The Specialist Committee on Uncertainty Analysis

Final Report and Recommendations to the 25th ITTC

1. INTRODUCTION

1.1 Membership and Meetings

The uncertainty analysis committee (UAC) was appointed by the 24th ITTC in Edinburgh, Scotland, 2005, and it consists of the following members shown in Figure 1:

- Dr. Joel T. Park (Chairman): Naval Surface Warfare Center Carderock Division, NSWCCD, West Bethesda, Maryland, USA.
- Dr. Ahmed Derradji-Aouat (Secretary): National Research Council Canada, Institute for Ocean Technology, NRC-IOT, Newfoundland and Labrador, Canada.
- Mr. Baoshan Wu: China Ship Scientific Research Centre, CSSRC, Wuxi, Jiangsu, China.
- Dr. Shigeru Nishio: Kobe University, Faculty of Maritime Sciences, Department of Maritime Safety Management, Kobe, Japan.
- Mr. Erwan Jacquin: Formerly a staff member of the Bassin d'Essais des Carènes, BEC, Val-de-Reuil, France.

Four (4) UAC meetings were held. The host Countries, host laboratories, and dates of the meetings were:

- France, BEC, March 30-31, 2006.
- China, CSSRC, October 23-25, 2006.
- Canada, NRC-IOT, June 7-8, 2007.
- USA, NSWCCD, January 30-February 1, 2008.

After the meeting in China, Mr. Erwan Jacquin left his position at BEC and the UAC. Neither the BEC nor the ITTC representative of Southern Europe appointed a new member to replace him.

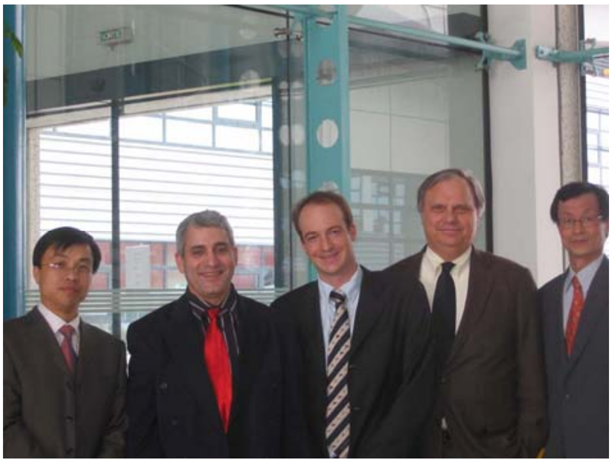


Figure 1 Photograph of Uncertainty Analysis Committee during its first meeting in France at BEC. Viewer's left to right are: Mr. Baoshan Wu (China), Dr. Ahmed Derradji-Aouat (Canada), Mr. Erwan Jacquin (France), Dr. Joel Park (USA), and Dr. Shigeru Nishio (Japan).

1.2 Terms of Reference

From the reference document provided by the 24th ITTC via the Advisory Council (AC), the UAC was tasked to develop 5 new procedures and revise another five (5) existing procedures. A total of 10 procedures were to be completed.

The five new uncertainty analysis procedures were for the following topics:

- Captive model testing
- Free running model testing
- Laser Doppler Velocimetry (LDV)
- Particle Imaging Velocimetry (PIV)
- Full-scale testing

Since the UAC started with only with 5 members, the departure of Mr. E. Jacquin from the BEC, France, led to the decision to eliminate the work for full-scale testing. Additionally, future full-scale test procedures may be derived from the procedures on captive and free-running model tests.

The five existing uncertainty analysis procedures that were to be revised are as follows:

- 7.5-02-01-01 Uncertainty Analysis in EFD, Uncertainty Assessment Methodology (19 pages)
- 7.5-02-01-02 Uncertainty Analysis in EFD, Guidelines for Resistance Towing Tank Tests (5 pages)
- 7.5-02-03-01.2 Propulsion, Performance Uncertainty Analysis, Example for Propulsion Test (26 pages)
- 7.5-02-03-02.2 Propulsion, Propulsor Uncertainty Analysis, Example for Open Water Test (15 pages)
- 7.5-02-07-03.2 Testing and Extrapolation Methods Loads and Responses, Ocean Engineering Analysis Procedure for Model Tests in Regular Waves (8 pages).

ITTC procedure 7.5-02-07-03.2 did not have an uncertainty component; therefore, nothing was available to review. The UAC may provide a document to the appropriate committee so it can be added to that existing procedure.

During the first meeting in France, the UAC concluded that ITTC 7.5-02-01-02 could be eliminated since it provided no new information in support of ITTC 7.5-02-01-02. The UAC subsequently submitted two revised procedures, ITTC (2008a, b). More importantly, during the first meeting in France, the UAC decided to adopt a more inclusive philosophy and follow the guidelines of the ISO (1995), also known as ISO-GUM (Guide to the Expression of Uncertainty in Measurements).

Application of the ISO (1995) to experimental hydrodynamics is a fundamental shift in thinking and in assessing uncertainties from what the ITTC historically had followed. Up to the 24th ITTC in 2005, the ITTC opted for the method of AIAA (1995), which was revised as AIAA (1999) for the development of ITTC UA procedures. AIAA (1999) is for wind tunnel testing. The UA standards for wind tunnel testing were considered applicable to experimental hydrodynamics and tow tank testing.

Starting in 2005 just after the creation of the UAC during the 24th ITTC, ITTC member organizations from geographic areas other than North America have demanded the use of ISO (1995) rather than AIAA (1999) or ASME (2005). Both AIAA and ASME are American organizations, and ISO was viewed as the legitimate international organization for guides and standards development.

Since the procedures for development by the UAC should be consistent with ISO (1995), the review of the 3 procedures 7.5-02-03-01.2, 7.5-2-03-02.2, and 7.5-02-07-03.2 were postponed after completion of ITTC (2008a). The development of the 2 new procedures on LDV and PIV were possible because the committee members were specialists in the subject areas as well as in ISO (1995).

An uncertainty analysis procedure for free-running model testing was not developed. Instead, the UAC provides an example on the application of uncertainty analysis to free-running models in section 12 of this report.

As a consequence for adoption of the ISO (1995) as the basis of ITTC (2008a), all existing and recommended ITTC UA procedures should be reviewed and revised accordingly. The 26th ITTC General and Specialist Committees should harmonize their existing UA procedures with ITTC (2008a). Any new UA procedures should also follow this new procedure as well. The UAC will provide assistance and guidance to various ITTC committees for harmonization of their existing and new procedures. Since ISO (1995) is concerned only with general guidelines for expressing uncertainties in measurements, specific disciplines such as experimental hydrodynamics should produce specific UA procedures and show how the ISO (1995) guidelines are implemented.

1.3 Additional Activities

The UAC proposed a new procedure on instrument calibration, ITTC (2008c). Although the ITTC terms of reference did not include a mandate for such a procedure, a procedure for UA Instrument Calibration is a fundamental extension to the general procedure (ITTC, 2008a).

In addition to the AC mandated tasks, the UAC played a proactive role in interacting and discussing UA related issues with other ITTC committees. Among these committees are the Specialist Committee on Powering, Performance Prediction, the Propulsion Committee, the Manoeuvring Committee, Resistance Committee, and the Seakeeping Committee. Some limited discussions with members of the Specialist Committee on Ice took place.

1.4 Symbols and Definitions

The basic and general definitions for metrology terms in ITTC (2008a) are the same as those given by the International Vocabulary for Metrology (VIM, 2007). This is also an ISO publication from the Bureau International des Poids et Mesures (BIPM) that is complimentary to the ISO (1995). Among these, are definitions for terms such as "measurand", "measurement", "error", "uncertainty", "repeatability", "reproducibility", and other expressions routinely mentioned in ISO (1995).

2. COMPLETED PROCEDURES

Five procedures were completed by the UAC, all based on ISO (1995) guidelines. The five procedures are:

- 7.5-02-01-01. Guide to the Expression of Uncertainty in Experimental Hydrodynamics.
- 7.5-02-01-02. Guidelines for Uncertainty Analysis in Resistance Towing Tank Tests.
- 7.5-01-03-01. Uncertainty Analysis: Instrument Calibrations.
- 7.5-01-03-02. Uncertainty Analysis: Laser Doppler Velocimetry (LDV).
- 7.5-01-03-03. Uncertainty Analysis: Particle Imaging Velocimetry (PIV).

3. STRUCTURE OF THE REPORT

This document is divided into four sections:

- Uncertainty Analysis section that includes general literature for the UA, its history, its importance, and why it is needed.
- Summary for the new and revised procedure completed by the UAC.
- Other activities section that includes interactions with other ITTC committees, discussion of UA application to free running model tests, and UA in fundamental equations for water properties (density, viscosity and vapour pressure).
- Conclusions and recommendations.

4. UNCERTAINTY ANALYSIS

4.1 Brief History of Uncertainty Analysis

Modern uncertainty analysis in North America evolved from a series of papers published by professors and their students from Stanford University, S. R. Kline, R. J. Moffat, and H. W. Coleman. The earliest paper was by Kline and McKlintock (1953). They introduced concepts such as single sample uncertainty, uncertainty interval, and the law of propagation of uncertainty. Kline and McKlintock (1953) also suggested describing the uncertainty with 20 to 1 odds. In current practice, uncertainty estimates are stated at the 95 % confidence level rather than 20 to 1 odds, which are equivalent.

In the international community, a group of experts formulated recommendation INC-1 (1980) "expression of uncertainty in measurements" under the sponsorship of Le Bureau International des Poids et Mesures (BIPM). Historically the creation of the BIPM goes back to Convention of the Metre (Convention du Mètre) 1875, 17 member nations signed a treaty in Paris (now that number is 51 nations are members and 27 states are associate members). The treaty, signed in 1875, gives authority to the General Conference on Weights and Measures (CGPM), the International Committee for Weights and Measures (CIPM) and the International Bureau of Weights and Measures (BIPM) to act in matters of world metrology, particularly concerning the demand for measurement standards of ever increasing accuracy, range and diversity, and the need to demonstrate equivalence between national measurement standards.

The recommendation INC-1 (1980) became the basis of the ISO (1995), which was first published in 1993. The text for the recommendation INC-1 (1980) is included the ISO (1995) in both French (original text) and English and Giacomo (1981).

At the BIPM, the ISO (1995) is the responsibility of the Working Group on the Expression of Uncertainty in Measurement (GUM), which is also known as Working Group 1 (WG1). WG1 is one of 2 working groups within the Joint Committee for Guides in Metrology (JCGM). The second working group (WG2) is responsible for the International Vocabulary of Basic and General Terms in Metrology, known as the VIM. The JCGM took the responsibility for both GUM and VIM documents from ISO TAG 4 (Technical Advisory Group 4). In the near future, the JCGM does not anticipate any revisions to the ISO (1995), but is in the process of providing supplements.

4.2 Basic Principles for the ISO GUM 1995 Uncertainty Analysis Guideline

The original concepts for the expression of uncertainty analysis were relatively simple and are summarized in the following 5 principles Giacomo (1981) and ISO (1995):

Principle 1. The uncertainty results may be grouped in 2 categories called Type A uncertainty and Type B uncertainty. They are defined as follows:

- Type A uncertainties are those evaluated by applying statistical methods to the results of a series of repeated measurements.
- Type B uncertainties are those evaluated by other means. The definition has been further refined by ISO (1995) to include prior experience and professional judgments, manufacturer's specifications, previous measurement data, calibrations, and reference data from handbooks.

Pre-GUM methodologies, the methodologies developed prior to ISO (1995), also classify total uncertainty into two components "random and systematic" in ASME 19.1-2005 or "precision and bias" in AIAA (1999).

No simple correspondence exists between the classification categories A and B and the classifications of the Pre-GUM methodologies. Recommendation INC-1 (1980) indicated that the term 'systematic uncertainty' can be misleading and should be avoided". ASME PTC 19.1-2005 has retained the terms "systematic" and "random" and avoided intentionally the use of terms "bias" and "precision".

Principle 2. The components in type A uncertainty are defined by the estimated variance, which includes the effect of the number of degrees of freedom (DOF).

Principle 3. The components in type B uncertainty are also approximated by a corresponding variance, in which its existence is assumed.

Principle 4. The combined uncertainty should be computed by the normal method for the combination of variances, now known as the law of propagation of uncertainty.

Principle 5. For particular applications, the combined uncertainty should be multiplied by a coverage factor to obtain an overall uncertainty value. The overall uncertainty is now called expanded uncertainty. For the 95 % confidence level, the commonly applied coverage factor is 2.

4.3 Pre-GUM Methodologies

The term Pre-GUM refers to methodologies developed prior to ISO (1995). Moffat (1982) introduced and defined the terms "bias" and "precision". The jitter diagram was proposed as a computational tool to propagate uncertainties by a central finite differencing method. This method is now included in the ISO (1995), section 5.1.3 on p. 19.

Moffat (1985) also introduced the term relative uncertainty for equations containing products of the terms (see equation 12, section 5.1.6, p. 20 of the ISO, 1995). Typical examples for such equations in experimental hydrodynamics are Reynolds number, *Re*, and Froude number, *Fr*.

The developmental progress of the uncertainty methodology in North America was described in a series of four (4) papers published in 1985: Abernethy and Benedict (1985), Kline (1985), Abernethy, et al. (1985), and Moffat (1985). The last 3 papers were originally presented in the Symposium on Uncertainty Analysis at the Winter Annual Meeting of the American Society of Mechanical Engineers (ASME) in Boston in 1983.

The following is a brief discussion regarding Pre-GUM schools of thought or standards. The focus is on two particular USA standards, ASME (2005) and AIAA (1999).

ASME PTC 19.1 was first published in 1985, since then, PTC 19.1 has been updated and revised twice. The last two revisions ASME PTC 19.1-1998 and ASME PTC 19.1-2005 are in harmony with the ISO (1995). The term "harmonization" employed by AMSE does not mean the same equations and same process. Harmonization is rather the manner how uncertainty analysis is assessed.

The ASME PTC 19.1 series (1985, 1998, and 2005) provided the mathematics necessary to calculate the two components of uncertainty. In ASME 1998, the terms "bias uncertainty" and "precision uncertainty" were applied. However, in ASME 2005, the terms "bias uncertainty" and "precision uncertainty" were intentionally avoided and replaced by "systematic" and "random" uncertainties.

In ASME PTC 19.1-1998, the term "total uncertainty" is defined, and at the 95 % confidence level the total uncertainty (U_t) is given by:

$$U_t^2 = B^2 + (t_{95}S)^2 \qquad (1a)$$

where B is the propagated bias uncertainty from the measurement, S is the propagated precision uncertainty, and t_{95} is the inverse Student t at the 95 % confidence level.

In ASME 19.1-2005, equation (1a) was changed. The term "combined uncertainty" has replaced the term "total uncertainty". Equation (1a) then becomes for the standard uncertainty:

$$u^2 = b^2 + s^2 \qquad (1b)$$

where $b = B/2$ and s are the systematic uncertainty component and the random uncertainty components, respectively.

Also in ASME (2005), the term "expanded uncertainty", U (Capital U), is defined as:

$$U = 2u \qquad (2)$$

where the coverage factor is 2 for the 95% confidence level.

Moffat (1988) provided a practical guide to engineering application of uncertainty analysis. He describes in detail the process from calibration of instrumentation to the analysis of the final experimental results. The bias component consisted of those from calibration, data acquisition, and data reduction, components that produce constant uncertainty value during testing. Acquisition of data during calibration with modern data acquisition systems includes some precision terms, but the calibration data becomes a fixed number when applied to the data analysis of the final experimental results. Moffat (1988) refers to this constant uncertainty as "fossilized uncertainty".

