INTERNATIONAL TOWING TANK CONFERENCE

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GUIDELINE

Updated / Edited by	Approved
Ocean Engineering Committee of 24th ITTC	24th ITTC 2005
Date 2005	Date 2005

Laboratory Modelling of Multidirectional Irregular Wave Spectra

1. PURPOSE OF GUIDELINE

The purpose of this recommended guideline is to ensure that laboratory generated directional waves are modelled and documented according to proper and well defined methods. A practical implementation and use within naval architecture and ocean engineering applications is essential.

It shall also point out particular challenges, limitations and uncertainties inherent in laboratory directional spectrum estimation.

Only the directional characteristics are considered. Thus the modelling of corresponding scalar power spectra, or the resulting wave time series statistics, are not described here, except where they are directly relevant for the directional modelling.

An overview of the most commonly used principles, methods and definitions is given in this procedure, together with some guidelines. It is not the intention to provide particular recipes for all steps in the wave generation and analysis, for which more details can be found in e.g. IAHR (1997).

2. SCOPE

2.1 Use of directional spectra

Real ocean waves are directional (short-crested). However, for practical and simplicity reasons, unidirectional waves have in the past traditionally been modelled in most applications within naval architecture and offshore engineering. Both numerically and experimentally, the generation, analysis and documentation of directional spectra are more complex. Also, the interpretation of model response results may be more challenging. Still, a significant development of experimental facilities and methods has taken place since the 1980's, especially within hydraulic and coastal engineering, and the use is expected to continue to increase. This is also supported by the fact that more ocean field data are becoming available.

The significance of using directional wave modelling, and which characteristics are essential, will depend on the actual application. The normal assumption is that for rigid bodies, wave loads are generally reduced, while for compliant systems, loads may sometimes also increase. The primary directional parameters are considered in many cases to include simply the mean direction, the spreading, and information on possible bimodal peaks. Further details may sometimes be relevant, and plots of estimated spectra often provide helpful information. More detailed experimental and numerical investigations are needed for quantifying these general considerations for a broad range of applications.

In many cases, unidirectional wave modelling will normally be a first step, since this is easier and "cleaner" to compare directly to numerical and theoretical models. Thus there is also a challenge to develop numerical wave and structural response models taking properly into account directional effects, which will be consistent in the comparison to experiments.

2.2 The inherent statistical nature

One of the basic challenges connected with directional spectrum modelling is that, in most applications, the characteristic directional parameters are of a statistical nature, and must be interpreted as such. Thus a "unique" sample estimate is difficult to define; estimated results are inherently subject to statistical errors, and there are basic and practical limitations in what resolution is in fact possible to document from a test. Certain characteristics of the estimates will then also be coloured by the method actually used, which is therefore important to document. This should be kept in mind, and the quantification of such errors and limitations is also a challenge.

3. MAIN DEFINITIONS

Directional frequency spectrum:

The combined frequency/directional spectrum can be written as:

$$S(f,\theta) = D(f,\theta)\, S(f)$$

Here $D(f,\theta)$ is the normalized directional spreading distribution, and $S(f)$ is the scalar power spectrum including all directions. If the spreading is independent of the frequency f, the spreading distribution is simplified as $D(\theta)$. In the following, this shall be assumed unless otherwise is noted. For models of the scalar power spectrum $S(f)$ we refer to the description in ITTC (2002).

Circular moments:

$$C_m \equiv \int_0^{2\pi} D(\theta) \exp(-j\, m\theta)\, d\theta$$

$$\equiv a_m + jb_m \equiv |C_m| \exp(j\varphi_m)$$

Here C_m is the complex moment of order m, a_m and b_m are its real components, and φ_m is its phase. Note that $C_0 = 1$.

Mean direction θ_0:

Two definitions are in common use:

$$\theta_{01} \equiv \arg(C_1) \equiv \arctan(b_1/a_1) \equiv \varphi_1$$

$$\theta_{02} \equiv \tfrac{1}{2} \arg(C_2) \equiv \tfrac{1}{2} \arctan(b_2/a_2) \equiv \tfrac{1}{2}\varphi_2$$

(notice the unit is radians). For symmetric distributions, these are identical. The second definition is less influenced by contributions outside $-\pi/2 > \theta_{02} > \pi/2$.

