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Guide to the Expression of Uncertainty in Experimental Hydrodynamics

1. PURPOSE OF PROCEDURE

This procedure is a summary of the guidelines for evaluation and expression of uncertainty in measurements for naval architecture experimental measurements, offshore technology testing, and experimental hydrodynamics. It is based on the comprehensive International Organization for Standardization (ISO) Guide to the Expression of Uncertainty in Measurement, also called GUM, now JCGM (2008a). Other relevant references include Taylor and Kuyatt (1994), AIAA S-071A-1999, AIAA G-045-2003, and ASME PTC 19.1-1998. Kacker et al. (2007) is a recent description of the evolution of the ISO GUM. The International Vocabulary of Basic and General Terms in Metrology or VIM (JCGM, 2008b) gives the definitions of terms relevant to the field of uncertainty in measurements. Four procedures for Resistance Testing (ITTC Procedure 7.5-02-01-02, 2008a), Calibration Uncertainty (ITTC7.5-01-03-01 Procedure , 2008b), Laser Doppler Velocimetry (ITTC Procedure 7.5-01-03-02, 2008c), and Particle Imaging Velocimetry (ITTC Procedure 7.5-01-03-03, 2008d) are examples for direct application of the guidelines outlined in this procedure.

2. SCOPE

This procedure is concerned mainly with the expression of uncertainty in the measurement of a well-defined physical quantity (called the measurand[1]) that can be characterized by a unique value. If the measurement of interest can be represented only as a distribution of values or it depends on other parameters, such as time, then the definition of measurand should include a set of quantities, which describes that distribution or that dependence.

In addition to uncertainty in measurements, this procedure is applicable to evaluation and expression of uncertainties associated with conceptual design, set up of actual experiments, methods of measurements, instruments calibrations, and Data Acquisition Systems (DAS). A general guideline is provided for the evaluation and expression of uncertainty in measurements, rather than a description of the details of a specific experiment. Therefore, development of procedures from this general guideline is necessary where the uncertainty in specific experiments is evaluated. Examples include the specific procedures for LDV and PIV measurements ITTC (2008b, c).

This procedure does not discuss how the uncertainty of a particular measurement result may be used for different purposes, such as drawing conclusions about the compatibility of the measurement result with other similar results, establish the tolerance limits in a given manufacturing process, or decide if a certain course of action may be safely taken. The use of uncertainty results to those ends is not within the scope of this procedure.

3. GENERAL

The word “uncertainty” means doubt, and therefore in its broadest sense “uncertainty of a measurement” means a “doubt about the validity of the result of that measurement”. The concept

[1] The definition of measurand is given in the following sections

of "uncertainty" as a quantifiable attribute is relatively new in the history of measurement. However, concepts of "error" and "error analysis" have long been a part of measurements in sciences, engineering, and metrology. When all of the known or suspected components of an error have been evaluated, and the appropriate corrections have been applied, an uncertainty still remains about the "truthfulness" of the stated result that is a doubt about how well the result of the measurement represents the "value" of the quantity being measured. The expression "true value" is not used in this procedure since the true value of a measurement may never be known.

4. SYMBOLS AND DEFINITIONS

The symbols used in this procedure are the same as those used in Annex J of JCGM (2008a). The basic and general definitions of metrology terms relevant to this procedure are given in the International Vocabulary for Metrology (JCGM, 2008b). Among these are definitions for terms such as measurand, error, uncertainty, and other expressions used routinely when performing uncertainty analysis on a measurement.

The difference between the definition of "repeatability of measurement results" and that of "reproducibility of measurement results" is important. The conditions for repeatability are:

a) The same measurement procedure
b) The same measuring instrument used under the same test "environmental" conditions
c) The same location (laboratory or field location)
d) Repetition over a short period of time, roughly, tests are performed in the same day

The term reproducibility of measurement results" is used when one or more of the above four repeatability conditions are not met. Examples include a different observer, a different test crew, a different laboratory, different environment such as laboratory room temperature, different test conditions, or different day. Usually, reproducibility has a higher uncertainty than repeatability.

4.1 Result of a measurement

The objective of a measurement is to determine the value of the measurand that is the value of the particular quantity to be measured. A measurement begins with an appropriate specification of the measurand, the method of measurement, and the measurement procedure. The result of a measurement is only an approximation or an estimate of the value of the true quantity to be measured, the measurand. Thus, the result of a measurement is complete only when accompanied by a quantitative statement of its uncertainty.

4.2 Measurement equation

The quantity Y being measured, defined as the measurand, is not measured directly, but it is determined from N other measured quantities X_1, X_2, …X_N.. Thus, the measurement equation or data reduction equation is

$$Y = f(X_1, X_2, X_3, \cdots X_N) \quad (1)$$

The function f includes along with the quantities $X_{(i,\, i = 1,2,, \ldots N)}$ corrections (or correction factors), as well as quantities that take into account other sources of variability, such as different observers, instrument calibrations, different laboratories, and times at which observations were made. Thus, the function f should express not only the physical law but also the measurement process, and in particular, it should contain all quantities that can contribute to the uncertainty of the measurand Y.