Coleman and Steele (1999) is the commonly used textbook on uncertainty analysis by the North American scientists and engineers. The first edition was published in 1989. H. W. Coleman was a USA representative on the committee for AGARD AR-304 (1994). AGARD (1994) was an international effort in the application of uncertainty analysis to wind tunnel testing, which was published a year after the first version of ISO in 1993. In AGARD (1994), total uncertainty was described by an equation similar to equation (1b) as follows:

$$U_f^2 = B_f^2 + P_f^2 \tag{3}$$

where B_f is the bias uncertainty and P_f the precision uncertainty for the propagated uncertainty for the function f or data reduction equations.

The law for propagation of uncertainty was expanded by AGARD (1994) for inclusion of perfectly correlated terms. ASME PTC 19.1-1998 also included relationships for perfectly correlated terms. However, ISO (1995) included a generalized law of propagation of uncertainty. The general propagation law accounts for any correlation term, from which the perfectly correlated version can be derived (see new procedure ITTC (2008a).

AIAA S-071A-1999 and its supplement AIAA G-045-2003 are derived from AGARD AR-304 (1994). ITTC uncertainty procedures were previously adapted from the AIAA (1995) version of AIAA S-071. Like ASME PTC 19.1-1998, AIAA S-071A-1999 has been harmonized with ISO (1995). In open literature, both AIAA and ASME standards are quite extensive, have many examples, and are references useful to ITTC UA procedure development.

In 1993, the National Institute of Standards and Technology (NIST, www.nist.gov/) published an abbreviated version of ISO (1995). NIST is the National Metrology Institute (NMI) in the USA. The NIST technical note 1297 was revised in 1994 (Taylor and Kuyatt, 1994). Technical Note 1297 is an outline of the implementation of ISO (1995) by NIST. Founded in 1901, NIST is a non-regulatory federal agency. Its mission is to promote U.S. innovation and industrial competitiveness by advancing measurement science, standards, and technology in ways that enhance economic security and improve our quality of life.

Expanded uncertainty is acknowledged as the reporting method for commercial, industrial, and regulatory applications. AIAA (1999) recommends traceability of calibrations to an NMI such as NIST. However by Taylor and Kuyatt (1994), NIST reports uncertainty as standard uncertainty. Like the NIST and its technical note 1297, the ITTC UAC provided general procedures ITTC (2008a, c) as examples of implementation of ISO (1995) concepts in experimental hydrodynamics and instrument calibration in tow tank and water tunnel testing. ITTC UAC recommended policy is reporting expanded uncertainty at the 95 % confidence level.

4.4 Road to Harmonization

The ASME (2005) revision of the PTC 19.1 is written with the objective to harmonize it with the ISO (1995). ISO (1995) recommends an expanded uncertainty at the 95 % confidence level in industrial applications while ASME PTC 19.1-2005 requires 95 % confidence level in reporting expanded uncertainty. The terms "combined uncertainty and expanded uncertainty" are applied in both standards. Both standards ASME PTC 19.1-2005 and ISO (1995) employ the term standard uncertainty, which is equivalent to one standard deviation.

However unlike ISO (1995), ASME PTC 19.1 emphasizes the following fundamental concept: The effect on the final test results determines the type of the uncertainty component. Systematic uncertainty component is from error sources whose effect is constant during the experiment. Random uncertainty component, however, is from error sources that cause scatters in the final results of the experiment.

ISO (1995) is based on the fundamental concept that uncertainty components (Type A and Type B) are classified on the method of computation. Type A uncertainty is computed as the standard deviation of the mean value of a series of measurements. However, Type B uncertainty is estimated from other methods, such as previous experience and engineering judgment including previous measurement data, calibration data, and manufacturing specifications.

ASME PTC 19.1-2005 did not use the terms "bias" and "precision" uncertainty components as ASME (1998). Instead the terms "systematic error" and "random error" are implemented. The AIAA (1999) still prefers the terms "bias" and "precision".

The road to full harmonization among standards is proven to be a difficult and long. NCSLI www.ncsli.org/index.cfm in its PR 12 document (2008) gave a comprehensive comparison between GUM and Pre GUM standards. No Post-GUM methodologies have been developed independent of ISO (2005). Nations follow ISO (1995), and the US organizations are working to harmonize their standards with that of the ISO (1995).

4.5 Importance of Uncertainty Analysis

The importance of uncertainty analysis in science and engineering has been described previously by a number of authors including AGARD (1994), AIAA (1999), Kline (1985), and Coleman and Steele (1999). Kline (1985) outlined 12 uses of uncertainty analysis and provided 7 case histories. The important considerations for ITTC are as follows.

Data Quality. Uncertainty analysis provides a measure of the quality of the data. Results may be compared between 2 or more laboratories or between repeat tests within the same laboratory. The results are comparable only if they are within the uncertainty of the measurement. Likewise, computational results may be compared to experimental results. The computational results will be valid only if they are within the uncertainty of the experiment. As a measure of the quality of the data, science and engineering journals as well clients are now requiring an uncertainty statement for the experimental and computational results. In particular, the ASME Journal of Fluids Engineering requires an uncertainty statement for all published papers.

Planning an Experiment. Uncertainty analysis is necessary for planning of an experiment, and/or improving the results of future experiments. Pre-test uncertainty analysis will identify the quality of the instrumentation needed for acquisition of the desired experimental results. In most experiments, the uncertainty is dominated by one or two sources (or instruments). Thus, pre-test uncertainty analysis will prevent investment in expensive instruments that do not impact the results significantly. In some cases, an uncertainty analysis will indicate that the desired results cannot be achieved and that the experiment should be abandoned. Kline (1985) cites an example of "A Hopeless Experiment Avoided".

Diagnostic Tool. Uncertainty analysis can be used as a diagnostic tool. In the planning stage of an experiment, uncertainty analysis will be used to identify major sources of uncertainties, why those sources exist, and devise methods for minimization of their effects.

Decision Mechanism. Uncertainty analysis can be applied as a decision mechanism. For example, a decision may be necessary for minimization of the effects of a major uncertainty source in an experiment. The need for a different load sensor may be driven by the desire for a reduction in the overall uncertainty. On the other hand, the new load sensor may be too expensive to buy. Therefore, the decision may be to keep the existing load sensor and accept the pre-calculated level of uncertainty.

Uncertainty Growth. Uncertainty growth is new concept that is introduced in the NCSLI PR 12, how uncertainty grows with time for a given component or a sensor (aging factors). The projected uncertainty growth has a direct impact on instrument inventories in the laboratory and facility operating budgets.

Better Understanding. Uncertainty analysis will provide a better understanding of the details and the execution of an experiment. It will provide focus and special care in the acquisition of critical data. Simply said, Kline (1985) on the basis of 30 years of laboratory experience states that uncertainty analysis is worth doing.

5. REPEATABILITY VERSUS REPRODUCIBILITY

5.1 Repeatability

In the conventional application of uncertainty analysis, uncertainty is estimated from a single test or experiment from single-sample uncertainty theory. In some cases, uncertainty is underestimated due to missing or uncontrolled elements in the analysis. Detection of such uncontrolled elements may be determined from repeat tests.

From ISO (1995), repeatability is defined as "closeness of the agreement between results of successive measurements of the same measurand carried out under the same conditions of measurement". Repeatability conditions are further defined as follows:

- Same measurement procedure
- Same observer/operator
- Same measuring instrument under the same conditions
- Same location or same laboratory
- Repetition over a short period of time. For experimental hydrodynamics, a short period of time may be tests performed on the same day.

As an example, repeat tests of carriage speed are illustrated in Figure 2. For laboratory A, the estimated uncertainty in carriage speed from the traditional metal wheel device is indicated by the error bars on the symbols. The expanded uncertainty in speed was estimated as ±0.00052 m/s or ±0.52 mm/s, at the 95 % confidence limit. The mean carriage speed from repeat runs is 2.0375 ±0.0014 m/s, ±0.069 %, with a coverage factor of 2.07. The expanded uncertainty from the repeatability test and metal wheel is then ±0.0015 m/s (±0.074%).

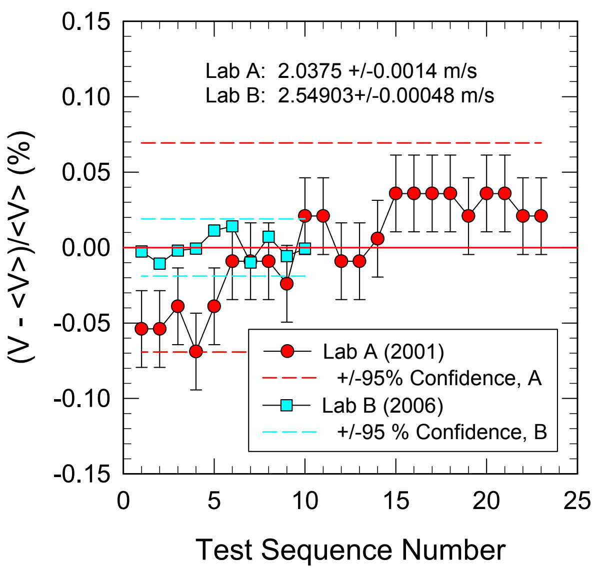


Figure 2 Comparison of repeatability between carriage speeds of 2 towing basins.

The second data series in Figure 2 is from a second laboratory, laboratory B. In that case, the result for 10 repeat runs was 2.54903 ±0.00048 m/s (±0.019 %) with a coverage factor of 2.26. The uncertainty in the speed measurement for Laboratory B is smaller than that from Laboratory A. For laboratory B, the uncertainty in the reference speed was not reported, but it is assumed to be similar to that of Laboratory A. The difference in results between the 2 laboratories is probably the control system for carriage speed. No uncertainty estimate is likely possible for the contribution of the control system. Consequently, any statement on uncertainty in carriage speed for Laboratory A should include repeatability in carriage speed. The estimate stated here is for a single measurement of the carriage speed that is based upon one standard deviation from multiple speeds in the repeat tests.

Another example of uncertainty from repeatability of an experiment is wave height measurement. The calibration result for an ultrasonic wave height gage from Chirozzi and Park (2005) is shown in Figure 3. Calibration results in Figure 3 are presented as residuals, where a residual is the difference between the data point and its value from a straight-line curve fit. The zero line in Figure 3 represents linear regression value. Further discussion of instrument calibration is in section 8.

In this case, calibration of the wave gage is accomplished by relocation of the sensor relative to the water on a shaft with precision pinhole locations with an estimated uncertainty of ±0.13 mm at the 95 % confidence limit. This uncertainty is smaller than the symbols in the figure. The dashed lines in the figure are the uncertainties at the 95 % confidence level from calibration theory, ITTC (2008c). As the figure indicates, the estimated uncertainty in calibration is within ±4.1 mm. The wave gage is highly linear with an *SEE* = 0.81 mm (standard error of estimate) and correlation coefficient of *r* = 0.999993. Since these measurements were acquired by relocation of the sensor relative to the water, the slope and intercept will have signs opposite to those of the calibration for the water surface moving relative to the fixed sensor.

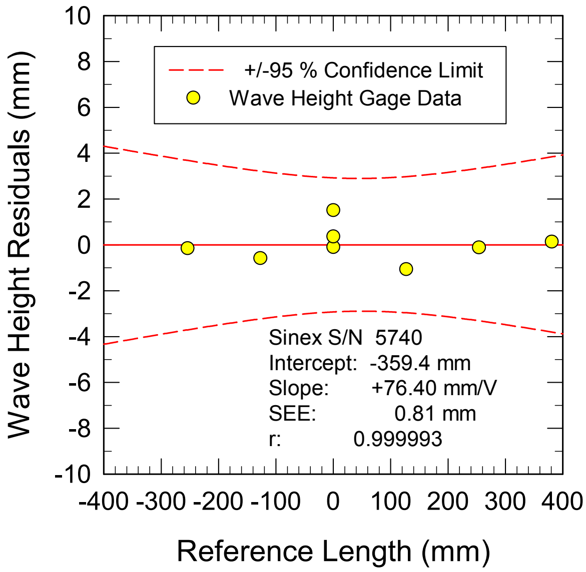


Figure 3 Residual plot of ultrasonic wave gage calibration from Chirozzi and Park (2004).

An actual wave measurement from the Manoeuvring and Seakeeping Basin (MASK) at David Taylor Model Basin (DTMB) is shown in Figure 4 for regular gravity waves with wave gage #6. The data are unpublished results from a test on March 11, 2004. The total run time was 42.2 s. Only 10 s of data are presented for clarity. The red symbols are outliers that were excluded from a linear regression analysis indicated by the solid red line in the figure.

A linear regression analysis was performed with a commercial curve fitting code with outliers removed. For execution of the code, the frequency was fixed with the value from the Fourier analysis. The form of the equation was as follows:

$$y = a + b\,sin(2\pi f + c) \qquad (4)$$

where *a*, *b*, and *c* are constants from the regression analysis, and *f* is the frequency from the Fourier analysis. The results are summarized in Table 1.

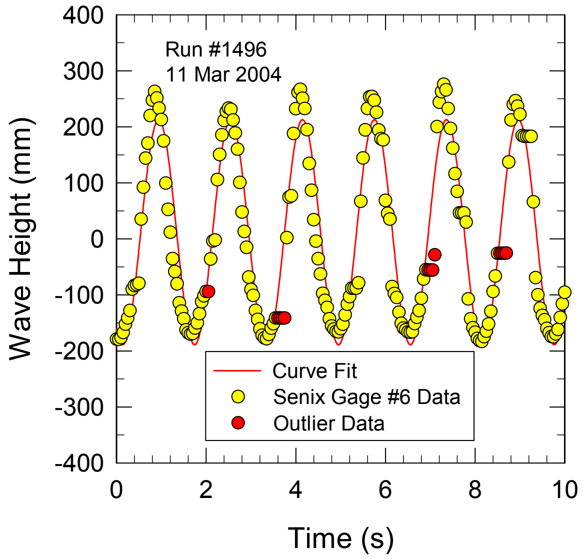


Figure 4 Regular wave as measured with an ultrasonic wave height gage.

Table 1 Summary of results for regular waves from regression analysis.

Symbol	Units	Value	Std Error
a	mm	11.8	1.58
b	mm	-201.0	2.06
c	rad	-5.253	0.0122
f	Hz	0.623	
r		0.967	
SEE	mm	40.3	

Thus, the uncertainty in the amplitude from Table 1 is (201.0.±4.1) mm (±2 %) at the 95 % level with a coverage factor of 2 in comparison to an amplitude of 201.1 mm from Fourier analysis. For a single wave train measurement from a single probe, the uncertainty in the amplitude is about the same as the uncertainty in the probe calibration.

Repeat measurements during the same day for wave gage #6 are indicated in Figure 5 for the wave amplitude from Fourier analysis. In this figure, sequence number 13 is the same data as Figure 4. As the figure indicates, the average of 14 runs very near that of Figure 4. A total of 15 measurements were acquired during the day, but one was an outlier. With the outlier excluded, the average wave amplitude for the day was 201 ±11 mm (±5.4 %) with a coverage factor of 2.16 at the 95 % confidence limit. In the figure, the error bars are the estimated uncertainty of ±3.5 mm within the measurement range of the wave amplitude from Figure 3. The average frequency was 0.6216 ±0.0058 Hz (±0.93 %).

