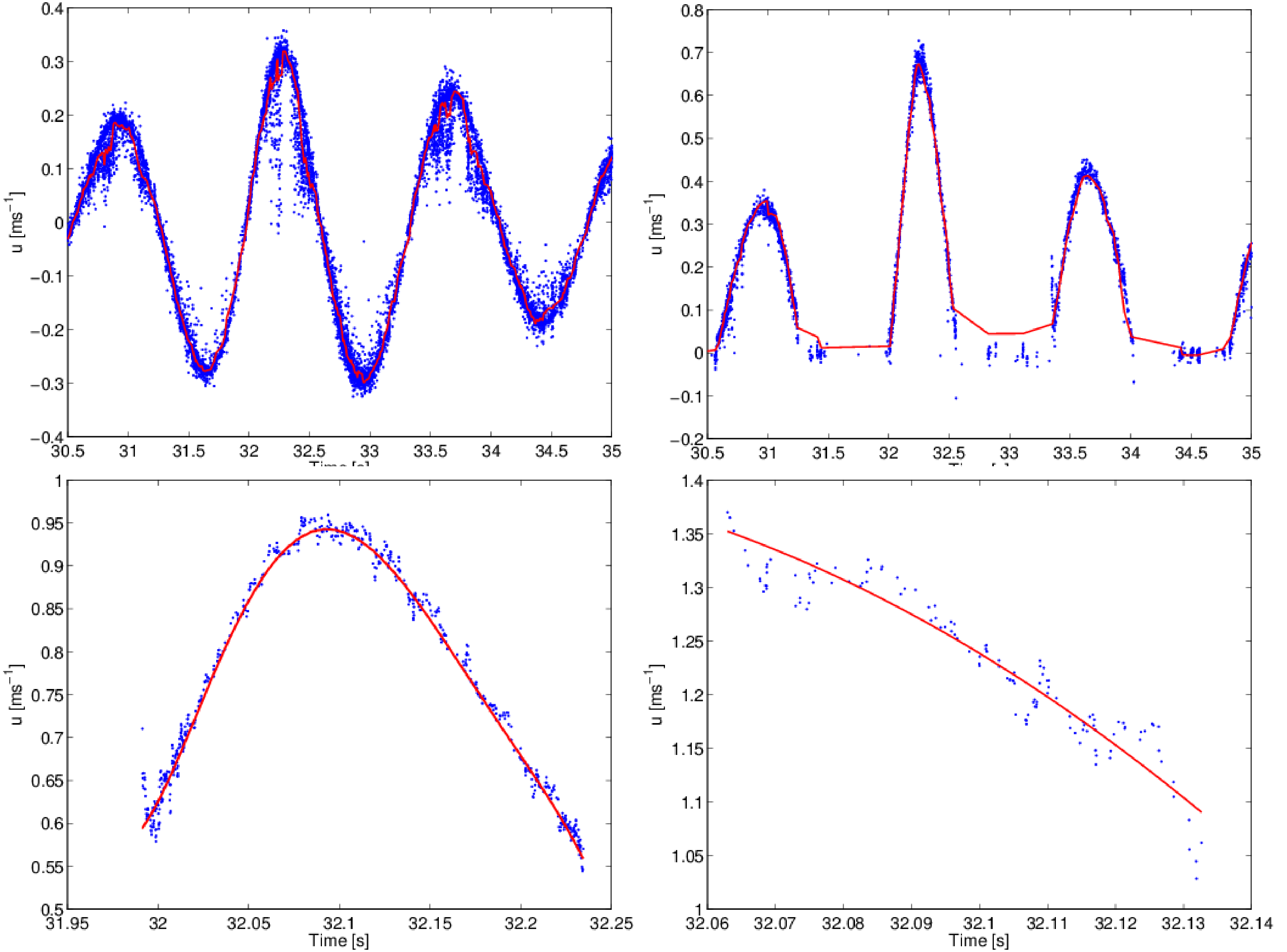
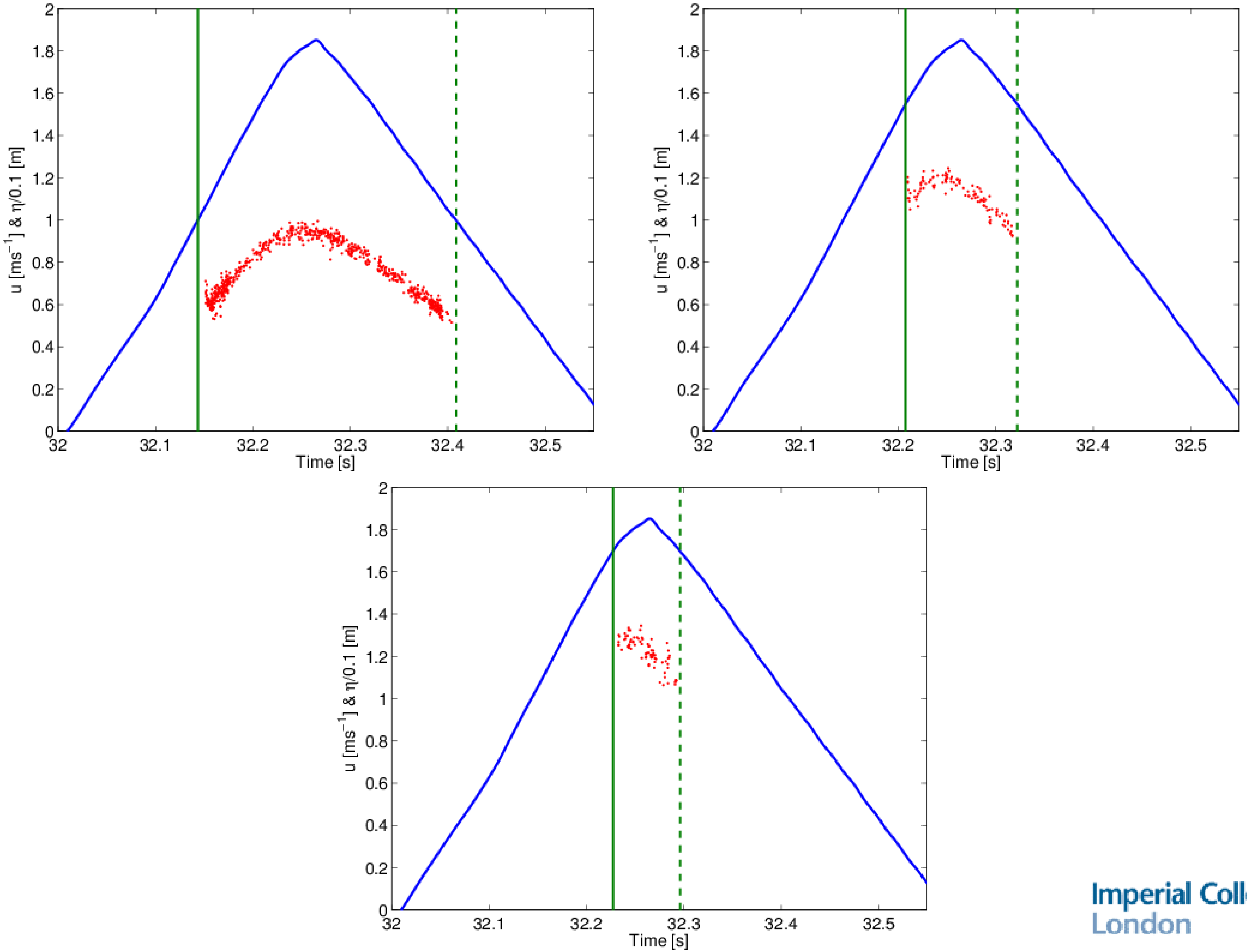
Example data records (LDA: Wave Case 2)