An estimate of the measurand (Y) is denoted by (y) and is obtained from equation (1) with the estimates x_1, x_2, ..., x_N for the values of the N quantities X_1, X_2, ..., X_N.. Therefore, the output estimate (y) becomes the result of the measurements:

$$y = f\left(x_1, x_2, x_3, \cdots x_n\right) \tag{2}$$

As an example, typical data reduction equations for propulsion performance from ITTC Procedure 7.5-02-03-01.1 (2002) are as follows:

Reynolds number:

$$Re_D = f(\rho, V, D, \mu) = \rho VD / \mu \tag{3}$$

Advance ratio:

$$J = f(V, n, D) = V / (nD) \tag{4}$$

Thrust coefficient:

$$K_T = f(T, \rho, D, n) = T / (\rho\, D^4 n^2) \tag{5}$$

Torque coefficient:

$$K_Q = f(Q, \rho, D, n) = Q / (\rho\, D^5 n^2) \tag{6}$$

where Q, T, ρ, μ, D, and n are torque (N.m), thrust (N), mass density of water (kg/m^3), viscosity of water (kg/m-s), propeller diameter (m), and rotational rate (1/s), respectively, and density and viscosity are functions of the temperature, t.

Therefore, an estimate for K_Q is obtained from estimates of the quantities Q, ρ, D, and n, while the estimates for K_T are obtained from quantities T, ρ, D, and n. The estimates for each quantity Q, T, ρ, D can be obtained from direct measurements or can be function of other quantities. The uncertainty in a measurement y, denoted by $u(y)$, arises from the uncertainties $u(x_i)$ in the input estimates x_i in equation (2). For example in equations (5) and (6), the uncertainties in K_Q and K_T are due to uncertainties in the estimations of Q, T, ρ, D, and n.

5. UNCERTAINTY CLASSIFICATION

JCGM (2008a) classifies uncertainties into three categories: Standard Uncertainty, Combined Uncertainty, and Expanded Uncertainty.

5.1 Standard uncertainty (*u*)

Uncertainty, however evaluated, is to be represented by an estimated standard deviation. This is defined as "standard uncertainty" with the symbol "*u*" and equal to the positive square root of the estimated variance.

The standard uncertainty of the result of a measurement consists of several components, which as per le Comité International des Poids et Mesures (Giacomo, 1981) can be grouped into two types. They are: Type A uncertainties and Type B uncertainties. Either type depends on the method for estimation of uncertainty.

- Type A: Uncertainty components obtained using a method based on statistical analysis of a series of observations.
- Type B: Uncertainty component obtained by other means (other than statistical analysis). Prior experience and professional judgements are part of type B uncertainties.

The purpose of Type A and Type B classification is a convenience for the distinction between the two different methods for uncertainty evaluation. No difference exists in the nature of each component resulting from either type of evaluation. Both types of uncertainties are based on probability distributions and the uncertainty components resulting from both types are quantified by standard deviations.

5.2 Combined standard uncertainty (u_c)

Combined standard uncertainty of the result of a measurement is obtained from the uncertainties of a number of other quantities. The combined uncertainty is computed via the law of propagation of uncertainty, which will be described in detail later in this procedure. The result is different if the quantities are correlated or uncorrelated (independent).

5.3 Expanded uncertainty (U)

Mathematically, expanded uncertainty is calculated as the combined uncertainty multiplied by a coverage factor, k. The coverage factor, k, includes an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

Thus, the numerical value for the coverage factor k should be chosen so that it would provide an interval $Y = y \pm U$ corresponding to a particular level of confidence. In experimental hydrodynamics, k corresponds usually to 95% confidence. All ITTC results will be reported with an expanded uncertainty at the 95 % confidence level.

The GUM indicates that a simpler approach is often adequate in measurement situations, where the probability distribution of measurements is approximately normal or Gaussian. If the number of degrees of freedom is significant ($\nu > 30$), the distribution may be assumed to be Gaussian, and k will be evaluated as 2. This assumption produces an interval ($Y = y \pm U$) having a level of confidence of approximately 95%. For a small number of samples, the inverse Student t at the 95 % confidence level is recommended. The Student t at the 95 % confidence level is shown in Figure 1, where the number of degrees of freedom is $\nu = n - 1$.