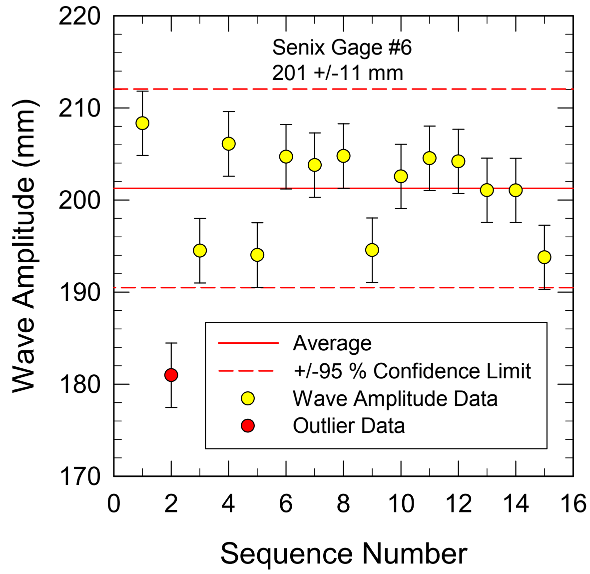


Figure 5 Repeat measurements of wave amplitude in a single day.

The wave field was measured with 6 gages. The Fourier analysis of the waves for the 6 gages is summarized in Table 2. As the analysis indicates, the wave amplitude and frequency for gage #6 are, respectively, 201.1 mm and 0.623 Hz for the measurements of Figure 4.

From Table 2, the mean wave amplitude for 5 of the 6 wave gages (3-8) is (201 ±29) mm (±14.6 %). The uncertainty in the measured wave height is significantly larger than the uncertainty in the calibration of the wave gage. In this case, the measurements from the different wave gages are more of a measure of the spatial variation in the wave height from the wave maker. The uncertainty in wave height can only be determined from repeat measurements since the uncertainty in wave height cannot be estimated from the control system of the wavemaker.

Table 2 Results of Fourier analysis for wave height measurements with ultrasonic gages for a single run of 42.2 s.

Probe #	*A* (mm)	*f* (Hz)
1	126.9	0.622
3	213.6	0.627
4	199.2	0.624
5	205.9	0.619
6	201.1	0.623
8	184.9	0.624
Average	200.9	0.6234
Std Dev	10.5	0.0029
t_{95}	2.78	2.78
U_{95}	29.3	0.0080

5.2 Reproducibility

Another check on the quality of the data is reproducibility. In general, the uncertainty for reproducibility will be larger than repeatability. From ISO (1995), reproducibility is defined as "closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement". The changed conditions include:

- Principle/reason of measurement,
- Method of measurement
- Observer/operator
- Measuring instrument
- Reference standard
- Location or laboratory
- Time of testing

For a long test, reproducibility should be checked. A representative test in a test series should be run at the beginning, middle, and end of test series. A reproducibility test is useful for verification of a test procedures and equipment and qualification of personnel.

An example is propeller test data from Donnelly and Park (2002). The same propeller has been tested since 1987 for verification of results from an open-water dynamometer at David Taylor Model Basin. Results for the thrust and torque coefficients are presented in Figure 6. As the figure indicates, a 4th-order polynomial fits the data quite well. The uncertainty in the coefficients is much smaller than the symbols. The error bars are quite evident in the residual plot of Figure 7. The dashed lines in the residual plots are the 95% prediction limit for the 2002 data.

The historical database in Figure 7 is from 1987 through 1998. The data includes 20 runs and 255 data points. In general, the historical data are in agreement with the 2002 data within the current uncertainty estimates.

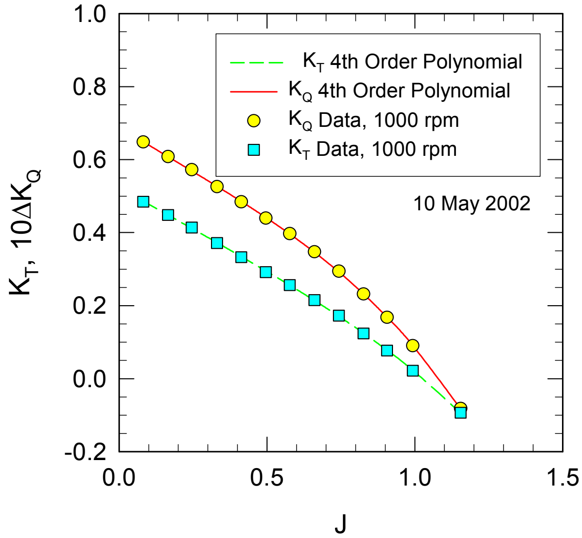
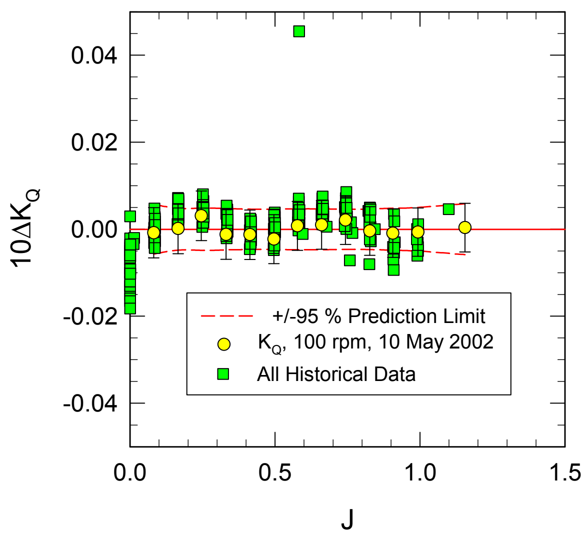
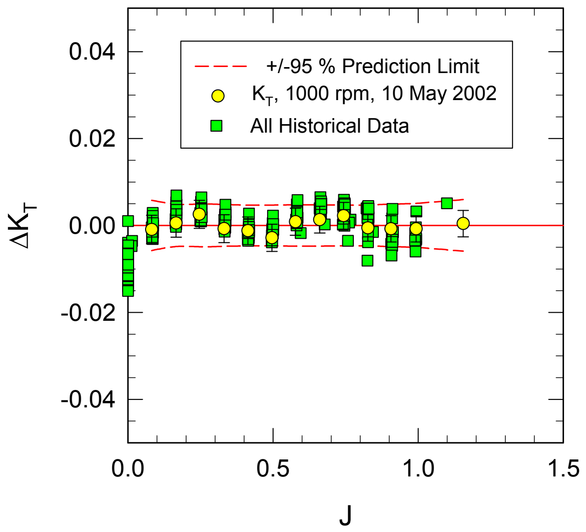


Figure 6 Propeller thrust and torque coefficients from open water dynamometer from Donnelly and Park (2002).



a. Torque coefficient



b. Thrust coefficient

Figure 7 Residual plots for propeller thrust and torque coefficients from open water dynamometer in comparison to historical data from Donnelly and Park (2002).

6. INTER-LABORATORY COMPARISON

As a better measure of a laboratory's uncertainty estimates, inter-laboratory comparisons are routinely performed. The method adopted by NMIs (National Metrology Institute) is the Youden plot (1959, 1960). The method requires the measurement of 2 similar test articles, A and B, by several laboratories, and then plotting the results of A versus B. For a naval hydrodynamics test, the test models (articles) may be 2 propellers in a propeller performance test or 2 ship hulls models in a resistance-towing test.

An example schematic of a Youden plot for flow-meters from the results of 5 laboratories is shown conceptually in Figure 8 from Mattingly (2001). In the method, vertical and horizontal dashed lines are drawn through the mean values of all laboratories. Then, a solid line is drawn at 45° (slope +1) through the crossing point of the dashed lines.

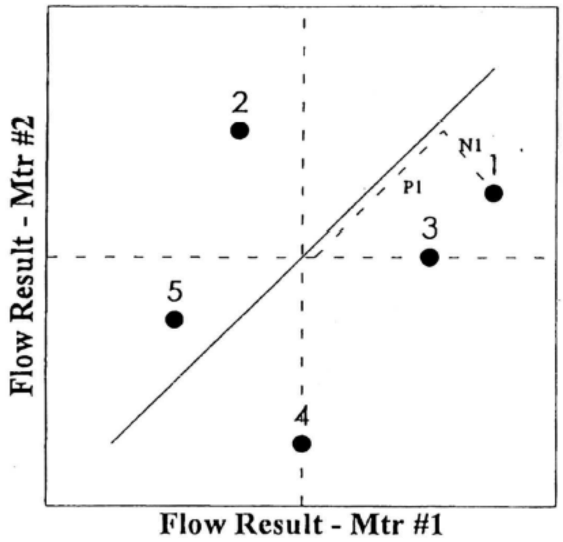


Figure 8 Youden plot for flow meter test from Mattingly (2001).

The data pattern in Figure 8 is as follows:

- NE and SW quadrants, systematic high and low values, respectively
- NW and SE quadrants, random high and low values, respectivelyUsually elliptic in shape are the random values along the minor axis and the systematic errors along the major axisIdeally, the pattern should be circular.

The variance of n laboratories normal to the 45° axis is given by:

$$s_r^2 = [1/(n-1)]\sum_{i=1}^{n} N_i^2 \tag{5a}$$

and parallel to the axis:

$$s_s^2 = [1/(n-1)]\sum_{i=1}^{n} P_i^2 \tag{5b}$$

where s_r and s_s may be interpreted as the random and systematic deviations of the data, respectively, and N_i and P_i are the respective normal and parallel components of the data projected onto the line with the slope of +1. The ratio of these two quantities is then the circularity of the data.

An actual inter-laboratory turbine meter test from Diritti, et al. (1993) is shown in Figure 9 and Figure 10. The data are from 7 European gas flow laboratories in England, France, Germany, and the Netherlands. Two pairs of turbine meters were tested with diameters of 250 and 150 mm.

In the Youden plot, meter 1 is plotted versus meter 2. The variable is a performance index, which is the ratio the meter flowrate to the reference flowrate. Ideally, the performance index is 1, as indicated by the dashed lines in the figures.

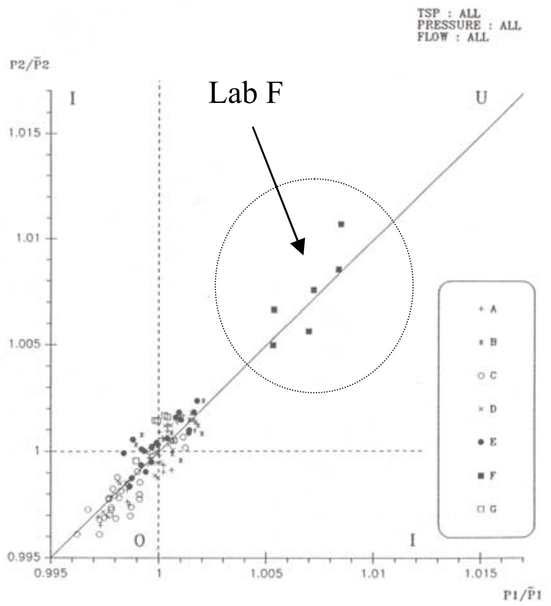


Figure 9 Youden plot of performance index for all laboratories from Diritti, et al. (1993).

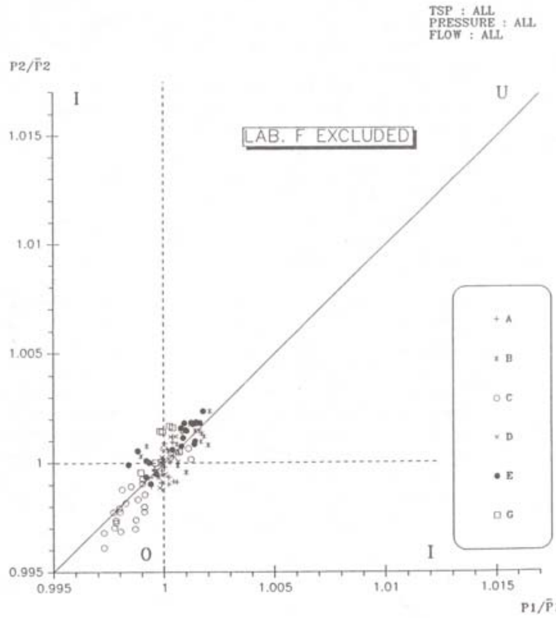


Figure 10 Youden plot of performance index with exclusion of Laboratory F, from Diritti, et al. (1993).

As Figure 9 indicates, Laboratory F is clearly is an outlier and was excluded in the analysis. The result without laboratory F is shown in Figure 10.

The flowrate uncertainty for these laboratories was typically stated as ±0.25 % at the 95 % confidence level. The day-to-day and week-to-week variation in flow for each facility was within ±0.15 %. With the exception of Laboratory F, the systematic differences between the laboratories were less than ±0.25 % at the 95 % confidence level. This result is especially phenomenal since tractability of the measurements in these facilities was to the NMIs in their respective countries.

7. GUIDE TO THE EXPRESSION OF UNCERTAINTY IN EXPERIMENTAL HYDRODYNAMICS

7.1 Introduction

The word "uncertainty" means doubt, and therefore in its broadest sense "uncertainty of a measurement" means a "doubt about the validity of the result of that measurement". The concept of "uncertainty" as a quantifiable attribute is relatively new in the history of measurement science. However, concepts of "error" and "error analysis" have long been a part of measurements in science, engineering, and metrology. When all of the known or suspected components of an error have been evaluated, and the appropriate corrections have been applied, an uncertainty still remains about the "truthfulness" of the stated result. That is, a doubt about how well the result of the measurement represents the "value" of the quantity being measured.

The objective of a measurement is to determine the value of the measurand, that is, the value of the particular quantity to be measured. A measurement begins with an appropriate specification of the measurand, the method of measurement, and the measurement procedure. The result of a measurement is only an approximation or an estimate of the value of the true quantity to be measured. Thus, the result of a measurement is complete only when accompanied by a quantitative statement of its uncertainty.

7.2 Measurement Equation

The quantity Y being measured, defined as the measurand, is not measured directly, but it is determined from N other measured quantities X_1, X_2, …X_N. Thus, the measurement equation can be presented as:

$$Y = f(X_1, X_2, X_3, \cdots X_N) \tag{6a}$$

The function f includes, along with the quantities X_i, the corrections (or correction factors) as well as quantities that take into account other sources of variability, such as different observers, instrument calibrations, different laboratories, and times at which observations were made.

An estimate of the measurand (Y) is denoted by (y) and is obtained from equation (6a) with the estimates x_i for the values of the N quantities X_i. Therefore, the estimate (y) becomes the result of the measurements:

$$y = f(x_1, x_2, x_3, \cdots x_N) \tag{6b}$$

The following are examples for typical measurement functions or data reduction equations of the propulsion performance functions of ITTC (2002).

Advance ratio:

$$J = f(V, n, D) = V/(nD) \tag{7a}$$

Thrust coefficient:

$$K_T = f(T, \rho, D, n) = T/(\rho D^4 n^2) \tag{7b}$$

Torque coefficient:

$$K_Q = f(Q, \rho, D, n) = Q/(\rho D^5 n^2) \tag{7c}$$

where Q, T, ρ, D, and n are torque (N.m), thrust (N), mass density of water (kg/m^3), propeller diameter (m), and rotational rate (1/s), respectively. Furthermore, the density of fresh water is a function of the temperature, t in °C.