Intermittent velocity records: high in wave crest



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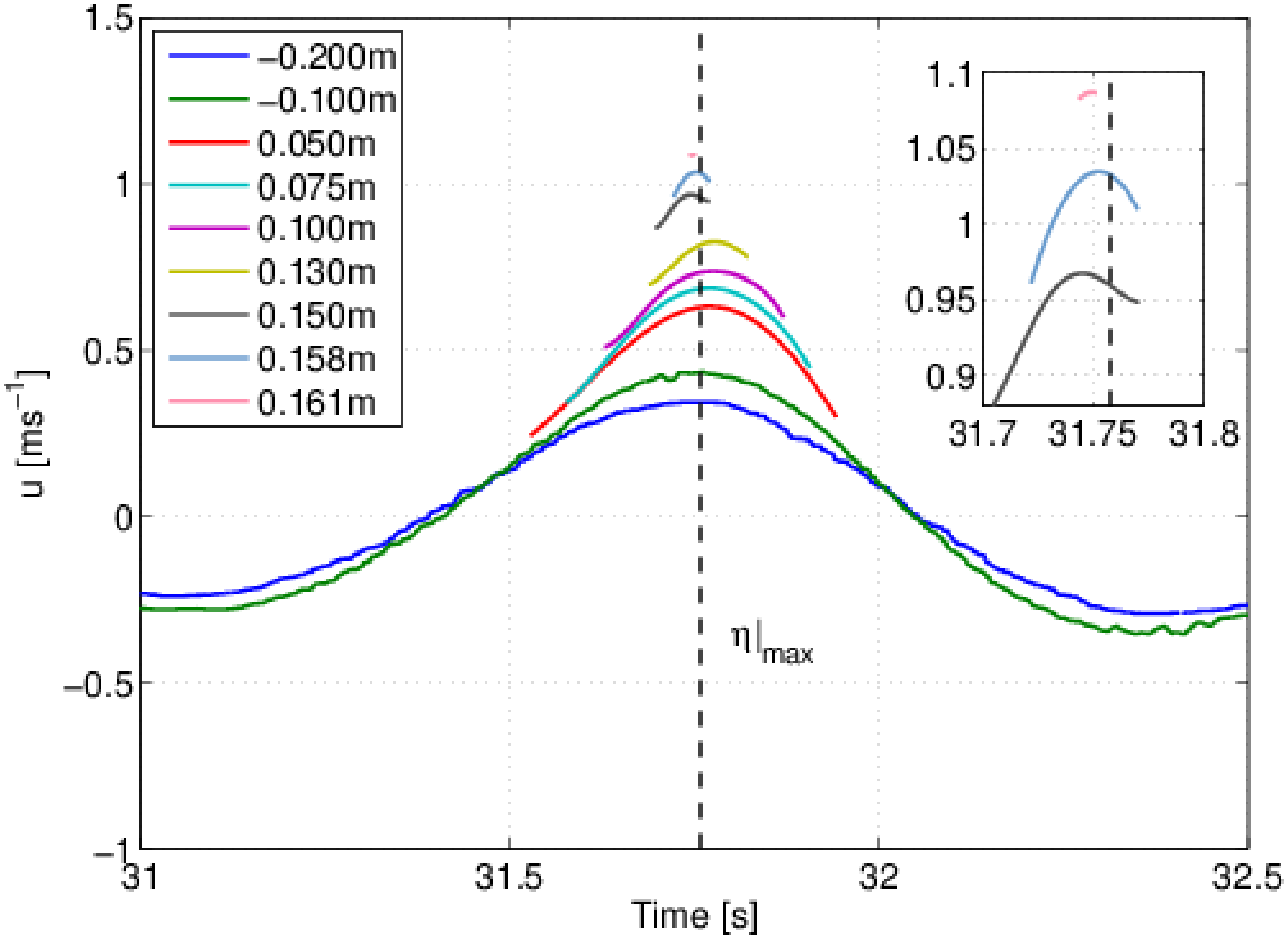
Horizontal velocities, *u*(*t*): Wave Case 1 - various elevations, -20.0m<*z*<+16.1m



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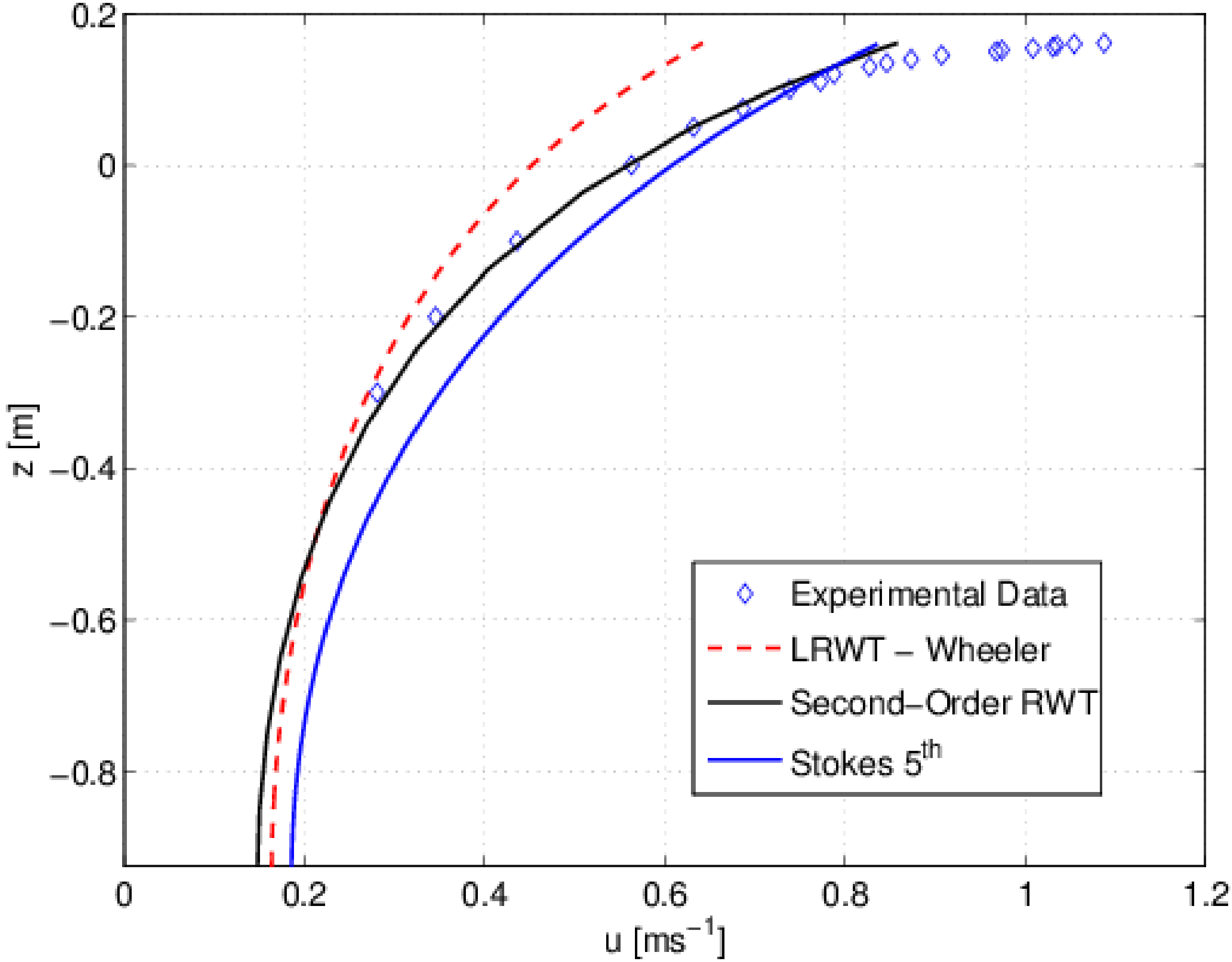
Comparisons to predicted velocities, *u*(*z*): Wave Case 1