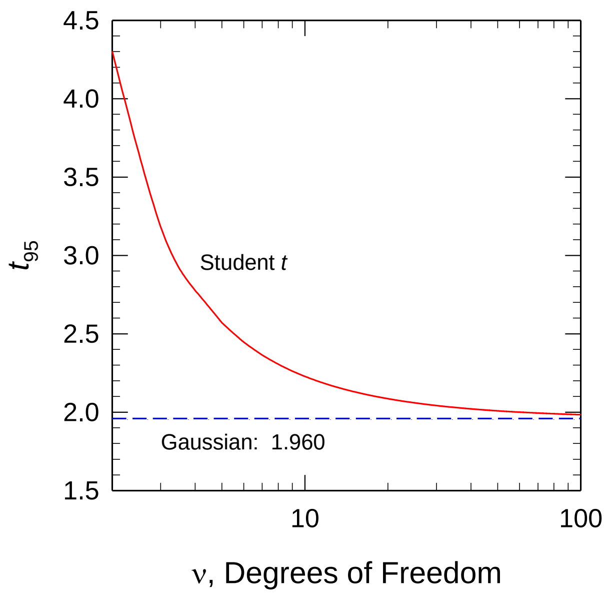

Figure 1: Inverse Student t at 95 % confidence level.

6. EVALUATION OF STANDARD UNCERTAINTY

6.1 Evaluation of uncertainty by Type A method

The best available estimate of the expected value of a quantity “q” that varies randomly and for which “n” observations have been obtained under the same conditions of repeatability is the arithmetic mean or average:

$$\bar{q} = (1/n)\sum_{k=1}^{n} q_k \tag{7}$$

Each individual observation has a different value from other observations due to the random variations of the influence quantities, or random effects. For a DAS, the data, q, is collected as a time series of a uniform sample interval of n samples. The mean value of the time series is then computed from equation (7).

The experimental variance of the observations, which estimates the variance of the normal probability distribution of "q" is:

$$s^2 = [1/(n-1)]\sum_{k=1}^{n}(q_k - \bar{q})^2 \tag{8}$$

This estimate of variance and its positive square root (s), termed the experimental standard deviation, characterize the variability of the observed values of q, or more specifically the dispersion of the values (q_k) about their mean.

For a stationary time series, the uncertainty of the mean value is dependent on the correlation in the signal. If there is no correlation (white noise) then the standard uncertainty can be estimated with:

$$u(\bar{q}) = s/\sqrt{n} \tag{9}$$

where n is the number of repeated observations, for a single measurement n = 1.

In a calibration test set-up, the variation in the measurement signal is mainly determined by the noise level of the DAS, which can be considered as white noise. For n = 100, the standard deviation of the mean from equation (9) is then a factor 10 smaller than the sample standard deviation. Consequently for high quality instrumentation in well controlled conditions, the Type A uncertainty is usually small in comparison to the Type B uncertainty.

In many experiments, however, measurement signals are oscillatory and thus contain correlation. Brouwer et al. (2013) give two methods to determine the uncertainty of the mean value for oscillatory time series. The first is a more elaborate method, based on the auto-correlation function of the signal. The second is a more easy and transparent method, based on splitting the signal into a number of equally-sized segments.

6.2 Evaluation of uncertainty by Type B method

Type B evaluation of standard uncertainty is usually based on judgment from all relevant information available, which may include:

- Previous measurement data,
- Experience and knowledge of the behaviour of relevant materials and/or instruments,
- Manufacturer's specifications,
- Data provided in calibration and other reports, which must be traceable to National Metrology Institutes (NMI), and
- Uncertainties assigned to reference data taken from handbooks. Typical examples in naval hydrodynamics include values obtained from equations for water mass density, viscosity, and vapour pressure from ITTC Procedure 7.5-02-01-03 (2011).

The proper use of the pool of available data and information for a Type B uncertainty requires an insight based on experience and general knowledge. It is skill that can be learned with practice. The Type B evaluation of standard uncertainty may be as reliable as a Type A uncertainty, especially in a measurement situation where a Type A evaluation is based on a comparatively small number of statistically independent observations. In general, all final results by the Type B method should be traceable to an NMI. Other methods may be applied in the design stages of a test or experiment.

7. EVALUATION OF COMBINED UNCERTAINTY

Combined uncertainty is evaluated by the law of "propagation of uncertainty". The general equation for combined standard uncertainty of a measurement result y, designated by $u_c(y)$, is from the JCGM (2008a):

$$u_c^2(y) = \sum_{i=1}^{N} (\partial f / \partial x_i)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\partial f / \partial x_i)(\partial f / \partial x_j) u(x_i, x_j) \tag{10a}$$

where N is the total number of input quantities from observations.

Equation (10a) is based on a first-order Taylor series approximation of the measurement equation and its estimate. The partial derivatives of f with respect to x_i and x_j are called sensitivity coefficients c_i and c_j:

$$c_i = \partial f / \partial x_i \ , \ c_j = \partial f / \partial x_j \tag{10b}$$

The general equation accounts for standard uncertainties in both uncorrelated (independent) and correlated measurement quantities (x_i and x_j). If the input quantities are correlated or dependent on each other, their degree of correlation is represented by the correlation coefficient $r(x_i , x_j)$:

$$r(x_i, x_j) = u(x_i, x_j) / [u(x_i)\ u(x_j)] \tag{10c}$$

The values for the correlation coefficient are symmetric $r(x_i , x_j) = r(x_j , x_i)$, their values range is: $-1 \le r\ (x_i\ , x_j) \le +1$. When equation (10a) is re-written in terms of sensitivity and correlation coefficients, it becomes:

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j u(x_i) u(x_j) r(x_i, x_j) \tag{11}$$

When the input quantities x_i and x_j are uncorrelated (independent), then $r(x_i , x_j) = 0$, and the total combined standard uncertainty is the square root of the sum of the squares of standard uncertainties:

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2\ u^2(x_i) \tag{12}$$

Essentially, equation (12) is the estimated standard deviation of the result for perfectly uncorrelated input quantities. Equation (12) is the most commonly applied version of the law of propagation of uncertainty.