Therefore, an estimate for K_Q is obtained from estimates of the quantities Q, ρ, D, and n, while the estimates for K_T are obtained from quantities T, ρ, D, and n. The estimates for each quantity Q, T, D are obtained from direct measurements while ρ is computed as a function of temperature, t in °C. The uncertainty in a measurement y, denoted by $u(y)$, arises from the uncertainties $u(x_i)$ in the input estimates x_i in the measurement function, equation (6b).

7.3 Classification of Uncertainty

ISO (1995) classifies uncertainties into three (3) categories: Standard Uncertainty, Combined Uncertainty, and Expanded Uncertainty.

Standard Uncertainty (u). Uncertainty, however evaluated, is to be represented by an estimated standard deviation. This is defined as “standard uncertainty” with the symbol “u (small letter u)” and equal to the positive square root of the estimated variance.

The standard uncertainty of the result of a measurement consists of several components, which can be grouped into two types. They are Type A uncertainty and Type B uncertainty as described in section 4.2.

The purpose of Type A and Type B classification is a convenient method for the distinction between the two different methods for assessing uncertainty. No difference exists in the nature of each component resulting from either type of evaluation. Both types of uncertainties are based on probability distributions and the uncertainty components resulting from both types are quantified by standard deviations. The value of Type B is approximated by a corresponding variance.

Combined Standard Uncertainty (u_c). Combined standard uncertainty of the result of a measurement function, f, is obtained from the uncertainties of a number of individual measurements, x_i. The combined uncertainty is computed via the law of propagation of uncertainty from the measurement function or data reduction equation, equation (6a). The result for independent or uncorrelated measurements is given by

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2\, u^2(x_i) \qquad (8a)$$

where c_i is the sensitivity coefficient, $\partial f/\partial x_i$. For perfectly correlated measurements, the combined uncertainty is then

$$u_c(y) = \sum_{i=1}^{N} c_i\, u(x_i) \qquad (8a)$$

Additional details are discussed in ITTC (2008a).

Expanded Uncertainty (U). Mathematically, expanded uncertainty is calculated as the combined uncertainty multiplied by a coverage factor, k. The coverage factor, k, includes an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

Thus, the numerical value for the coverage factor k should be chosen so that it would provide an interval $Y = y \pm U$ corresponding to a particular level of confidence. In experimental hydrodynamics, k corresponds usually to 95% confidence. All ITTC results will be reported with an expanded uncertainty at the 95 % confidence level.

The ISO (1995) indicates that a simpler approach is often adequate in measurement situations, where the probability distribution of measurements is approximately normal or Gaussian. This is effectively based on the central limit theorem in statistics. If the number of degrees of freedom is significant ($\nu > 30$), the distribution may be assumed to be Gaussian, and k will be evaluated as 2 for the 95% level of confidence. For a smaller number of samples, the value of k is then the inverse Student t distribution at the 95 % confidence level. Detailed information and equations needed to perform UA calculations, including standard, combined, and expanded uncertainties, are given ITTC (2008a).

7.4 Outliers

Sometimes data occur outside the expected range of values, and should be excluded from the calculation of the mean value and estimated uncertainty. Such data are referred to as outliers. If an outlier is detected, the specific cause should be identified before the outlier is excluded. Outliers may be identified by several methods, which are described in ITTC (2008a). One of these methods is the hypothesis *t*-test:

Hypothesis *t*-test. The conventional method for outliers is the *t*-test hypothesis testing. The details of the methodology may be found in a standard statistics text such as Ross (2004). The *t*-test for a random variable is as follows:

$$T = (q_i - \overline{q}) / s \tag{9}$$

Accept if $T \le t_{95,n-1}$, otherwise reject.

where q_i is the measurement, $\overline{q}$ is the mean, s is the standard deviation, and $t_{95,n-1}$ is the inverse Student *t* for a 2-tailed probability density function (pdf) at the 95 % confidence level and cumulative probability $p > 0.975$. In practical terms, any *T*-value that exceeds 2 is suspected as an outlier at the 95 % confidence level.

7.5 Pre-test and post-test uncertainty analysis

Before the first data point is taken in a test program, the use of the data should be known such as the measurement functions, also known as data reduction equations. For example for a data acquisition system (DAS), the function should include the measurement equations and data for conversion of the digitally acquired data to physical units from calibrations. Finally, uncertainty analysis should be included in the data processing codes.

A pre-test uncertainty analysis should be performed during the planning stage, and throughout various design phases. The pre-test uncertainty should include primarily Type B uncertainties unless data are available from previous tests for an estimate of the Type A uncertainty.

In the pre-test stage, all elements of the Type B uncertainty should be applied. In particular, manufacturers specifications may be included for an assessment of adequacy of a particular instrument for the test before the device is purchased. Selection of an instrument may involve economic trade-offs between cost and performance (see section 4.5 on the importance of UA).

For the post-test uncertainty analysis, the post-processing code should provide sufficient data on uncertainty analysis for the final report of the test program. In this case, data will include results from both the Type A and Type B methods. All of the elemental uncertainties should be based upon measurements that are traceable to an NMI. That is, all measurements should be based upon documented uncertainties. Such evaluation should include no guesses or manufacturer's specifications.

Finally, the contributions of the elemental uncertainties should be compared to the combined uncertainty. Such comparison will identify the important elemental uncertainties that have significant contribution to the overall uncertainty.

7.6 Reporting uncertainty

The main directive for reporting uncertainties is that all information necessary for a re-evaluation of the measurement should be available to others when and if needed. When uncertainty of a result is evaluated on the basis of published documents such as instrument calibration certificates, these publications should be referenced. Test results should be consistent with the measurement procedure actually applied. If experiments are performed with instruments that are subjected to periodic calibration and/or legal inspections, the instruments should conform to the specifications that apply.

In practice, the amount of information necessary to document UA depends on its intended use. For example, ISO (1995) requires a list of all uncertainty components, standard uncertainty, combined uncertainty, and expanded uncertainty. Uncertainty estimates should be documented fully how they were evaluated. Furthermore, expanded uncertainty should be reported at the 95 % confidence level in ITTC applications, and the basis for selecting the coverage factor value, *k*, should be explained.

8. INSTRUMENT CALIBRATION

8.1 Introduction

Contemporary laboratories acquire data with a DAS. For conversion to engineering or physical units, instrumentation connected to these systems must be calibrated. This section describes methods for applying UA to these calibrations. Most instrumentation is highly linear; consequently, the calibration includes a linear fit to the data. Usually, most of the uncertainty is associated with the data scatter in the regression fit. The ITTC (2008c) provides more details for linear and non-linear curve fits with examples.

Torque transducers, load cells, and block gages are typically calibrated in a calibration stand by mass. Uncertainty analyses for force and torque calibration by mass are discussed with example calibration results.

8.2 End-to-End Calibration

Usually, the uncertainty in the reference standard should be small relative to the data scatter in the calibration. All calibrations should be "through system calibration" or "end-to-end calibration" with the same data acquisition system and software applied during the test.

A schematic of the end-to-end calibration process is shown in Figure 11. A known measured physical input is applied to the instrumentation system such as roll angle, for example. The physical input is then measured by an instrument with an NMI traceable calibration. The physical input is converted to a voltage by an electronic instrument. Amplification is then applied to the signal so that the expected voltage range matches the range of the AD (analogue to digital) converter. The output from the amplifier is then processed by a low-pass filter, which matches the frequency range of the electronic instrument. The filtered signal is digitized by the AD converter at a data rate, which is consistent with the Nyquist sampling theorem (Otnes and Enochson, 1972, and Bendat and Piersol, 2000). Finally, the data are processed by software and output the data in voltage units.

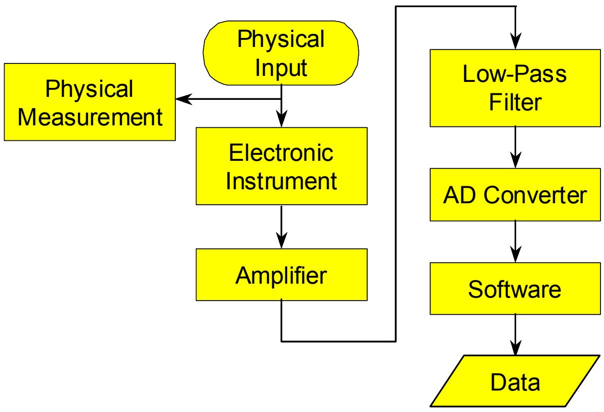


Figure 11 End-to-end calibration schematic.

8.3 Linear Calibration

For conversion of digital Volts to physical or engineering units, the intercept and slope are computed from linear regression analysis. The uncertainty in the calibration curve is determined from a statistical calibration theory described by Scheffe (1973) and Carroll, et al. (1988). Details of the method are described in

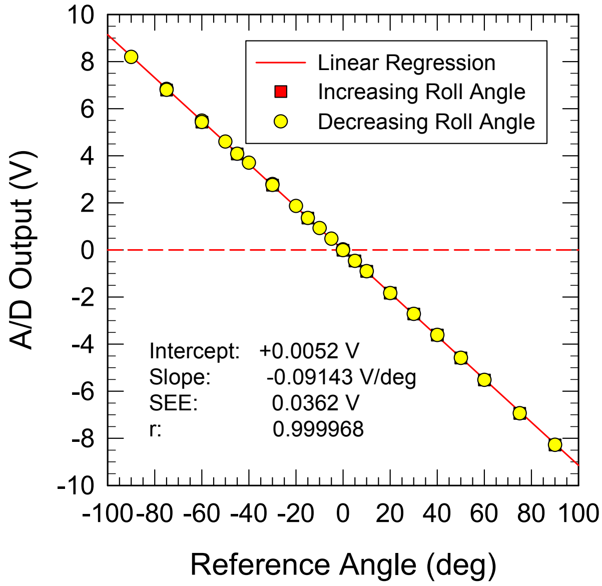
ITTC (2008c), only an example is presented in this section.

An example plot is shown in Figure 12 for calibration of a commercial vertical gyroscope in roll from Chirozzi and Park (2005). The reference angle was an electronic protractor with a measurement uncertainty of ±0.2° at the 95% confidence limit. The manufacturer rates the gyroscope with an uncertainty of ±1.0°.

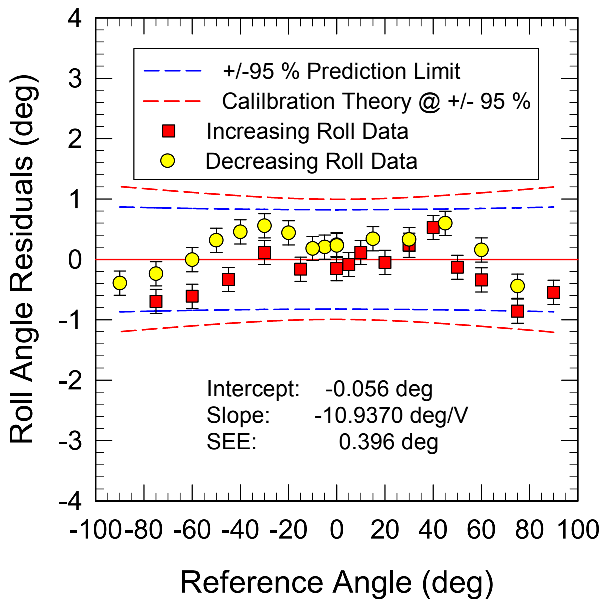
From Figure 12a all data points lie on a straight line. The error bars in such a plot are smaller than the symbols. The residual plot in Figure 12b yields significantly more information about the statistical character of the data. As the plot indicates, the data for increasing angle are systematically different from the decreasing angle. The plot indicates a slight hysteresis in the data not evident in the linear plot of Figure 12a. The error bars are readily apparent in the residual plot and in this case are the uncertainty in the reference measurement standard at the 95 % confidence limit (±0.2°).

The dashed lines in the residual plot are the prediction limits from the curve fit at the 95 % confidence level. The blue dashed line is the conventional prediction limit while the red dashed line is from calibration theory. As the figure shows, the uncertainty from calibration theory is similar to the value claimed by the manufacturer.

A useful variation in Figure 12b is a plot of standardized residuals, where the residuals are normalized with the *SEE*. In that format, outliers are more easily identified, where an outlier is usually a value greater than 2 × *SEE*. In this example, *SEE* = 0.396 so that the conventional 95 % confidence limit is 2 × *SEE* = 0.79, which is near the conventional 95 % prediction limit. However, the uncertainty from calibration theory is near 3 × *SEE* = 1.2. From calibration theory, 3 × *SEE* is considered a better estimate.



a. Linear plot



b. Residuals plot

Figure 12 Calibration data for vertical gyroscope in roll.

8.4 Force and Mass Calibration

In many applications in naval hydrodynamics, force and torque are calibrated on a calibration stand with masses. In that case, mass is related to force by the following equation from ASTM E74-02.

$$F = mg(1 - \rho_a / \rho_w) \tag{10a}$$

where m is the mass, g is local acceleration of gravity, ρ_a is air density, and ρ_w is the density of the weight.

The last term of equation (10) is a buoyancy correction term. Local gravity can differ from standard gravity, 9.80665 m/s^2, on the order of 0.1 %, and the buoyancy correction is typically 0.017 %.

Mass sets commonly applied to force calibrations have a specification on the order of 0.01 %, such as an OIML Class M1, NIST Class F, or ASTM Class 6. The detailed characteristics for these weight classes are described in OIML R 111 (2004), NIST (1990), and ASTM E617-97, respectively. Consequently, the correction for local gravity can be 10 times the uncertainty in the reference mass.

Load cells, dynamometers, and force balances are calibrated by addition or removal of weights from the calibration stand. The total mass is the sum of the individual masses:

$$m = \sum_{i=1}^{n} m_i \tag{10b}$$

OIML (2004) and ASTM E740-02 performance specifications recommend that the uncertainty calculations in weights should be perfectly correlated. The combined uncertainty is the sum of the uncertainties for the individual masses. The expanded uncertainty is from OIML (2004):

$$U_m = u_m / 3 \tag{10c}$$

where u_m is the nominal rated uncertainty. In many of the previous ITTC UA procedures, the uncertainty in mass has been erroneously reported as an uncorrelated uncertainty or the square root of the sum of squares.

8.5 Uncertainty in Pulse Count

In naval hydrodynamic applications, rotational rate is a commonly measured parameter. In particular, two applications are shaft rotational rate in propeller performance and towing carriage speed. Rotational rate is measured from a pulse-generating device such as an optical encoder or steel gear with a magnetic pickup. These devices are inherently digital. Data acquisition cards typically include a 16-bit analog to digital converter, counter ports, and accurate timing. The rotational rate is measured as:

$$\omega = n / (pt) \tag{11a}$$

where ω is the rotational rate, n the number of pulses, p the number of pulses per revolution, and t is the time.

From equation (10), the relative uncertainty in the rotational rate is

$$u_\omega / \omega = \sqrt{(u_n / n)^2 + (u_t / t)^2} \tag{11b}$$

The number of pulses per revolution, p, is assumed to be precisely known; therefore, its uncertainty is zero. The AD should have calibration traceability to an NMI. The uncertainty in the pulse count is computed from a uniform probability distribution function, ISO (1995) as:

$$u_n = 1/\sqrt{12} = 0.29 \tag{11c}$$

where the pulse count is ±1/2 pulse.