– solutions matched to η_{max}



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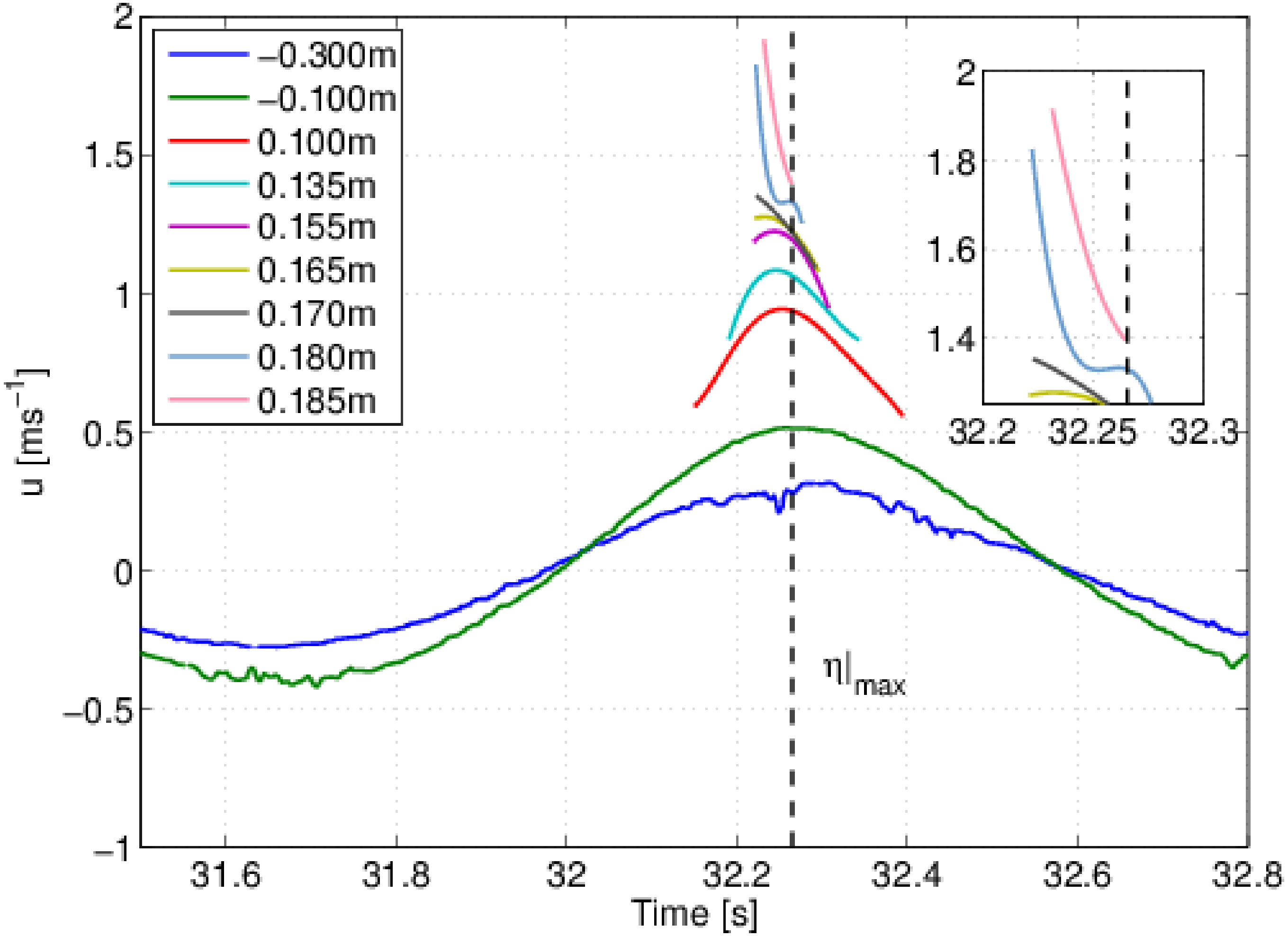
Horizontal velocities, *u*(*t*): Wave Case 2 - various elevations, -30.0m<*z*<18.5m