Spreading parameter σ_θ:

For the standard deviation, two definitions are commonly in use:

$$\sigma_{\theta 1} = [2(1 - |C_1|)]^{1/2}$$

$$\sigma_{\theta 2} = \tfrac{1}{2}[2(1 - |C_2|)]^{1/2}$$

The second definition is less influenced by contributions outside $-\pi/2 > \theta_{02} > \pi/2$. There are also other, model-dependent spreading parameters, defined in connection with actual models in Chapter 4.

A detailed list with additional directional parameters can be found in IAHR (1997).

4. MODELLING OF MULTIDIRECTIONAL IRREGULAR WAVE SPECTRA

4.1 Input spectra and parameters

4.1.1 Uni-modal directional distribution models:

For single-peaked directional spreading, the most commonly used model is the cosine-shaped type, see below. There are also other models suggested in the literature. They are all symmetric, and in analysis of actual measured data it may be difficult to distinguish some from the others, depending on the actual spectral resolution. Thus the main characteristics often reduces to the mean and the spreading (standard deviation or similar). But the actual input model should always be documented.

Cosine model:

$$D(\theta) = A_1 \cos^{2s}[(\theta - \theta_0)/2], \quad -\pi > \theta - \theta_0 > \pi$$

where $A_1 = \{2^{2s-1} \Gamma^2(s+1) / [\pi \Gamma(2s+1)]\}$

Here the exponent s defines the spreading. A similar version often used is:

$$D(\theta) = A_2 \cos^N(\theta - \theta_0), \qquad -\pi/2 > \theta - \theta_0 > \pi/2$$

where $A_2 = \{\Gamma(N+1) / [\sqrt{\pi}\,\Gamma(N+½)]\}$; and N describing the spreading.

Normal (Gaussian) model:

$$D(\theta) = [1/\sqrt{(2\pi)}\sigma_\theta] \exp[-(\theta - \theta_0)^2 / (2\sigma_\theta^2)]$$

The shape is quite similar to the cosine model (for the same standard deviation σ_θ).

Wrapped Normal:

Strictly speaking, the Normal model is defined for an infinite linear domain, and not on a circle. For narrow distributions, this is no practical problem, but a wrapped circular model is basically more correct.

Poisson model:

The Poisson model is the simplest single-peak shape of a “Maximum Entropy” model (see 4.4.2 below). It has a sharper peak but longer tails than the cosine model (for the same standard deviation σ_θ).

Other models:

Some additional models are presented and compared to the above models in an interesting review by Krogstad & Barstow (1999).

4.1.2 Multi-modal distributions:

Multi-modal distributions are normally specified as a combination of uni-modal shapes defined in 4.1.1 above.

4.1.3 Frequency-dependent spreading

Mitsuyasu model

Here a cosine distribution is assumed for each frequency f, but with a frequency-dependent spreading exponent s:

$$s(f) = s_p\ (f/f_p)^5 \qquad \text{for } f < f_p$$

$$= s_p\ (f/f_p)^{-2.5} \qquad \text{for } f > f_p$$

where s_p and f_p are the spreading exponent and the frequency at the spectral peak, respectively.

Wind and swell

Sea states are sometimes defined as a combination of a wind sea component and a long-periodic swell component. They are often specified as a combination of two uni-directional spectra, collinear or in different directions. A more appropriate model is the combination of two separate directional frequency spectra $S(f,\theta)= S_{wi}(f,\theta) + S_{sw}(f,\theta)$. Here the directional distributions of each component may be frequency independent.

4.1.4 Record duration

The accuracy and directional resolution in the final analysis increases with increasing record duration. In field data, stationary records longer than 1 hour are rarely available, but for laboratory modelling, longer records, e.g. three hours (full scale), are recommended.