9. LASER DOPPLER VELOCIMETRY

9.1 Introduction

Laser Doppler velocimetry (LDV), also known as the laser Doppler anemometer (LDA), is an important tool for non-invasive local velocity measurements in naval hydrodynamics. ITTC procedure (2008d) has been developed for calibration of LDV and evaluation of uncertainty in its measurement of flow velocity.

Two methods of calibration are typically applied in LDV calibration. One is based on the optics of the system and the electronic processor for conversion of the light signal to velocity. The other method is a direct velocity calibration method from the velocity of a rotating disk or wheel. In general, the preferred method is the rotating disk.

9.2 Rotating Disk Method

A spinning disk is the primary standard for velocity calibration. In this case, the LDV is calibrated directly in velocity units. Since LDV processors are highly accurate, the method is essentially measurement of the laser beam intersection angle. The velocity from a spinning disk is:

$$V = r\omega = 2\pi f_{\mathrm{r}} r \tag{12a}$$

where r is the disk radius, ω is the rotational speed in radians/s, and f_{r} is the rotational frequency in Hz (revolutions/s).

The combined uncertainty in the velocity of the spinning disk is:

$$u_V = \sqrt{(2\pi r u_{f_{\mathrm{r}}})^2 + (2\pi f_{\mathrm{r}} u_r)^2} \tag{12b}$$

From the calibration theory in section 8.3 and ITTC (2008c), the velocity from LDV as measured by the rotating disk as the reference velocity is given by linear regression analysis for a range of velocities

$$V_{\mathrm{LDV}} = a + bV \tag{13}$$

Nominally, $a = 0$ and $b = 1$.

9.3 LDV Calibration Results

The spinning disk may be one of the following three types:

Spinning Wire. In this case, a single particle from the diameter of a fine wire travels through the probe volume of known radius.

Disk Edge. The probe volume is located on the curved surface of a rotating cylinder where the diameter is precisely measured.

Disk Face. The probe volume is located on the flat surface of a spinning disk. In this case, probe location is determined by the traversing system. One advantage is that both the vertical and axial velocity components may be calibrated by location of the probe on the disk.

Either of these types of devices may be mounted on the shaft of a digitally controlled motor with a high-resolution optical encoder.

Four elements contribute to the uncertainty in the velocity; they are:

- Rotational speed, f_{r}
- Radius, r
- Standard deviation of the velocity time series from the LDV processor.
- Curve fit from calibration theory

Results from the uncertainty analysis for a rotating sand disk are presented from Park, et al. (2002) in Figure 13 for a radius of 100 mm. As the figure indicates, the uncertainty in rotational velocity is dominant. The uncertainty in velocity is nearly constant at ±0.025 m/s for the expanded uncertainty at the 95 % confidence level. In this case, the uncertainty is from manufacturer's specification for their motor controller. Direct measurement of the rotational speed would likely reduce the uncertainty.

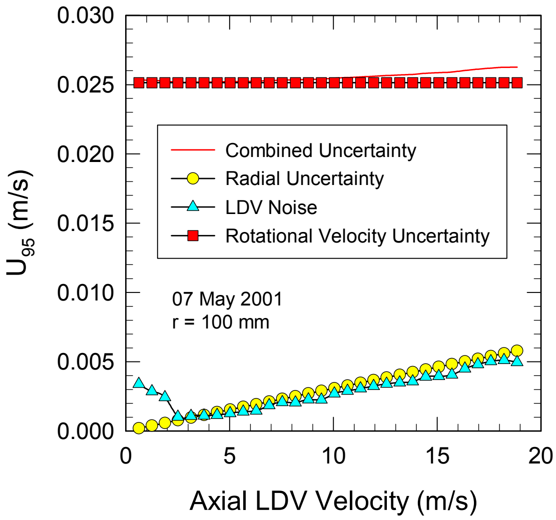


Figure 13 Velocity uncertainty of rotating disk for r = 100 mm.

Additional analysis demonstrates that r = 50 mm has a significantly lower uncertainty by almost half. The procedure also describes the results of similar calibration methods in other laboratories. The lowest expanded uncertainty was ±0.055 % at 20 m/s.

10. UNCERTAINTY ANALYSIS FOR PIV MEASUREMENTS

10.1 Introduction

This new procedure ITTC (2008e) describes the use of uncertainty analysis on flow field measurements from particle image velocimetry (PIV). The procedure was developed on the basis of the experimental guideline of the Visualization Society of Japan (VSJ, 2002), which was the result of an organized project on PIV standardization by Nishio, et al. (1999).

The present procedure is limited to the PIV measurement itself. The uncertainties of model test are not included. The total evaluation of an experiment should consider the contribution of model test uncertainty separately.

10.2 Data Flow in Measurement System

The target experiment is the velocity field measurement of 2-D water flow, where the measurement area and uniform flow speed are 320 x 330 mm^2 and 0.5 m/s, respectively. The target measurement consists of the following sub-systems: (1) Calibration, (2) Flow Visualization, (3) Image Detection, and (4) Data Processing. The evaluation of the measurement includes the coupling between the sub-systems.

The data flow in measurement system is shown in Figure 14. The measurement starts from the digital image of the visualized flow field, the pulsing time of illumination, and the distance of reference points. The final targets of the measurement are the flow velocity, the measurement location, and the measurement time. Possible uncertainty sources are also shown in Figure 14, and the uncertainties propagate to the final measured target through the data flow.

10.3 Summary of Uncertainties

The uncertainties of measurement are summarized in Table 3. Root-sum-square is employed for the combined uncertainties per ITTC (2008a). The uncertainties for the measurement parameters α, ΔX, Δt, and δu are considered separately, and their propagation to the final measurement velocity, u, is shown. The uncertainty propagation to the measurement target for position, x, and time, t, is also included. The uncertainties of u, x, and t are analysed independently. When accumulation for total performance of the measurement system by the uncertainty for the flow speed is required, the following equation for the combined uncertainty is applied, where u_u, u_x, and u_t represent the standard uncertainties of u, x, and t, respectively.

$$u_c = \sqrt{{u_u}^2 + (u_x \partial u / \partial x)^2 + (u_t \partial u / \partial t)^2} \qquad (14)$$

The major uncertainty sources may be evaluated in conjunction with the combined uncertainties. The largest uncertainty in this case is the mis-matching error of ±0.20 pixels, which appeared in the image analysis procedure. Its contribution to the uncertainty in velocity is ±26 mm/s in comparison to the combined uncertainty for all elements of ±27 mm/s. The next two largest error sources are the sub-pixel analysis, ±0.03 pixels or ±4.0 mm/s, and the uncertainty of calibration from the lens aberration, ±4.11 pixels or ±2.5 mm/s. This kind of analysis can contribute to the improvement of measurement systems. For this example, any significant improvement in uncertainty must focus on the mis-matching error. For the uniform flow, the expanded uncertainty in velocity is (0.500 ±0.054) m/s or ±11 % from a coverage factor of 2.

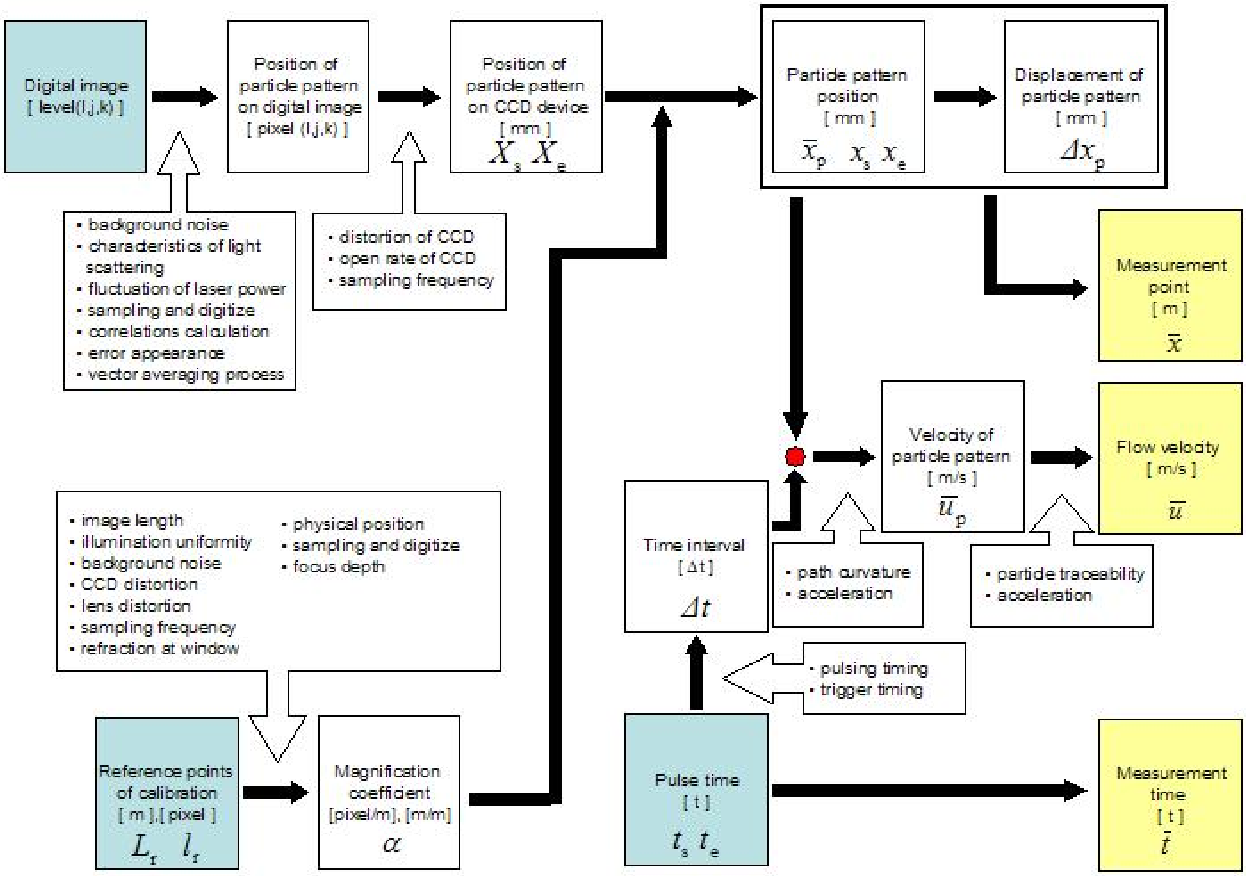


Figure 14 Data flow of PIV measurement

Table 3 Summary of uncertainties for fluid velocity (u), position (x), and time (t)

Parameter	Category	Error sources	$u(x_i)$ (unit)	c_i (unit)	$c_iu(x_i)$ (unit)
α		Magnification factor	0.00165 (mm/pix)	1580.0 (pix/s)	2.61
ΔX		Image displacement	0.204 (pix)	132.0 (mm/pix/s)	26.8
Δt		Image interval	5.39E-09 (s)	1.2 (mm/s^2)	6.47E-09
δu		Experiment	0.732 (mm/s)	1.0	0.732
			Combined uncertainty	u_u	26.9 (mm/s)

Parameter	Category	Error sources	$u(x_i)$ (unit)	c_i (unit)	$c_iu(x_i)$ (unit)
X_s, X_e	Acquisition	Digital error	0.5 (pix)	0.316 (mm/pix)	0.158
		Non-uniformity of distribution	8.0 (pix)	0.316 (mm/pix)	2.53
X_0	Calibration	Origin correlation	2.0 (pix)	0.316 (mm/pix)	0.632
α		Magnification factor	0.00178 (mm/pix)	506.0 (pix)	0.906
			Combined uncertainty	u_x	2.76 (mm)

Parameter	Category	Error sources	$u(x_i)$ (unit)	c_i (unit)	$c_iu(x_i)$
t_s, t_e	Acquisition	Delay generator	2.00E-09 (s)	1.0	2.00E-09
		Pulse time	5.00E-09 (s)	1.0	5.00E-09
			Combined uncertainty	u_t	5.39E-09 (s)

Standard uncertainty: $u(x_i)$ Combined uncertainty: u_c
Sensitivity coefficient: $c_i = \partial f/\partial x_i$

11. UNCERTAINTY ANALYSIS PROCEDURES FOR CAPTIVE MODEL TESTS

11.1 Introduction

In the narrow sense, the terms captive model test and free-running model test are usually related to model tests for ship manoeuvrability. In the general sense and from the viewpoint of measurement in captive model tests, the global forces and moments acted on ship models are measured, and in free-running model tests the global motion parameters of ship models are measured. The 26th ITTC-UAC will coordinate with other committees in the development of guidelines for uncertainty analysis in measurements for captive and free-running model tests. However for now, the 25th ITTC UAC has revised the uncertainty analysis for the resistance test in towing tanks, ITTC (2008b), as an example of the application ISO (1995) to captive model testing.

An uncertainty analysis procedure for forces and moments using PMM (Planar Motion Mechanism) test has been reviewed for the Manoeuvring Committee. The PMM procedure is discussed in Appendix B.

The remainder of this section summarizes ITTC (2008b). The procedure provides the formulas for the uncertainty estimates associated with resistance testing including the following: Froude number (Fr), Reynolds number (Re), total resistance coefficient (C_T), frictional resistance (C_F), calibration, model geometry, and form factor (k). Only model geometry and form factor are discussed in the following sections.

11.2 Purpose

The purpose of the ITTC (2008b) procedure is to provide guidelines for implementation of uncertainty analysis in model scale towing tank resistance tests that follow the ITTC (2002a, 20008a). Uncertainties related to extrapolation and full-scale predictions are not taken into consideration. This general guideline does not go into some specific details, such as turbulence stimulation, drag of appendages, blockage and wall effect of tank, scaling effect on form factor, etc.

11.3 The Measurement Equation

The objective of measurement in resistance towing tank tests is to obtain the relationship between residuary resistance coefficient C_R and Froude number Fr of a ship model and, if required, the form factor k. The direct measurement of the tests is the total resistance (R_T) as well as the running attitudes of a ship model at each speed. The form factor k is obtained through regression analysis of data at low Froude numbers ($Fr \leq 0.2$, if no separation is present) by the straight-line plot of C_T/C_F versus Fr^n/C_F by Prohaska's method (1966) in van Manen and van Oossanen (1988) and ITTC (2002a),

$$C_T / C_F - 1 = k + b(Fr^n / C_F) \tag{15}$$

where b is the slope from linear regression analysis, k the intercept, and n the power exponent of Froude number and usually, $4 \leq n \leq 6$.

11.4 Measurement System

From uncertainty analysis, the whole test system for resistance test, in a general sense, may be grouped under five blocks of No. 1 to No. 5, as shown in Figure 15. Each block is related to one group of uncertainty sources. In a narrower sense, the measurement system just includes three blocks No. 2 to No. 4.

11.5 Uncertainty Analysis Related to Hull Geometry

The uncertainties of hull geometric parameters are usually propagated through measurement functions or data reduction equations. Length of the hull is included in the calculation of Reynolds number and Froude number. In uncertainty analysis, such a length parameter is a characteristic length, and the resistance is more related to the real size of hull than to the real value of a specific length. The size of a model is usually estimated by its displacement volume, i.e., the characteristic length L can be taken as:

$$L \propto \sqrt[3]{\nabla} \tag{16}$$

and the relative uncertainty of the length can be approximated as:

$$u_L / L = u_\nabla / (3\nabla) \tag{17a}$$

where ∇ is the moulded displacement volume of the hull model.