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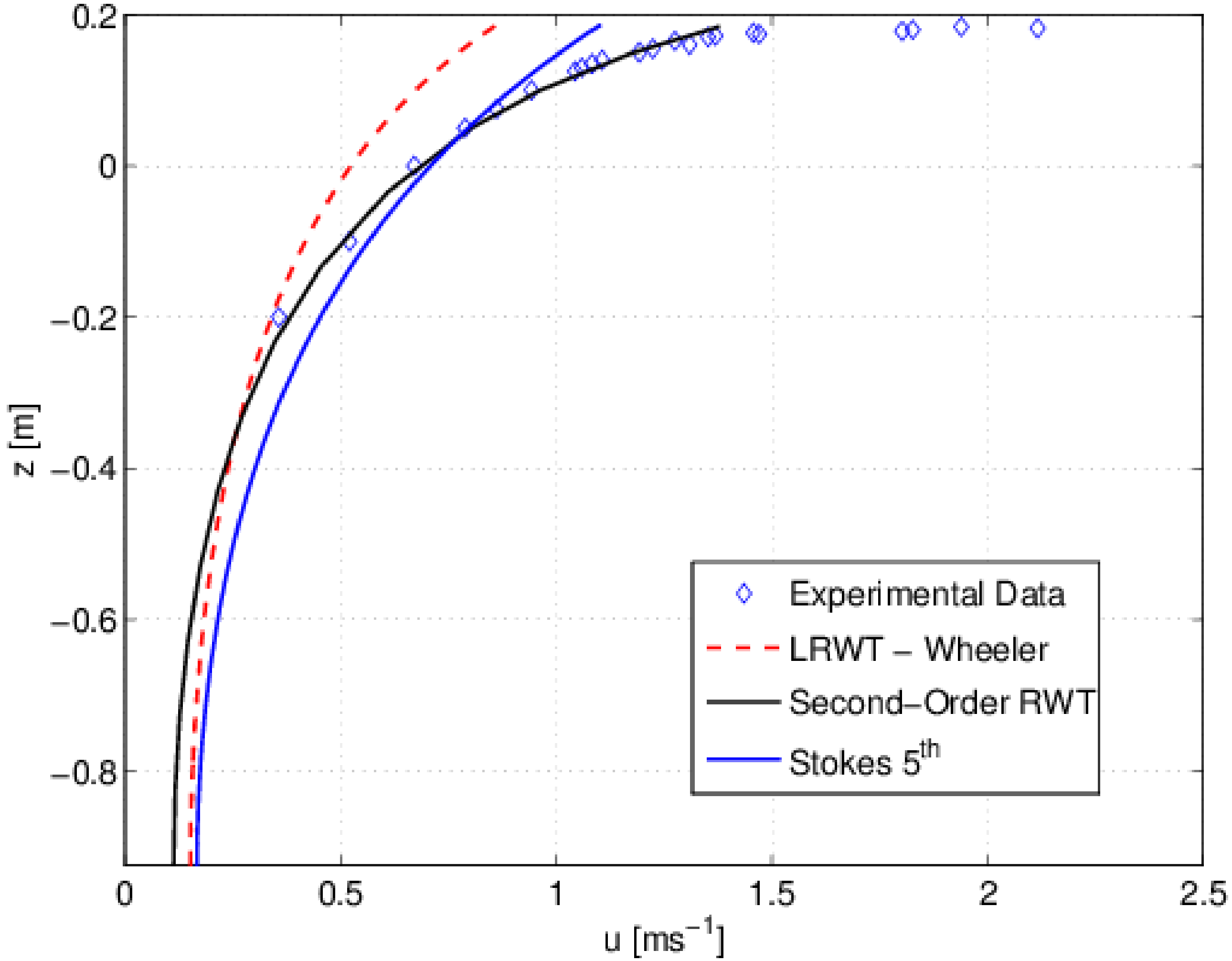
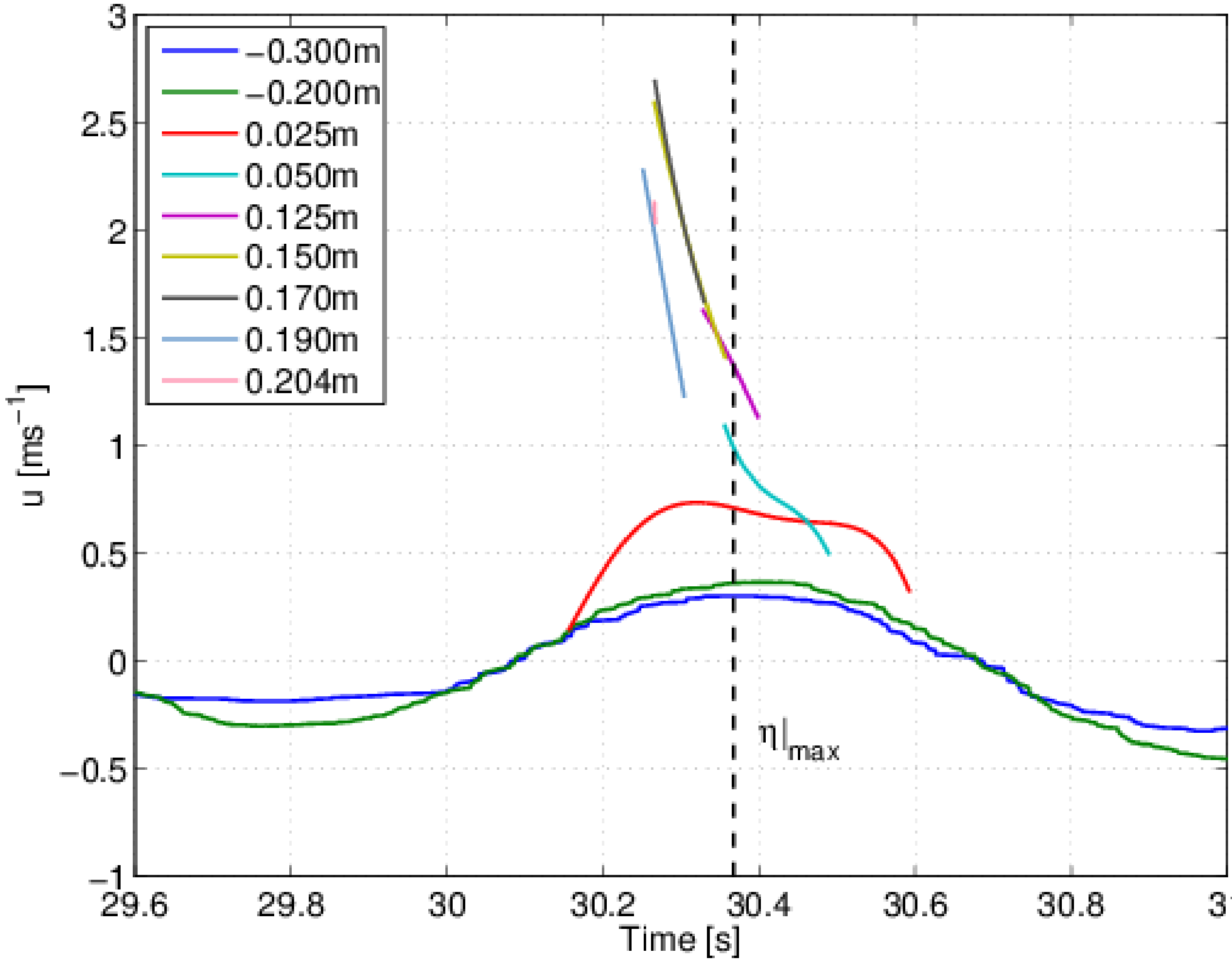
Comparisons to predicted velocities, *u*(*z*): Wave Case 2