If the input quantities x_i and x_j are fully correlated, then $r(x_i, x_j) = 1$ and the total combined standard uncertainty is simply the linear sum of the standard uncertainties.

$$u_c(y) = \sum_{i=1}^{N} c_i u(x_i) \tag{13}$$

The most common application of this equation in experimental hydrodynamics is the calibration of force with mass. Since the masses are calibrated against the same reference standard, the uncertainties of the masses are correlated. Therefore the combined uncertainty is the sum of the uncertainties of the individual masses.

8. EVALUATION OF EXPANDED UNCERTAINTY

The combined standard uncertainty $u_c(y)$ is universally applied in the expression of the uncertainty of a measurement result. Expanded uncertainty, U, from the combined uncertainty $u_c(y)$ multiplied by a coverage factor, k, is:

$$U = k\, u_c(y) \tag{14}$$

The result of a measurement should be expressed as $Y = y \pm U$, or the best estimate of the value attributable to the measurand Y is between (y - U and y + U). The interval $y \pm U$ may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to Y.

From practical viewpoint, in experimental hydrodynamics and flow measurements, an interval with a level of confidence of 95% (1 chance in 20) is justifiable. If a normal probability density function (pdf) for the measurement result is assumed, then the value of 2 for the coverage factor is applied for the 95% confidence level for an acceptable number of repeated observations.

Theoretically for specification of the value for the coverage factor for a specific level of confidence, detailed knowledge of the probability distribution function of the measurement result and its combined standard uncertainty are needed. In most towing tank and water tank experiments, the t-distribution may be assumed for a small number of observations, and the value for coverage factor can be obtained directly from the plot in Figure 1. If the number of degrees of freedom is high enough (ν > 30), the Student t becomes very close to Gaussian. For ν =30, $k = t_{95}$ = 2.042, in comparison to the Gaussian value of 1.960.

If the probability distribution functions of the input quantities $X_{(i,\, i\, =\, 1,2,,\, \ldots N)}$ upon which the measurand Y depends are normal, then the resulting distribution of Y will also be normal. If the distribution of the input quantities X_i, is not normal distributions, the Central Limit Theorem allows the mean value of Y to be approximated by a normal distribution from IS0 GUM (1995).

9. SENSITIVITY COEFFICIENTS

A numerical method (or computer routine) based on the functional central differencing scheme was proposed by Moffat (1982) for calculation of the sensitivity coefficients c_i in equation (10). The method also is included in the JCGM (2008a). If the uncertainty $u_i(y)$ is represented by the functional difference, Z_i:

$$\begin{aligned} u_i(y) = c_i u(x_i) = Z_i = \\ (1/2)[f\left(x_1, x_2, \cdots x_i + u(x_i), \cdots x_N\right) \\ - f\left(x_1, x_2, \cdots x_i - u(x_i), \cdots x_N\right)] \end{aligned} \tag{15a}$$

Then, the sensitivity coefficients, c_i, are:

$$c_i = Z_i / u(x_i) \tag{15b}$$

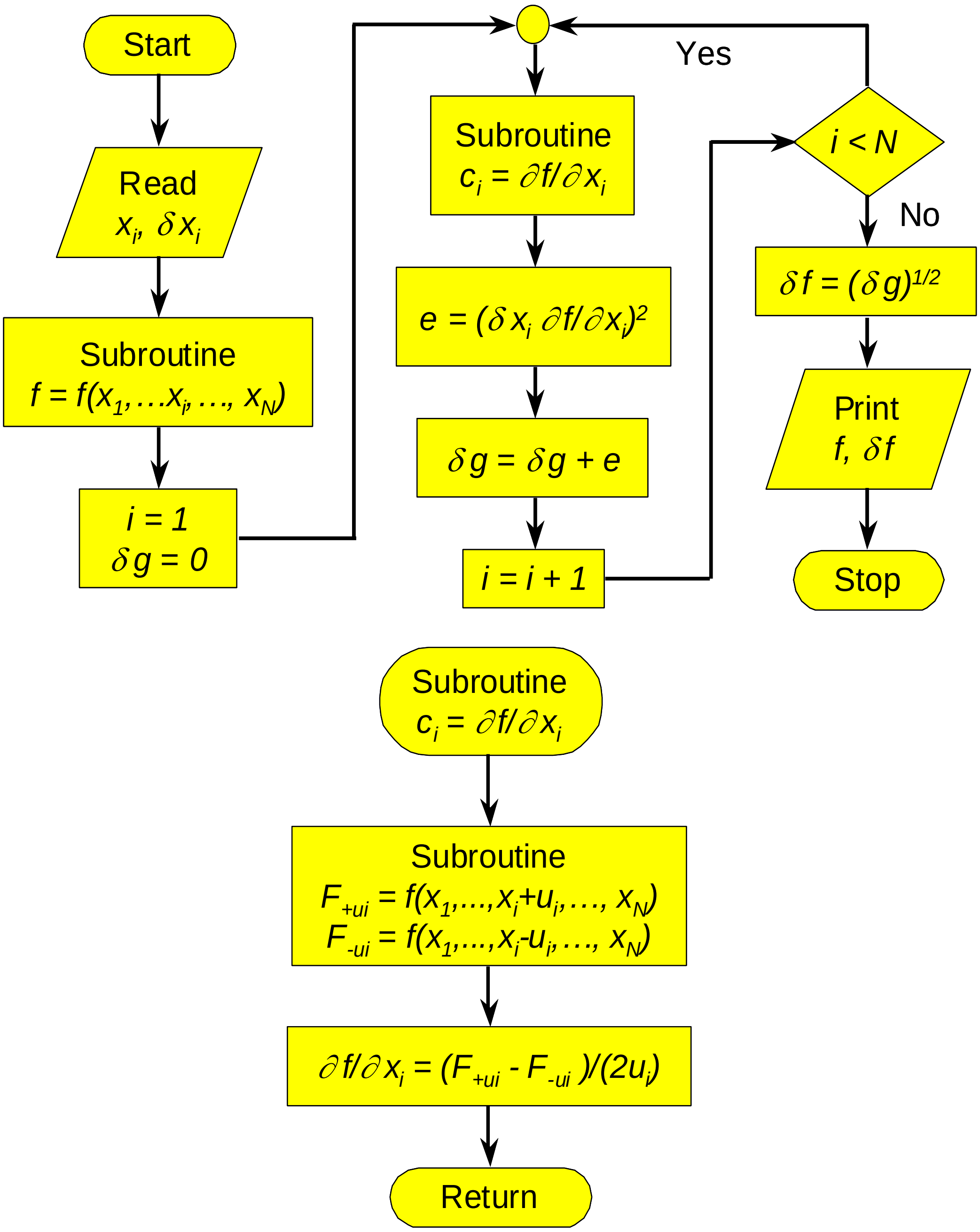

Figure 2: Flow chart for numerical determination of sensitivity coefficients (Moffat, 1982)

A flow chart for the central differencing method is given in Figure 2. The details of the central differencing method were given by Moffat (1982). The method in the program, whose flow chart is in Figure 2, may be described as Central Differencing for Evaluation of Sensitivity Coefficients or "Jitter Program per Moffat, (1982)". It eliminates the need for development lengthy tables of partial derivatives for parameters in data reduction equations.

10. RELATIVE UNCERTAINTY

In hydrodynamics, the data reduction equations are typically a product of terms of the form

$$Y = cX_1^{p1} X_2^{p2} \cdots X_N^{pN} \tag{16}$$

Then, the relative combined uncertainty from equation (12) is

$$[u_c(y)/y]^2 = \sum_{i=1}^{N} [p_i u(x_i)/x_i]^2 \tag{17}$$

where $y \neq 0$ and $x_i \neq 0$.

The following are examples for relative uncertainties of well defined equations in experimental hydrodynamics.

10.1 Propeller equations

The relative uncertainties in the propeller equations are obtained from equation (17) and equations (3) to (6):

Reynolds number:

$$[u_c(Re_D)/Re_D]^2 = (u_\rho/\rho)^2 + (u_V/V)^2 + (u_D/D)^2 + (-u_\mu/\mu)^2 \tag{18}$$

Advance ratio:

$$[u_c(J)/J]^2 = (u_V/V)^2 + (-u_n/n)^2 + (-u_D/D)^2 \tag{19}$$

Thrust coefficient:

$$[u_c(K_T)/K_T]^2 = (u_T/T)^2 + 4(u_n/n)^2 + (\partial\rho/\partial t)^2 (u_t/\rho)^2 + 16(u_D/D)^2 \tag{20}$$

Torque coefficient:

$$[u_c(K_Q)/K_Q]^2 = (u_Q/Q)^2 + 4(u_n/n)^2 + [(\partial\rho/\partial t)(u_t/\rho)]^2 + 25(u_D/D)^2 \tag{21}$$

10.2 Resistance equation

The data reduction equation for total resistance is from ITTC Procedure 7.5-02-01-02 (2008a)

$$C_T = 2R_T/(\rho S V^2) \tag{22}$$

The relative uncertainty in C_T is then:

$$[u_c(C_T)/C_T]^2 = (2u_V/V)^2 + (u_{RT}/R_T)^2 [(\partial\rho/\partial t)(u_t/\rho)]^2 + (u_S/S)^2 \tag{23}$$

11. SIGNIFICANT DIGITS

For a measurement result, the number of digits after the decimal point should be the same as those after the decimal point reported for its associated combined uncertainty u_C.