However, the wetted surface area of hull is the most important geometric parameter in data reduction of a resistance test. The value of the real wetted surface of hull in test is not only determined by manufacturing but also by the model ballasting, running attitudes, and environmental effects. The combined standard uncertainty of the wetted area due to model ballasting is as follows:

$$u_S = (\kappa L_{WL} \cdot \nabla / A_W)\sqrt{(u_\Delta / \Delta)^2 + (u_\rho / \rho)^2} \tag{17b}$$

where A_W is the area of hull water-plane, (κL_{WL}) the expanded length of hull waterline, and $\Delta = \nabla \cdot \rho$.

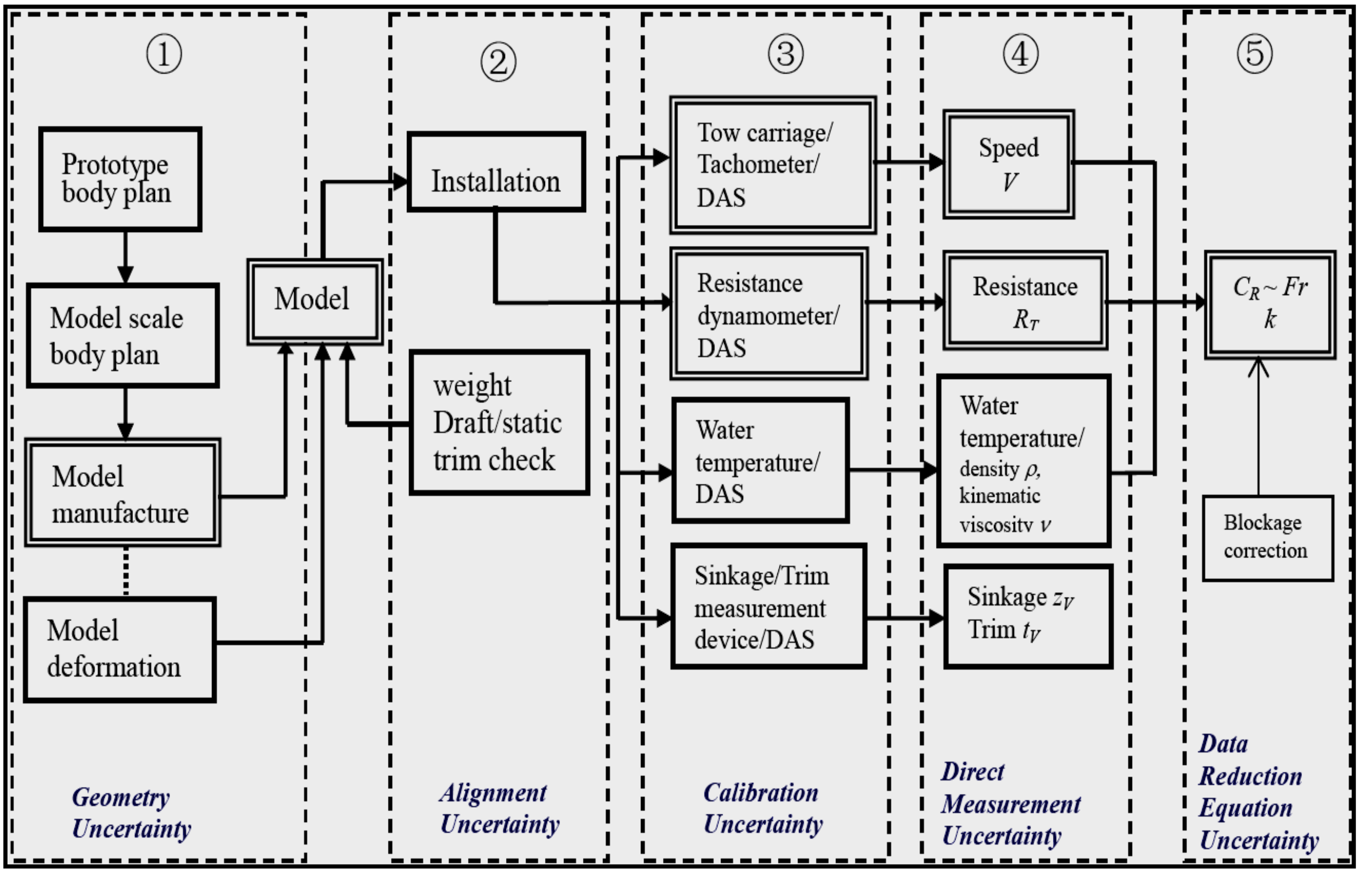


Figure 15 Schematic diagram of whole system for resistance testing.

11.6 Uncertainty Analysis of the Form Factor

Equation (15) can be re-written as

$$y = k + bx \tag{18}$$

where $y = C_T / C_F - 1, x = Fr^n / C_F$. The slope b and intercept k of fitting curve are determined by linear regression analysis as described in the ITTC (2008c). The standard uncertainty of the form factor k and the intercept are estimated from linear regression analysis as described in ITTC (2008c).

An example result for form factor from the China Ship Scientific Research Center (CSSRC) is shown in Figure 16. From the method in ITTC (2008c), the value of the form factor is 0.1574 with an expanded uncertainty of ±0.0097 (±6.2 %).

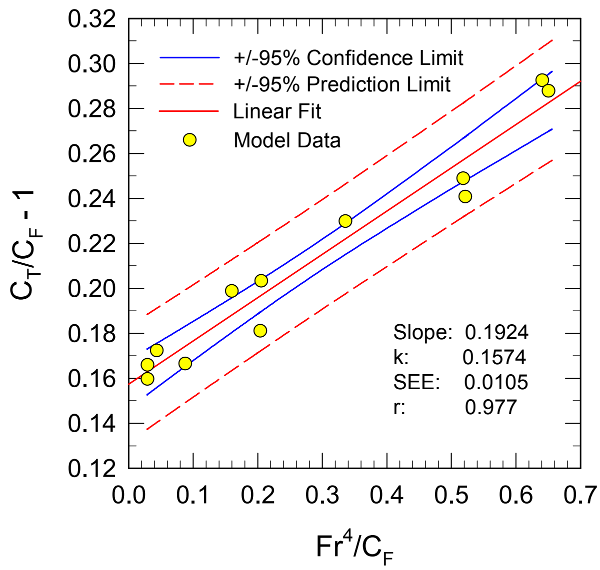


Figure 16 Curve fit for form factor.

11.7 General UA for Captive Model Testing

The following recommendation and suggestions are presented for application of this general guideline to a specific example of resistance model test as follows.

Testing Process. Uncertainty depends on the entire towing tank testing process, and any changes in the process can significantly affect the uncertainty of the test results.

Assessment Methodology. Uncertainty assessment methodology should be applied in all phases of the towing tank testing process including design, planning, calibration, execution and post-test analyses. Uncertainty analysis should be included in the data processing codes.

Simplified Analysis. Simplified analysis by prior knowledge, such as a database, tempered with engineering judgment is suggested. Dominant uncertainty components should be identified, and effort focused on those sources for possible reduction in uncertainty.

End-to-End Calibration. Through system or end-to-end calibrations should be performed with the same DAS and software for the test. A database of the calibrations should be maintained so that new calibrations can be compared to previous ones.

Benchmark Test. A laboratory should have a benchmark test with uncertainty estimates that is repeated periodically. A benchmark test will insure that the equipment, procedures, and uncertainty estimates are adequate.

Reference Test Condition. A reference test condition in a test series should be repeated about 10 times in sequence as a better measure of uncertainty and check on uncertainty estimates. Also, reproducibility of test results for a representative test condition should be checked in a long test with a duration of more than one day with a test at the beginning, middle, and end of the test series.

Reporting Uncertainty. A final report on test documentation should include an uncertainty statement with the following information: towing tank test process, measurement systems, data streams in block diagrams, equipment, and procedures.

12. FREE-RUNNING MODEL TESTS

12.1 Introduction

The following is a discussion for application of ISO (1995) to free-running model testing. The section discusses instrument calibration, model speed, and circles manoeuvres.

12.2 Instrument Calibration

The general procedure for calibration of instruments is outlined in ITTC (2008c) and discussed in section 8. Since a free-running model will have an onboard computer for data acquisition, calibration should be performed end-to-end as illustrated by the schematic in Figure 11. On-board measurements may include rudder angle, propeller shaft speed, and onboard instruments could include, accelerometers, compass, and vertical gyroscope for the measurement of pitch and roll angles. An example for calibration of a roll sensor for a free-running model is presented in Figure 12. Additional examples are in ITTC (2008c).

Another example is calibration of an accelerometer, which has some characteristics different from other transducers. Under static conditions, an accelerometer is an accurate inclinometer. An accelerometer is calibrated with local gravity, g, as the reference by inclination of the accelerometer. The relationships between local g and inclination angle for transverse and longitudinal acceleration are:

$$g = sin\,\theta \qquad (19a)$$

and for vertical acceleration the equation is

$$g = \cos\alpha - 1 \qquad (19b)$$

where g is local acceleration of gravity and α and θ are the tilt angles. The uncertainties in g from the tilt angle are then for the transverse and longitudinal components

$$u_g = (\cos\theta)u_\theta \qquad (20a)$$

and for the vertical component

$$u_g = (\sin\alpha)u_\alpha \qquad (20b)$$

Figure 17 shows the results from equations (19) for tilt angle uncertainties of ±0.2° and ±0.05°. The manufacturer's specification was nominally ±0.1 % for a 1 g range device or an uncertainty of ±0.001 g. As this figure indicates, the uncertainty in the inclinometer should be better than ±0.05° at the 95 % confidence level.

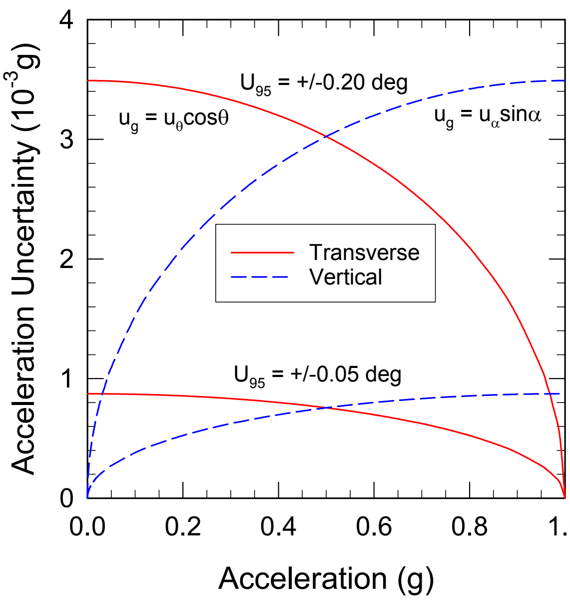


Figure 17 Uncertainty in local g from uncertainty tilt angle for accelerometer calibration.

The calibration results for a tri-axial accelerometer in the transverse direction are presented in Figure 18. The transducer was calibrated in approximately equal increments of local g. Results are shown for calibrations by two engineers with different instruments at five months apart. In the earlier calibration, the uncertainty in the angle was smaller than the symbols. The more recent calibration was performed by an instrument with an uncertainty of ±0.2° as indicated by the error bars.

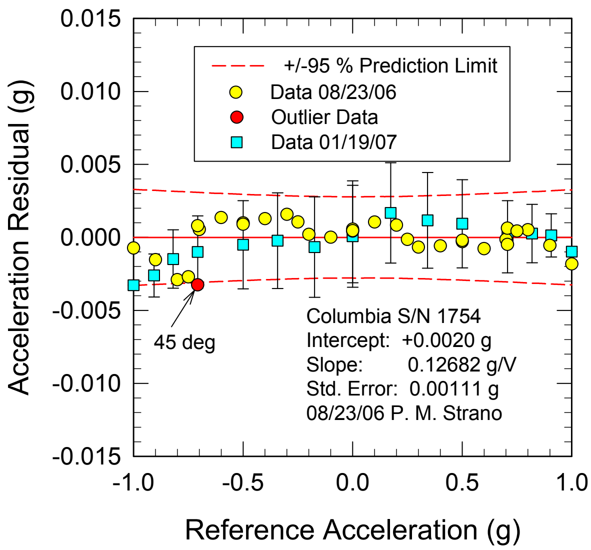


Figure 18 Calibration of a tri-axial accelerometer in the transverse direction from Chirozzi, et al. (2007).

12.3 Model Speed

For a free-running model, model speed is normally set by model propeller shaft speed. Shaft speed is monitored by an optical encoder. Since the encoder is inherently digital, it should be calibrated as described in ITTC (2008c). The calibration of the shaft speed sensor should be an end-to-end. In this case, the model motor should be driven at various set points, and the shaft speed measured.

In some cases, the signal from an encoder is converted to an analogue signal by a frequency to voltage (f-v) converter. An f-v converter is highly linear at calibration but is subject to drift. A better estimate of the uncertainty requires repeat calibration. An example of such a calibration with a function generator is shown in Figure 19 from Chirozzi, et al. (2007). In this case, the f-v converter has drifted in both slope and intercept relative to the earlier calibration. For the earlier calibration, the uncertainty was within ±0.1 rpm (revolutions /min) and a correlation coefficient of 0.999999993, but the largest difference in subsequent calibration was 1 rpm, 10 times the uncertainty in the previous calibration.

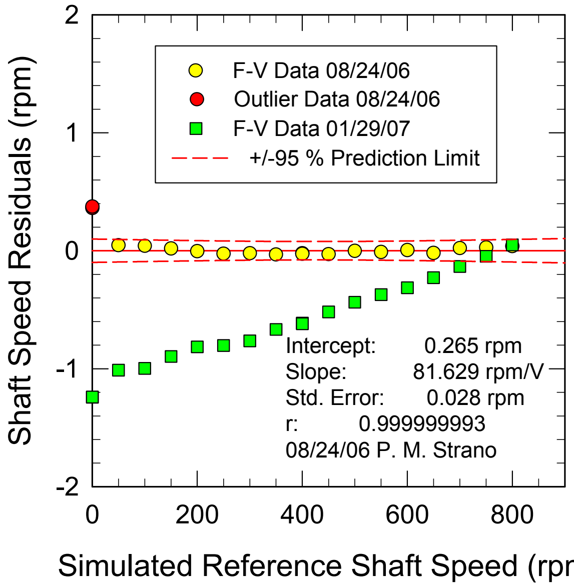


Figure 19 Calibration of an f-v converter for propeller shaft speed by a function generator.

Recently, model speed has been calibrated with a laser based indoor global positioning system (IGPS) or tracker. Typically, a second-order polynomial is the best curve fit to the data. For the measurements, none are NMI traceable; however, the uncertainty in NMI traceability is probably small in comparison to data scatter in the curve fit.

An example calibration is presented in Figure 20. In this case, a third-order polynomial was a better fit. This model had receivers located on both the stern and bow. The residuals are based on the curve fit for the stern receiver. The results for both the stern and bow are in good agreement. However, the stopwatch data are not consistent, in particular at the higher speeds where the response time for the stopwatch operator is shorter. In any case, the results indicate an uncertainty in speed of ±0.030 m/s from a conventional 95 % prediction limit, which is relatively constant over the calibration range.