– solutions matched to η_{max}



Horizontal velocities, *u*(*t*): Wave Case 3 - various elevations, -30.0m<*z*<20.4m



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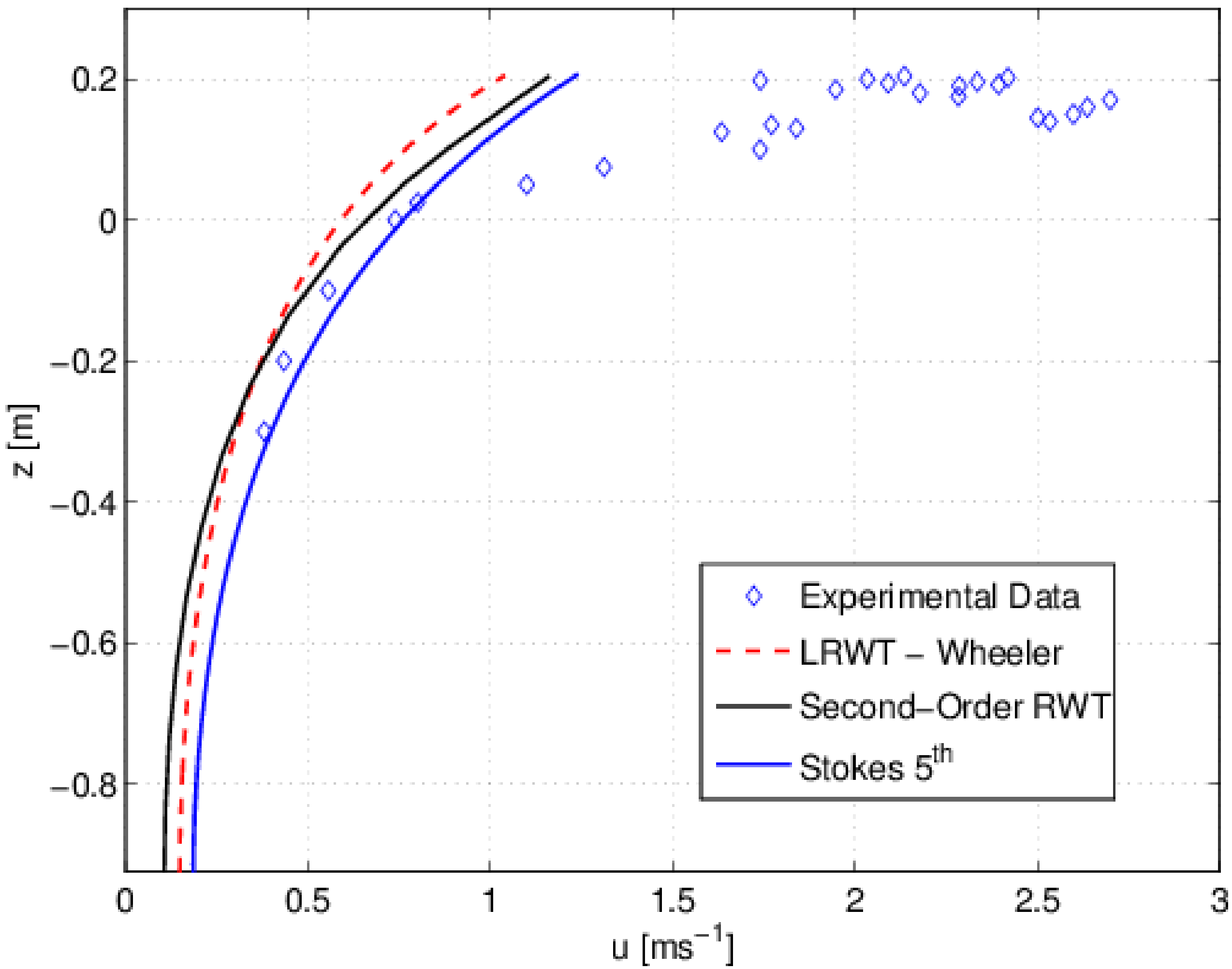
Comparisons to predicted velocities, *u*(*z*): Wave Case 3