In general, the uncertainty should be reported to 2 significant digits. For example: Consider a mass (m = 100.2147 ± 0.0079) kg, where the number of digits after the symbol ± is the numerical value of expanded uncertainty (U). The expanded uncertainty is computed from value of the combined uncertainty, u_C = 0.0035 kg and a coverage factor, k = 2.26 where k is based on the *t*-distribution for $\nu = n - 1 = 9$ degrees of freedom and an interval estimated with a level of confidence of 95 percent. The number of digits after the decimal point is 5, in both the estimated value for *m* and its associated combined uncertainty ($u_c = 0.0035$ kg).

12. OUTLIERS

Sometimes data occurs outside the expected range of values and should be excluded from the calculation of the mean value and estimated uncertainty. Such data are referred to as outliers. If an outlier is detected, the specific cause should be identified before it is excluded. Several methods may be applied in the determination of outliers. Additional information on outliers as applied to calibration is contained in the procedure on Calibration Uncertainty in ITTC (2008b).

12.1 Hypothesis *t*-test

The conventional method for outliers is the *t*-test from hypothesis testing. The details of the methodology may be found in a standard statistics text such as Ross (2004). Then the *T* statistic is defined as:

$$T = (q_i - \bar{q})/s \qquad (24)$$

Accept as valid if $T \le t_{95,n-1}$

Reject as outlier if $T > t_{95,n-1}$

That is, *q* is an outlier, where $t_{95,n\text{-}1}$ is the inverse Student *t* for a 2-tailed probability density function (pdf) at the 95 % confidence level and the cumulative probability is $p > 0.975$. In practical terms, any *T* that exceeds 2 may be considered as an outlier at the 95 % confidence level.

12.2 Chauvenet's criterion

A less stringent test is given by Chauvenet's criterion from Coleman and Steele (1999). By this criterion a data point is rejected as an outlier if the inverse Gaussian, Z, for a 2-tailed pdf is

$$Z = (q_i - \bar{q})/s \qquad (26)$$

Reject if $Z > z_{1-1/(4n)}$

As an example for $n = 10$, then $p > 0.975$ and $z = 1.960$. A plot of Chauvenet's criterion is presented in Figure 3.

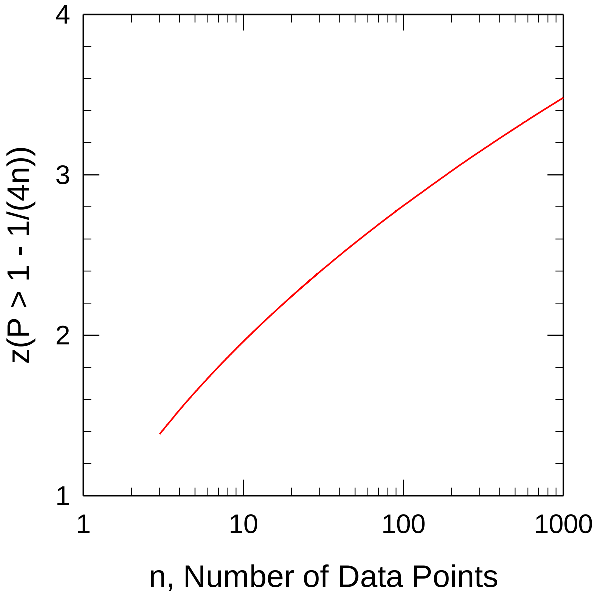

Figure 3: Chauvenet's rejection criterion

12.3 Higher-order central moments

Another useful concept for outliers is the higher-order central moments, which are defined from Papoulis (1965) as

$$m_p / s^p = [1/(ns^p)] \sum_{i=1}^{n} (q_i - \bar{q})^p \qquad (27)$$

where they are non-dimensionalized with the standard deviation. The central moments may also be applied as a measure of how close to Gaussian a process is.

For a Gaussian pdf, the higher-order central moments are as follows:

For p odd:

$$m_{2j-1} / s^{2j-1} = 0 \tag{28}$$

For p even

$$m_{2j} / s^{2j} = 1 \times 3 \times 5 \times \cdots \times (2j-1) \tag{29}$$

The commonly applied higher-order central moments are the third-order, defined as skewness factor (S) and the fourth-order defined as flatness factor (F). Thus for a Gaussian pdf, S = 0 and F = 3. The fourth-order moment has also been defined as kurtosis, K, where $K = F – 3 =$ 0. For significant deviations from these values, either the pdf is non-Gaussian or contains outliers. Any time series with on the order of F > 5 to 10 should be investigated for outliers. With a DAS, the mean, standard deviation, skewness factor, and flatness factor may be computed routinely in almost real time for a time series.