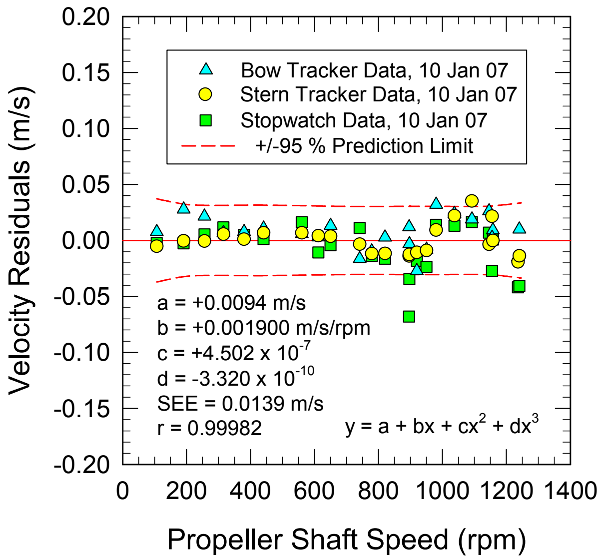


Figure 20 Velocity calibration for free-running model with a comparison of tracker and stopwatch data.

Basing model speed on towing carriage speed seriously under estimates the uncertainty in model speed for a free-running model. An example calculation is presented in ITTC 7.5-02-07-02.1 (2005), Appendix A. In the example, the estimated uncertainty is ±0.002 m/s. This method under estimates the uncertainty in speed by over a factor of 10. The erroneous assumption is that the model speed is the same as the carriage speed. The uncertainty in model speed is also dependent on the model operator's ability in control of the model speed relative to the carriage. A better estimate in the model speed is obtained by the methods previously described with the propeller shaft speed constant.

From the uncertainty estimate in velocity, the uncertainty in Froude number may be computed from ITTC (2008b). The uncertainty in Fr will be dominated by the uncertainty in velocity since the uncertainty in local g and length of the model, L, will be small. The uncertainty in Fr will then be

$$u_{Fr} = u_V / \sqrt{gL} \tag{21}$$

12.4 Circle Manoeuvres

With the IGPS, the characteristics of a free-running model in a circle manoeuvres may be evaluated. A typical result for a model track in a port turn is shown Figure 21 for the bow receiver. From the figure, the diameter and centre of the circle may be may be computed by a Gauss-Newton algorithm described by Forbes (1989).

The beginning of the circle is determined by an on-board compass when the compass heading changes by 90° from the initial straight line run. A complete circle is attained when the model has competed a 360° turn.

Throughout the manoeuvre, the velocity may be computed from either finite differencing at each time step or the end-points of a steady condition. The velocity during the track of Figure 21 is presented for finite differences at approximately 1 s time steps in Figure 22. Shown for comparison is the velocity from the model shaft speed.

In this case, the initial velocity from the straight-line end-point speed was $Fr = 0.370$ in comparison to the shaft-speed calculation of 0.369. The average steady speed in the turn was 0.262 ±0.042 m/s from the velocities at 1 s time steps. The speed loss in the turn was then 29 %. The uncertainty in the steady speed in the turn will be lower for the calculation from the circumference of the circle.

From analytic geometry, the radius of a circle in Cartesian coordinates is given by

$$r^2 = (x - x_0)^2 + (y - y_0)^2 \tag{22}$$

where r is the radius, x and y are the circle coordinates, and x_0 and y_0 is the coordinate for the centre of the circle. The centre coordinate and radius are computed from a Gauss-Newton method described by Forbes (1989).

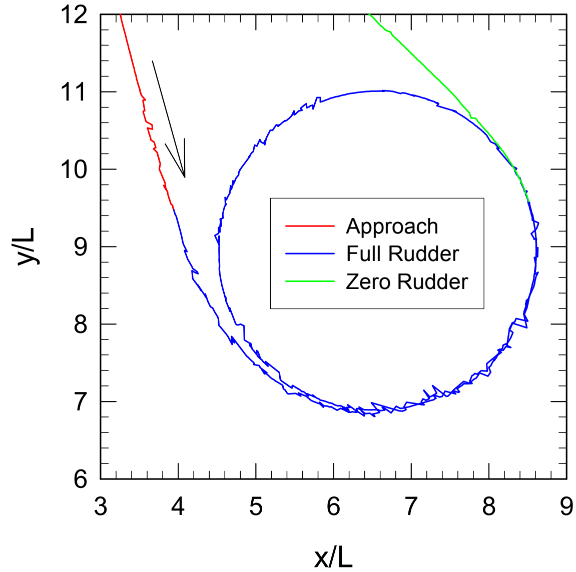


Figure 21 Track of a free-running model in a port circle manoeuvre at an initial $Fr = 0.370$.

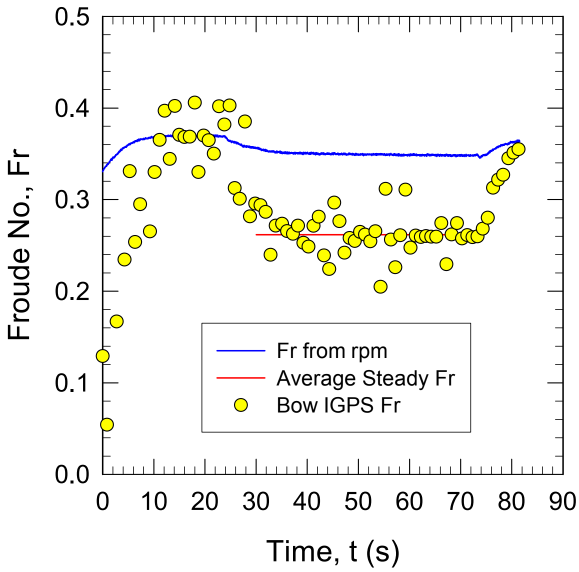


Figure 22 Free-running model velocity in a port circle turn at an initial $Fr = 0.370$.

The results from the coordinates in Figure 21 are presented in Figure 23. For the example, the mean non-dimensional radius, as computed by the Type A uncertainty method, is 2.0550 ±0.0018 (±0.090 %) at the 95 % confidence level with a coverage factor of 2, sample size of 736, and approximately 360° turn. However as discussed previously, a better measure of the uncertainty for the test condition should be computed from 10 repeat tests.

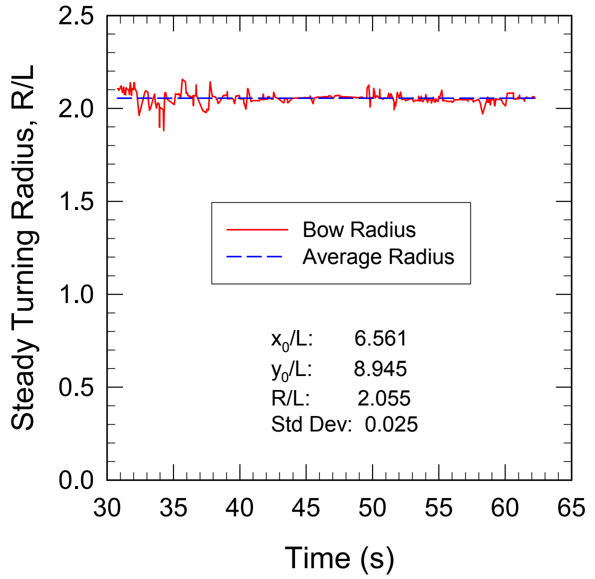


Figure 23 Steady turning radius from Gauss-Newton method.

13. UNCERTAINTY IN WATER PROPERTIES

At present, water density and viscosity may be computed from ITTC (1999). From these properties, their uncertainties may be determined by propagation of the uncertainty in water temperature. These properties should be updated with a statement of the uncertainty of the equations. Additionally, the source of the equations should be documented. Cavitation index is an important parameter in propeller performance; consequently, the vapour pressure should be added to the properties in this procedure.

One source on the properties of water is the International Association for the Properties of Water and Steam (IAPWS). The latest model for density and vapour pressure is IAWPS (1997), and for the viscosity IAWPS (2003). The uncertainties in these properties are summarized in Table 4.

Table 4 Uncertainty in fresh water properties from IAWPS.

Quantity	Symbol	Units	U_{95} (%)
Vapour Pressure	p_V	Pa	<0.03
Viscosity	μ	kg/ms	1.0
Density	ρ	kg/m^3	0.003

14. CONCLUSIONS

The following are the main conclusions from the work of the Specialist Committee on Uncertainty Analysis (UAC). All newly developed or revised ITTC UA procedures conform to the requirements of the IS0 (1995).

Five procedures were completed by the UAC as follows:

- 7.5-02-01-01, “Guide to the Expression of Uncertainty in Experimental Hydrodynamics”.
- 7.5-02-01-02, “Guidelines for Uncertainty Analysis in Resistance Towing Tank Tests”.
- 7.5-01-03-01, “Uncertainty Analysis: Instrument Calibrations”.
- 7.5-01-03-02, “Uncertainty Analysis: Laser Doppler Velocimetry (LDV)”.
- 7.5-01-03-03, “Uncertainty Analysis: Particle Imaging Velocimetry (PIV)”.

The list of symbols in Appendix J of the ISO (1995) was applied by the 25th ITTC UAC. The list was reformatted per the ITTC formatting requirements, and it is presented in Appendix A.

Meetings with members of the Powering Performance Prediction specialist committee resulted in the conclusion that the selection of the method of testing in tow tanks should not be based on UA. The purpose of UA is to highlight the accuracy or uncertainty in measured values, and not to recommend particular test methods.

The UAC discussed in detail a new procedure on forces and moments. Tow tank testing with PMM (Planar Motion Mechanism) has been evaluated as an example application. The UAC comments are included in Appendix B.

15. RECOMMENDATIONS

The following are the recommendations to the 25th ITTC from the Specialist Committee on Uncertainty Analysis (UAC)

Adopt the three new uncertainty analysis procedures as follows:

- ITTC Procedure 7.5-01-03-01, "Uncertainty Analysis: Instrument Calibration".
- ITTC Procedure 7.5-01-03-02, "Uncertainty Analysis: Laser Doppler Velocimetry Calibration".
- ITTC Procedure 7.5-01-03-03, "Uncertainty Analysis: Particle Imaging Velocimetry".

Adopt the two revised procedures as follows:

- ITTC Procedure 7.5-02-01-02, "Guidelines for Uncertainty Analysis in Resistance Towing Tanks Tests" Revision 2.
- ITTC Procedure 7.5-02-01-01, "Guide to the Expression of Uncertainty in Experimental Hydrodynamics" Revision 01

Adopt ISO (1995), "Guide to Expression of Uncertainty in Measurement", as the scientific basis for all existing, recommended, and future ITTC UA procedures.

Adopt the list of symbols in Appendix A for UA procedures. In addition, the International Vocabulary for Metrology (VIM, 2007) should be adopted as the dictionary for definitions of basic and general terms used in the ITTC UA procedures.

All benchmark test data should include uncertainty analysis statement and be reviewed by the UAC.

16. REFERENCES

Abernethy, R. B. and Benedict, R. P., 1985, "Measurement Uncertainty: A Standard Methodology", ISA Transactions, Vol. 24, No. 1, pp. 75-79.

Abernethy, R. B., Benedict, R. P., and Dowdell, R. B., 1985, "ASME Measurement Uncertainty", ASME Journal of Fluids Engineering, Vol. 107, No. 2, pp. 161-164.

AGARD AR-304, 1994, "Quality Assessment for Wind Tunnel Testing", Advisory Group for Aerospace Research and Development, North Atlantic Treaty Organization, Neuilly-sur-Seine, France.

AIAA S-071A-1999, "Assessment of Experimental Uncertainty With Application to Wind Tunnel Testing," American Institute of Aeronautics and Astronautics, Reston, Virginia, USA.

AIAA G-045-2003, "Assessing Experimental Uncertainty—Supplement to AIAA S-071A-1999", American Institute of Aeronautics and Astronautics, Reston, Virginia, USA.

ASME PTC 19.1-2005, "Test Uncertainty", American Society of Mechanical Engineers, New York.

ASTM E74-02, 2002, "Standard Practice of Force-Measuring Instruments for Verifying the Force Indication of Testing Machines," American Society for Testing and Materials, West Conshohocken, Pennsylvania, USA.

ASTM E617-97, 1997, "Standard Specification for Laboratory Weights and Precision Mass Standards", American Society for Testing and Materials, West Conshohocken, Pennsylvania, USA.

Bendat, J. S., and Piersol, A. G., 2000, Random Data: Analysis and Measurement Procedures, Third Edition, John Wiley and Sons, Inc., New York, New York, USA.

Carroll, R. J., Speigelman, C. H., and Sacks, J., 1988, "A Quick and Easy Multiple-Use Calibration-Curve Procedure", Technometrics, Vol. 30, No. 2, pp. 137-141.

Chirozzi, B. D. and Park, J. T., 2004, Unpublished results, Naval Surface Warfare Center Carderock Division, West Bethesda, Maryland, USA.

Chirozzi, B. D., and Park, J. T., 2005, Unpublished Data, Naval Surface Warfare Center Carderock Division, West Bethesda, Maryland, USA.

Chirozzi, B. D., Strano, P. M., and Park, J. T., 2007, Unpublished Data, Naval Surface Warfare Center Carderock Division, West Bethesda, Maryland, USA.

Coleman, H. W. and Steele, Jr., W. G.,1999, Experimentation and Uncertainty Analysis for Engineers, John Wiley, and Sons, Inc., New York.

Diritti, M., Arietti, M., Bellinga, H., Cannizzo, M., Delhez, F. J., Deneuve, F., Donat, D., Frankvoort, W., Harbrink, B., Van Der Kam, P. M. A., Kerkmann, W., Norman, R., and Rombouts, P., 1993, "Intercomparison Exercise of High Pressure Test Facilities within GERG", Technical Monograph, GERG TM, Groupe Européen de Recherches Gazières, Brussels, Belgium.

Donnelly, M. J. and Park, J. T., 2002, Unpublished results, Naval Surface Warfare Center Carderock Division, West Bethesda, Maryland USA.

Forbes, A. B., 1989, "Least-Squares Best-Fit Geometric Elements", NPL Report DITC 140/89, National Physical Laboratory, Teddington, UK.

Giacomo, P., 1981, "News from the BIPM", Metrologia, Vol. 17, No. 2, pp. 69-74.

ISO, 1995, "Guide to the Expression of Uncertainty in Measurement", International Organization for Standardization, Genève, Switzerland.

IAWPS (1997), "Release on the IAWPS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam", International Association for the Properties of Water and Steam, Erlangen, Germany.

IAWPS (2003), "Revised Release on the IAPS Formulation 1985 for the Viscosity of Ordinary Water", International Association for the Properties of Water and Steam, Vejle, Denmark.

ITTC, 1999, "Density and Viscosity of Water", ITTC Procedure 7.5-02-01-03.

ITTC, 2002a, "Resistance Test", ITTC Procedure 7.5-02-02-01, Revision 01.

ITTC, 2002b, " Model Manufacture, Ship Models", ITTC Procedure 7.5-01-01-01, Revision 01.

ITTC, 2008a, "Guide to the Expression of Uncertainty in Experimental Hydrodynamics", ITTC Procedure 7.5-02-01-01, Revision 01.

ITTC, 2008b, "Guidelines for Uncertainty Analysis in Resistance Towing Tank Tests", ITTC Procedure 7.5-02-01-02, Revision 02.

ITTC, 2008c, "Uncertainty Analysis - Instrument Calibration", ITTC Procedure 7.5-01-03-01.

ITTC 2008d, "Uncertainty Analysis - Laser Doppler Velocimetry Calibration", ITTC Procedure 7.5-01-03-02.

ITTC 2008e, "Uncertainty Analysis - Particle Imaging Velocimetry", ITTC Procedure7.5-01-03-03.

Kline, S. J. and McClintock, F. A., 1953, "Describing Uncertainties in Single-Sample Experiments", ASME Mechanical Engineering, Vol. 75, No. 1, pp. 3-8.