– solutions matched to η_{max}



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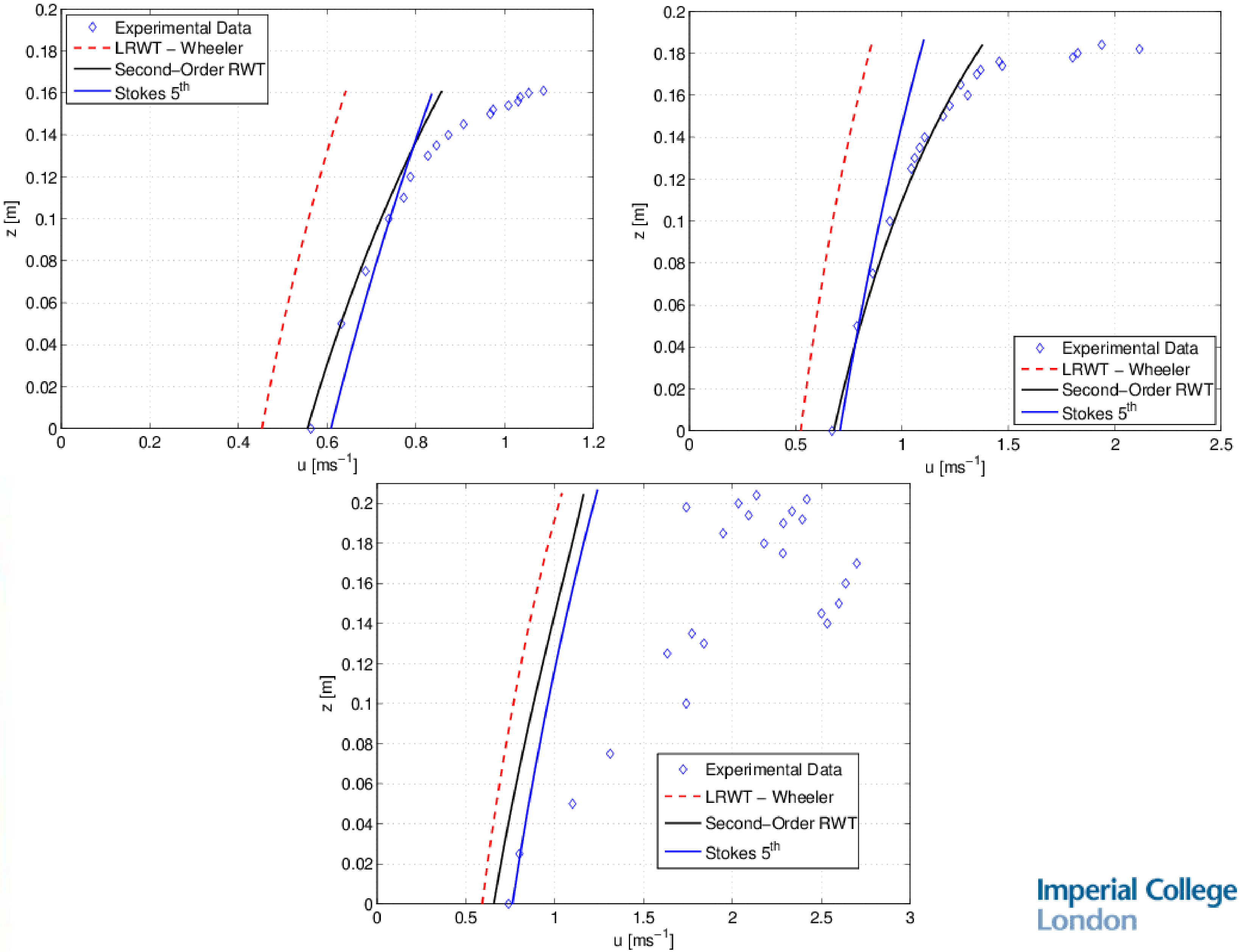
Crest kinematics, *u*(*z*) for *z*>0



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Concluding Remarks #1

Crest height statistics:

- Very long random wave simulations undertaken
- Significant departures from existing $O(a^2)$ design solutions
- Emphasised the importance of:
  - Nonlinear amplifications (beyond 2nd order)
  - Wave breaking
- Critically dependent upon:
  - Sea state steepness ($\frac{1}{2}H_s k_p$)
  - Directional spread
  - Effective water depth ($k_p d$)
- Largest waves are more uni-directional
- Average shape differs from linear predictions

Rigorous control of generated wave components is essential

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Concluding Remarks #2

Kinematics measurements:

- Detailed observations above SWL
- Highlight the inadequacy of the commonly applied solutions
- u_{max} as $z \rightarrow \eta_c$ significantly increased
  - Relevant for wave slamming & wave-in-deck loading
  - Phase velocity is not an effective upper bound: $u_{max} > c$
- u_{max} for z<SWL reduced
  - Relevant for sub-structure loads & sea bed pipelines
  - Present designs may be overly conservative

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Concluding Remarks #3

Future developments:

Formulation of an empirically based wave model:

- **Wave profiles**
- **Wave crest statistics**
- **Wave kinematics**

Inclusive of nonlinearity, directionality and wave breaking

The true benefits of physical model testing lies in:

- **Its combination with theoretical / numerical models**
- **Not as a simple means of validation / calibration**
- **Rather:**
  - **The provision of the underlying physics**
  - **The identification of critical effects**

Taken together they can provide solutions to some of the most challenging wave & wave-loading problems

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