13. INTER-LABORATORY COMPARISONS

As a better measure of a laboratory's uncertainty estimates, inter-laboratory comparisons are routinely performed. The method adopted by the NMIs is the Youden plot (1959). The method requires the measurement of 2 similar test articles, A and B, by several laboratories and then plotting the results of A versus B. For a naval hydrodynamics test, the test models (articles) may be 2 propellers in a propeller performance test or 2 ship hulls in a resistance towing test.

A schematic of a Youden plot for flowmeters for the results from 5 laboratories is shown in Figure 4 from Mattingly (2001). In the method, vertical and horizontal dashed lines are drawn through mean values of all laboratories. Then, a solid line is drawn at 45° through the crossing point of the dashed lines.

The data pattern is then as follows:

- NE and SW quadrants, systematic high and low values
- NW and SE quadrants random values both high and low
- Usually elliptic in shape with random values along minor axis and systematic errors along major axis
- Ideally, the pattern should be circular.

The variance of n laboratories normal to the 45° axis is given by

$$s_r^2 = [1/(n-1)] \sum_{i=1}^{n} N_i^2 \tag{30a}$$

and parallel to the axis

$$s_s^2 = [1/(n-1)] \sum_{i=1}^{n} P_i^2 \tag{30b}$$

where s_r and s_s may be interpreted as the random and systematic deviations of the data, respectively, and N_i and P_i are the respective normal and parallel components of the data projected onto the line with the slope of +1. The ratio of these two quantities is then the circularity of the data:

$$c = s_s / s_r \tag{30c}$$

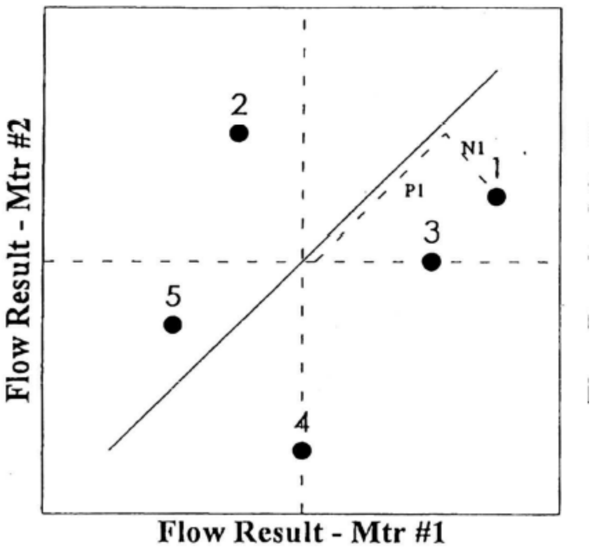

Figure 4: Youden plot for flow meter test

14. SPECIAL CASES

14.1 UA for mass measurements

During calibration of force instruments, such as load cells and dynamometers, the force is changed by addition or removal of weights from the calibration fixture. The total mass is the sum of the individual masses:

$$m = \sum_{i=1}^{N} m_i \tag{31a}$$

The weight set is usually calibrated as a set at the same time against the same reference standard. OIML (2004) and ASTM E740-02 performance specifications recommend that the uncertainty in weights is perfectly correlated. The standard uncertainty in the total mass is from equation (13):

$$u_m = \sum_{i=1}^{N} u_i \tag{31b}$$

The expanded uncertainty for the calibrated mass is required by OIML (2004) to be:

$$U_m = \delta m / 3 \tag{31c}$$

where δm is the rated tolerance.

For many naval hydrodynamics laboratories, the masses have a tolerance of 0.01 %.

14.2 UA for instrument calibration

Electronic instruments must be calibrated by a reference standard that is traceable to an NMI. Such calibration is necessary for conversion of voltage units to physical units. Most instruments in experimental hydrodynamics are highly linear. Consequently, the calibration includes a linear fit of the data.

Two types of instrument calibrations exist. These are end-to-end calibration or bench individual instrument calibrations. For example, end to end means that both the measurement sensor (such as load cell) and Data Acquisition Card (DAC) are calibrated together as one unit. Otherwise, the load sensor and DAC are calibrated separately.

Usually the uncertainty in instrument calibration is associated with the data scatter in the regression fit. The NMI traceable reference standard for the calibration should have an uncertainty that is small in comparison to the uncertainty from the curve fit. A separate Calibration Uncertainty procedure (ITTC, 2008b) describes in detail the uncertainties associated with both linear and non-linear curve fitting.

14.3 Repeat tests

In some cases, the methodology outlined in this procedure does not adequately define the uncertainty of a test. Frequently, tests in naval hydrodynamics contain an uncontrolled element that is not included in the uncertainty estimate. Consequently, repeat tests, at least 10, are suggested for a representative condition as a better estimate of the uncertainty. Ten tests should provide a reasonable estimate of the standard deviation. The standard deviation is computed from equation (8). Since this will provide an estimate for tests which are performed only once, equation (9) should not be applied.