4.1.5 Stationarity

It is generally assumed that stationary conditions are to be modelled. However, if modelling of time-varying conditions is wanted, it can be specified by a sequence of shorter stationary conditions.

4.2 Generation

4.2.1 Synthesizing method

There are basically two different philosophies for the synthesization of multidirectional irregular wave input signals: Single-summation and Double-summation.

Single-summation method

Control signals to the wavemaker are made to produce wave elevation time series signals $\eta(\boldsymbol{r},t)$ at a location $\boldsymbol{r}\equiv(x,y)$ according to a single summation over NF discrete frequencies f_m:

$$\eta(\boldsymbol{r},t) = \sum_{m=-NF}^{NF} F_m \cdot \exp[\,j(2\pi f_m t - \boldsymbol{k}(f_m)\bullet\boldsymbol{r})]\ \Delta f$$

Here F_m is a complex amplitude with a uniformly distributed random phase and a modulus given by:

$$|F_m|^2\ \Delta f = S(f); \quad \text{or} \quad \mathrm{E}[|F_m|^2]\ \Delta f = S(f)$$

(the latter formulation means statistical average).

$\boldsymbol{k}(f_m)$ = wave vector, one for each frequency, with a direction θ_m randomly drawn from the actual input distribution model, and with a modulus $|\boldsymbol{k}_m|$ = $2\pi/\lambda_m$, λ_m = wave length. The summation is normally done by FFT, with typically around 10000 frequencies depending on the record length.

Double-summation method

Control signals are generated according to a double summation over NF frequencies f and ND directions θ:

$NF \quad ND$

$$\eta(\boldsymbol{r},t) = \sum_{m=-NF} \sum_{n=1} F_{mn} \cdot \exp[\, j(2\pi f_m t - \boldsymbol{k}_{mn} \bullet \boldsymbol{r})]\, \Delta f \cdot \Delta\theta$$

Here F_{mn} is a complex amplitude with a uniformly distributed random phase and a modulus given by:

$$|F_{mn}|^2 \, \Delta f \, \Delta\theta = S(f) \cdot D(f,\theta)$$

Here $\boldsymbol{k}(f_{mn})$ = wave vector, many for each frequency, with a direction θ_{mn} randomly drawn from the actual input distribution model, and with a modulus $|\boldsymbol{k}_{mn}| = 2\pi/\lambda_m$, λ_m = wave length. The number ND of directions per frequency must be high in order to avoid non-ergodic conditions in the sea. Experience has shown that ND=100 works fine. The final summation over frequency f is normally done by FFT, with typically around 10000 frequencies depending on the record length.

The double-summation method generates a wave field with “natural” statistical variations in space. The single-summation method gives a constant, or deterministic, spectrum $S(f)$ in space (if possible laboratory reflections and diffraction are disregarded).

4.3 Laboratory measurement

4.3.1 Wave elevation array

The most commonly used measuring device for estimation of directional spectra in a laboratory wave basin is an array of staffs for recording of the wave elevation $\eta(\boldsymbol{r},t)$, arranged in a particular manner. The optimal range for the inter-spacing between the staffs is more or less given by the actual wave length range - thus it is typically a fraction (≈1/10 – 1/2) of the dominant wavelengths. Too coarse arrays lead to aliasing effects, while too small arrays give very small signals relative to noise. The detailed array arrangement can vary from basin to basin – linear array as well as circular (including triangular) configurations are frequently in use. The spatial resolution of the estimates generally increases by increased number of staffs, but this rule must also be seen in relation to basic limitations due to statistical variability from short records etc.

4.3.2 Particle velocities

Another common laboratory method is the combination of the horizontal particle velocity components (u_x & u_y) and elevation measurements at the same location. This resembles, in principle, measurements by pitch-and-roll buoys used in the field. The maximum possible directional resolution is less than what is in principle possible from a large array, since only the first two complex circular moments C_1, C_2 can be derived, but for short-duration records there may be only small differences.