Kline, S. J., 1985, "The Purpose of Uncertainty Analysis", ASME Journal of Fluids Engineering, Vol. 107, No. 2, pp. 153-160.

Mattingly, G. E., 2001, "Flow Measurement Proficiency Testing for Key Comparisons of Flow Standards among National Measurement Institutes and for Establishing Traceability to National Flow Standards", Proceedings of the ISA 2001 Conference, Houston, Texas, USA.

Moffatt, R. J., 1982, "Contributions to the Theory of Single-Sample Uncertainty Analysis", ASME Journal of Fluids Engineering", Vol. 104, No. 2, pp. 250-260.

Moffat, R. J., 1985, "Using Uncertainty Analysis in Planning an Experiment", Journal of Fluids Engineering, Vol. 107, No. 2, pp. 173-178.

Moffat, R. J., 1988, "Describing the Uncertainties in Experimental Results", Experimental Thermal and Fluid Science, Vol. 1, No. 1, pp. 3-17.

NIST Handbook 105-1, 1990, "Specifications and Tolerances for Reference Standards and Field Standards and Measures," National Institute of Standards and Technology, Gaithersburg, Maryland, USA.

OIML R 111-1, 2004, "Weights of Classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3}, and M_3, Part 1: Metrological and technical requirements," Organisation Internationale de Métrologie Légale, Paris, France.

Otnes, R. K., and Enochson, L., 1972, Digital Time Series Analysis, John Wiley and Sons, Inc., New York, New York, USA.

Park, J. T., Cutbirth, J. M., and Brewer, W. H., 2002, "Hydrodynamic Performance of the Large Cavitation Channel (LCC)", Technical Report NSWCCD-50-TR—2002/068, Naval Surface Warfare Center Carderock Division, West Bethesda, Maryland USA.

Ross, S. M., 2004, Introduction to Probability and Statistics for Engineers and Scientists, Third Edition, Elsevier Academic Press, Amsterdam.

Scheffé, H., 1973, "A Statistical Theory of Calibration", The Annals of Statistics, Vol. 1, No. 1, pp. 1-37.

Taylor, B. N. and Kuyatt, C., 1994, "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results", NIST Technical Note 1297, National Institute of Standards and Science, Gaithersburg, Maryland, USA.

Visualization Society of Japan, 2002, Handbook of Particle Image Velocimetry, Morikita Publishing Co. Ltd. (in Japanese).

VIM, 2007, "International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM)", ISO/IEC Guide 99:2007, International Organization for Standardization, Genève, Switzerland.

Youden, W. J., 1959, "Graphical Diagnosis of Interlaboratory Test Results", Industrial Quality Control, Vol. XV, No. 11, pp. 133-1 to 137-5.

Youden, W. J., 1960, "The Sample, The Procedure, and The Laboratory", Analytical Chemistry, Vol. 32, No. 13, pp. 138-23A to 147-37A.

17. APPENDIX A: UNCERTAINTY ANALYSIS SYMBOLS

Main Reference: ISO (1995), "Guide to the Expression of Uncertainty in Measurement"

ITTC Symbol	Name	Description or Explanation
c_i	Sensitivity coefficient	$c_i = \partial f / \partial x_i$.
f	Function	Functional relationship between measurand Y and input quantities X_i on which Y depends, and between output estimate y and input estimates x_i on which y depends.
$\partial f / \partial x_i$	Partial derivative	Partial derivative of f with respect to input quantity x_i
k	Coverage factor	For calculation of expanded uncertainty $U = ku_c(y)$
k_p	Coverage factor for probability p	For calculation of expanded uncertainty $U_p = k_p u_c(y)$
n	Number of repeated observations	
N	Number of input quantities	Number of input quantities X_i on which the measurand Y depends
p	Probability	Level of confidence: $0 \le p \le 1$.
q	Random quantity	
$\overline{q}$	Arithmetic mean or average	
q_k	kth observation of q	k^{th} independent repeated observation of randomly varying quantity q
$r(x_i,x_j)$	Estimated correlation coefficient	$r(x_i, x_j) = u(x_i, x_j)/(u(x_i)\, u(x_j))$
s_p^2	Pooled estimate of variance	
s_p	Pooled experimental standard deviation	Positive square root of s_p^2
$s^2(\overline{q})$	Experimental variance of the mean	$s^2(\overline{q}) = s^2(q_k)/n$; estimated variance obtained from a Type A evaluation
$s(\overline{q})$	Experimental standard deviation of the mean	Positive square root of $s^2(\overline{q})$
$s^2(q_k)$	Experimental variance from repeated observations	

ITTC Symbol	Name	Definition or Explanation
$s(q_k)$	Experimental standard deviation of repeated observations	Positive square root of $s^2(q_k)$
$s^2(\overline{X}_i)$	Experimental variance of input mean	From mean $\overline{X}_i$, determined from n independent repeated observations $X_{i,k}$, estimated variance obtained from a Type A evaluation.
$s(\overline{X}_i)$	Standard deviation of input mean	Positive square root of $s^2(\overline{X}_1)$
$s(\overline{q},\overline{r})$	Estimate of covariance of means	
$s(\overline{X}_i,\overline{X}_j)$	Estimate of covariance of input means	
$t_p(v)$	Inverse Student t	Student t-distribution for v degrees of freedom corresponding to a given probability p
$t_p(v_{\mathrm{eff}})$	Inverse Student t for effective degrees of freedom	Student t-distribution for v_{eff} degrees of freedom corresponding to a given probability p in calculation of expanded uncertainty U_p
$u^2(x_i)$	Estimated variance	Associated with input estimate x_i that estimates input quantity X_i
$u(x_i)$	Standard deviation	Positive square root of $u^2(x_i)$
$u(x_i,x_j)$	Estimated covariance	
$u_c^2(y)$	Combined variance	Combined variance associated with output estimate y
$u_c(y)$	Combined standard uncertainty	Positive square root of $u_c^2(y)$
$u_{cA}(y)$	Combined standard uncertainty from Type A	From Type A evaluations alone
$u_{cB}(y)$	Combined standard uncertainty from Type B	From Type B evaluations alone
$u_c(y_i)$	Combined standard uncertainty	Combined standard uncertainty of output estimate y_i when two or more measurands or output quantities are determined in the same measurement
$u_i^2(y)$	Component of combined variance	$u_i^2(y) \equiv [c_i u(x_i)]^2$
$u_i(y)$	Component of combined standard uncertainty	$u_i(y) \equiv \lvert c_i \rvert u(x_i)$
$u_c(y)/\lvert y \rvert$	Relative combined standard uncertainty	
$[u(x_i)/x_i]^2$	Estimated relative variance associated with input estimate	

ITTC Symbol	Name	Definition or Explanation
$u(x_i)/\|x_i\|$	Relative standard uncertainty	
$[u_c(y)/y]^2$	Relative combined variance of output estimate	
$\frac{u(x_i,x_j)}{\|x_i x_j\|}$	Estimated relative covariance	
U	Expanded uncertainty	$U = ku_c(y)$
U_p	Expanded uncertainty of probability p	$U_p = k_p u_c(y)$
x_i	Estimate of input quantity	
X_i	Input quantity	ith input quantity on which measurand Y depends
$\overline{X}_i$	Average of input quantity	Estimate of the value of input quantity X_i, equal to the arithmetic mean or average of n independent repeated observations $X_{i,k}$ of X_i
$X_{i,k}$		kth independent repeated observation of X_i
Y	Estimate of measurand	
y_i	Estimate of measurand	Two or more measurands are determined in the same measurement
Y	Measurand	
Greek		
$\frac{\Delta u(x_i)}{u(x_i)}$	Estimated relative uncertainty of standard uncertainty	
μ_q	Expectation or mean	
ν	Degrees of freedom	
ν_I	Degrees of freedom	For standard uncertainty $u(x_i)$ of input estimate x_i
ν_{eff}	Effective degrees of freedom	In combined uncertainty $u_c(y)$ with $k_p = t_p(\nu_{\text{eff}})$ for calculation of expanded uncertainty U_p
ν_{effA}	Effective degrees of freedom from Type A	For combined standard uncertainty determined from standard uncertainties obtained from Type A evaluations alone
ν_{effB}	Effective degrees of freedom from Type B	For combined standard uncertainty determined from standard uncertainties obtained from Type B evaluations alone
σ^2	Variance	Estimated by $s^2(q_k)$
σ	Standard deviation	Estimated by $s(q_k)$

18. APPENDIX B: UNCERTAINTY IN FORCES AND MOMENTS FOR CAPTIVE MODEL TESTING - PMM APPLICATION

18.1 Introduction

The ITTC manoeuvring general committee developed a procedure for uncertainties in forces and moments in captive model testing. The UA committee reviewed the PMM (Planar Motion Mechanism) testing as an example application.

This new procedure for forces and moments is a modification to the UA section that exists in ITTC procedure 7.5-02-06-02, Revision 02, 2005. The UA section in this procedure was eliminated and replaced by a recommended new procedure, which is the subject of this review.

The PMM proposed procedure for forces and moment is quite detailed, but some additional details and clarification are necessary for the procedure to be understandable by a wider audience. Technical comments about this procedure are outlined below. Editorial comments are not included here.

The review focuses on the fundamentals of uncertainty analysis as stated in the previous UA general procedure ITTC 7.5-02-01-01, 1999 and its basis in AIAA S-071-1995. The comments here are not based on ISO (1995); however, the PMM procedure should be revised for consistency with ITTC (2008a) and ISO (1995). The following comments may also be applicable to previous ITTC UA procedures and examples

18.2 Technical Comments

Jitter Method. Due to the complexity of the equations, uncertainty should be propagated by the jitter method as outlined in Moffat (1982) and ISO (1995). The analysis is essentially a central finite differencing method. The procedure would be simplified, and the need for the tables of sensitivity coefficients would be eliminated. The three tables of sensitivity coefficients contain between 12 and 14 coefficients each for the three primary data reduction equations. Moffat (1982) recommends analytical computation of the sensitivity coefficients only for simple equations and the jitter method for more complex equations.

Assumptions. Too many assumptions are made concerning uncertainty estimates. The word “estimated” appears 28 times and the word “assumed” nine (9) times. All measurements should be traceable to a National Metrology Institute (NMI) with a known uncertainty. In the USA, the NMI is NIST (National Institute for Standards and Technology). NIST is nowhere mentioned in this procedure.

Model Length. The uncertainty in model length is assumed based upon an ITTC requirement of ±1 mm from ITTC (2002b). The uncertainty should be based upon on a manufacturing tolerance traceable to an NMI or direct measurement with an instrument traceable to an NMI. Laser based measurement systems are now available for direct measurement of model manufacturing accuracy. The PMM procedure contains data on David Taylor (DTMB) Model 5512. All DTMB ship models are currently measured with a laser based measurement system.

Drift Angle. Uncertainty in model alignment is assumed to be ±0.03°. This measurement should be based upon an angular measurement traceable to an NMI with a known uncertainty.

Mass Uncertainty. In several locations in the report, the uncertainty in mass is estimated as the RSS (Root Sum Square) of the masses, which assumes that the uncertainty in the masses is uncorrelated. In general, the uncertainty in mass is correlated, and the uncertainty in mass is the sum of the uncertainties. Typically, all masses are calibrated against the same reference standard. Consequently, the uncertainty in mass is correlated, and equation (8b) should be applied for the combined uncertainty. This issue is discussed in OIML R111-1 (2004) and section 8.4 of this report. Additionally, the PMM procedure does not identify the class of masses for calibration.

Force. Local g and buoyancy are not included in the calculation of force from mass. See ASTM E74-02 (2002) and section 8.4 of this report for further discussion. In any case, most of the uncertainty is in the calibration of the strain gage amplifier. The uncertainty in force calibration in the PMM procedure probably under-estimates the calibration uncertainty.

Calibration and Acquisition. The terminology in the procedure, calibration and acquisition, is somewhat confusing. Different terminology is suggested. As apparently applied in this procedure, these are two parts of the calibration process. Calibration consists of three parts: (1) uncertainty in the reference standard for the calibration of individual calibration points, (2) the uncertainty in the curve fit from linear regression analysis, (3) Type A (precision) uncertainty in the mean value of the data points if the calibration data are acquired from a time series by a DAS. The UAC has written a procedure, which describes the process. The new UA procedure, ITTC (2008c) is summarized in section 8 of this report. In the PMM procedure, the uncertainty in the curve fit is defined as 2× *SEE*. This method describes the uncertainty at the time of calibration and does not define the uncertainty in application to the test. Application to future events is describe by statisticians as the prediction limit. If *SEE* is applied, the UAC is recommending 3× *SEE* as the prediction limit.

Water Density and Temperature. In the PMM procedure, the uncertainty in the temperature probe is stated to be ±0.2 °C. The specific type of probe, amplifier, and NMI traceability are not documented. Attainment of a temperature uncertainty of ±0.l °C is non-trivial. Usually, a probe is connected to an electronic amplifier, which includes linearization. An uncertainty estimate for the calibration of the temperature electronics is not included. Realistically, the uncertainty in temperature is more likely ±0.5 to ±1 °C. The procedure number for the density equation is not stated (ITTC, 2002a).

Precision Limit. In general, data are acquired with a DAS. Data is then recorded as a time series, the uncertainty in the mean values is computed from 12 repeat where the standard deviation is divided by the square root of 12, the number of repeat tests. In some cases, the true uncertainty can be estimated only with repeat experiments due to uncontrolled elements in the test. In hydrodynamic test facilities, repeat tests at all conditions is cost prohibitive. In that case, a repeat test is performed for a representative test. In such a test, an estimate is then obtained for the standard deviation. The estimated uncertainty is then 2 times the standard deviation for other test conditions since only one sample is taken. The PMM procedure should clarify if this is the case or whether 12 tests were repeated for a better estimate of the mean value.

Carriage Speed**.** The uncertainty estimate is described for carriage speed, and the measurement details are outlined. Although in principle, the description is correct, an alternative approach is recommended. The sources of the uncertainty in the length and time have not been identified. Also, errors apparently exist in the calculations, which are not discussed here.

Carriage speed is reported as 2× *SEE* from their measurements. Only 3 speeds are listed in their table. An alternate approach is suggested. The uncertainty in the carriage velocity could be defined by repeat runs as describe by Forgach (2002). At least 10 repeat runs should be completed at each speed, or 10 speeds of approximately equal increments. Figure 2 is an example of repeat runs at a single carriage speed. By the repeat method at the same speed, an estimate can be made from Table B1 based on data from the PMM procedure. Three repeat runs exist in this table for each of 3 speeds.

Due to the limited number of runs, the coverage factor is determined from the Student *t* at the 95 % confidence limit and 2 degrees of freedom. As the table indicates, the uncertainty is speed dependent. Also, the uncertainty estimate is about half the value reported in the PMM procedure, ±0.010 m/s in comparison to ±0.0053 m/s by the present method.

Table B1 Uncertainty in carriage speed from repeat runs.

i	U_{ci} (m/s)	Mean	Std Dev	t_{95}	U_{95} (m/s)	U_{95} (%)
1	0.7840	0.7826	0.00123	4.30	0.00531	0.68
2	0.7823					
3	0.7816					
4	1.5601	1.5589	0.00125	4.30	0.00539	0.35
5	1.5590					
6	1.5576					
7	2.2631	2.2626	0.00064	4.30	0.00277	0.12
8	2.2619					
9	2.2629					