Forgach (2002) provides such an example. In his report, the expanded uncertainty estimate for carriage speed based upon rotation of a metal wheel was ±0.00052 m/s. However, the expanded uncertainty from 23 repeat runs (2 standard deviations) was ±0.0015 m/s or 3 times the uncertainty estimate from the wheel speed. The speed for this case was 2.036 ± 0.0016 m/s (± 0.08 %) for the expanded uncertainty including both the uncertainty in repeat runs and the wheel speed. In this example, the uncontrolled variable was an estimate of the uncertainty contribution from the carriage speed controller, which included a manual setting by a carriage operator.

15. PRE-TEST AND POST-TEST UNCERTAINTY ANALYSIS

Before the first data point is taken in a test, the data reduction equations should be known. A data reduction program for the DAS should include the measurement equations, data for conversion of the digitally acquired data to physical units from calibrations (traceable to an NMI), and finally uncertainty analysis should be included in the data processing codes. The codes should include the details of the uncertainty analysis:

- Elemental uncertainties, $u_i(y) = c_i u(x_i)$ and their relative importance to the combined uncertainty, $u_c(y)$
- Combined and expanded uncertainty, u_c and U
- Calibration factors for conversion from digital units to physical units
- Contributions to the uncertainty by Type A and Type B methods.

A pre-test uncertainty analysis should be performed during the planning and designing phases of the test with the same computer code applied during the test. The pre-test uncertainty will only include Type B uncertainties. In this phase, all elements of the Type B uncertainty should be applied. In particular, manufacturer's specifications may be included for an assessment of adequacy of a particular instrument for the test before the device is purchased. Selection of an instrument may involve economic tradeoffs between cost and performance.

For the post-test uncertainty analysis after the data are acquired, the post-processing code should provide sufficient data on uncertainty analysis for the final report of the test. In this case, data will include results from both the Type A and Type B methods. All of the elemental uncertainties should be based upon measurements that are traceable to an NMI. That is, all measurements should be accompanied by documented uncertainties. These should contain no guesses or manufacturer's specifications unless the manufacturer supplies a calibration certificate that is NMI traceable.

Finally, the contributions of the elemental uncertainties $ui(y)$ should be compared to the combined uncertainty, $u_c(y)$. Such comparison will identify the important contributors to the combined uncertainty. These results should be compared to the pre-test uncertainty analysis. In this manner, the expected performance should be verified. Are the results of the pre-test and post-test uncertainty analysis consistent? Finally, the results should be reviewed for potential improvements or reduction in the uncertainty for future tests.

16. REPORTING UNCERTAINTY

The main directive for reporting uncertainties is that all information necessary for a re-evaluation of the measurement should be available to others when and if needed. When uncertainty of a result is evaluated on the basis of published documents, such as the case of instrument calibration results that are reported on a manufacturer certificate, these publications should be referenced, and insure that they are consistent with the measurement procedure actually used. If experiments are performed with instruments that are subjected to periodic calibration and/or legal inspections, the instruments should conform to the specifications that apply.

In practice, the amount of information necessary to document uncertainties in a measurement result depends on its intended use. The following is a list for the base guideline in reporting uncertainty.

- Describe clearly the method used to obtain the measurement result and its uncertainty.
- List all uncertainty components and document fully how they were evaluated: these are standard uncertainty, combined uncertainty, and expanded uncertainty. Expanded uncertainty should be reported at the 95 % level and the basis of the coverage factor, *k*, documented.
- The final measured values should be documented as $y \pm U$ (U/y in percent, $|y| \neq 0$).
- Present the data and uncertainty analysis in such a way that each of its important steps can be readily followed. The calculation of the reported result can be independently repeated if necessary.
- Give all corrections and constants used in the analysis and their sources. JCGM (2008a) gives specific guidance on how to report the numerical values of a measurement result (y) and its associated standard uncertainties, combined standard uncertainty, and expanded uncertainty.

17. LIST OF SYMBOLS

c_i	Sensitivity coefficient, $c_i = \partial f/\partial x_i$	
C_T	Total resistance coefficient, equation (22)	
D	Diameter of propeller	m
f	Function of measurement variables or data reduction equation	
J	Advance ratio, equation (4)	1
k	Coverage factor, usually 2	1
K_Q	Torque coefficient, equation (6)	1
K_T	Thrust coefficient, equation (5)	1
n	Number of samples or observations	1
n	Also, propeller rotational frequency	Hz
N	Number of input quantitities	1
p	Probability	1
Q	Torque	Nm
r	Correlation coefficient, eq. (10c)	1
R_T	Total resistance	N
Re	Reynolds number, equation (3)	1
s	Standard deviation, equation (8)	
S	Surface area	m^2
t	Water temperature	°C
$t_{p,\nu}$	Inverse Student t	1

T	t-value for hypothesis test	1
T	Also, thrust	N
u	Standard uncertainty, $u = s/\sqrt{n}$	
u_c	Combined standard uncertainty	
U	Expanded uncertainty, $U = ku_c$	
V	Velocity	m/s
μ	Absolute viscosity	kg/(m s)
ν	Degrees of freedom	1
ν	Also, kinematic viscosity, $\nu=\mu/\rho$	m^2/s
ρ	Water density	kg/m^3

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