4.3.3 Pressure array

An array of pressure sensors is sometimes used in the same manner as for elevation. In finite and shallow water, the sensors may be mounted on the bottom

4.4 Analysis and documentation

4.4.1 Cross-spectral analysis

The common way to analyse directional spectra from combination of irregular elevation, velocity or pressure records is by means of cross-spectra between available pairs of measuring channels. Inherently, this means that spectral averaging is an essential factor in the analysis, and statistical variability must be taken into account. From the cross-spectrum estimates, directional spectra and/or directional parameters (including some of the circular moments C_m) can then be derived by various methods, some of which are addressed in 4.4.2 below.

4.4.2 Estimation methods

Basic characteristics of some frequently used methods are briefly described in the following. More details are given in IAHR (1997), where also additional methods are described. Furthermore, new versions or updates of the methods are frequently being established. It is important to recall that the various methods are all different ways of trying to extract information from a statistical data set (cross spectra) with sampling variability, and that it may sometimes be hard to judge which is "best". In actual applications, it is important to document which method was used.

Parameters from circular moments

A simple, but robust and often satisfactory way of estimating the directional spectrum from measurements is by simple parameters such as the mean direction θ_0 and standard deviation σ_θ. As shown in Chapter 3, there are two common definition sets for these parameters, based on the estimated first and second circular moment, C_1 & C_2, respectively. Use of both sets can be helpful, since the first moment is more influenced by possible basin reflections.

In addition, information on the deviation from a symmetric cosine (or Normal) distribution shape is also of interest, in particular to check possible secondary peaks in the spectrum. Parameters that partly describe this include the skewness and kurtosis, which can also be derived from C_1 & C_2.

Distributions by parametric models

Plots of estimated directional spectra are often made by assuming a parametric model, e.g. a cosine model or a Normal model. Parameters are estimated and then used in the plotting. The shapes will be influenced by the actual model assumed. The approach is used assuming uni-modal as well as bi-modal spectra. However, for the fine resolution of multi-peaked distributions, other methods may be preferred.

Note that the generation of directional plots directly based on a Fourier sum of the circular moments C_m is not recommended if only C_1 & C_2 are estimated, due to significant truncation effects

Maximum Entropy Method (MEM)

Several of the most commonly used laboratory methods for estimation of distribution shapes are based upon the principle of Maximum Entropy from the theory of probability. The approach makes use of the similarity between the directional distribution function and a probability density function. Various types have been developed since the 1980's; some of them are quite advanced. They seem to reproduce multiple spectral peaks reasonably well, but as for parametric models (and all other methods as well), the estimates will be coloured by the actual method and the way it is applied. Some versions may produce too high and too many peaks in the spectra, although the most advanced versions are more reliable.

Maximum Likelihood Method (MLM)

Another widely used group of methods is based on the principle of Maximum Likelihood, originally applied in seismic detection. It is normally easier to use and more computationally efficient than MEM.

Bayesian method

This is based on the Bayesian technique in probability theory. It is relatively complex in numerical implementation, but at the same time it is basically a powerful method and takes consistently into account the statistical nature of the estimation problem. Any kind of single- or multi-peaked spectrum can be analysed. It is best fitted for use with multi-gauge arrays, and not so well for single-point estimation. No a priori shape or shape characteristics is inherently assumed in the Bayesian principle.

4.4.3 Multi-modal peaks; directional resolution

It should be noted that regardless of the estimation methods described above, the real resolution of multiple peaks is also limited by the statistical errors in the cross-spectral estimates, given by the record length. Thus testing the method using "ideal" cross-spectra only is helpful, but should be accompanied by tests on "real" records.

4.5 Spatial homogeneity; reflections

In laboratory modelling, there are two basic challenges in generating a spatially homogenous multidirectional sea state. First, the finite lengths of multi-flap wave-makers mean that the range of possible directions at a given point in space may depend on the actual location, at least at the outer regions of the field. There are methods in order to reduce these effects, e.g. by making use of sidewall reflections. Furthermore, reflections from beaches and sidewalls may also represent a problem, and one should try to keep these low. In any case, it will be helpful to document the magnitudes of the above effects.

5. KEY PARAMETERS

See Chapter 3 – Definitions.

6. REFERENCES

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