RESISTANCE SESSION

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REPORT OF RESISTANCE COMMITTEE

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INTRODUCTION

The terms of reference recommended by the 10th I.T.T.C. were "To study the fundamentals of ship resistance with attention to wavemaking resistance and the relationship between the various components of resistance".

The Resistance Committee has held five formal meetings each lasting two days: in Paris in November 1963, in Wageningen in May 1964, in Hamburg in September 1964, in Paris in June 1965 and in Feltham in January 1966.

A questionnaire was circulated to all member organizations in order to discover the extent of the use of blockage corrections and of turbulence detection techniques, and to ascertain the frequency of occurrence of tank "storms". The results are presented in this report.

With the direct encouragement and support of the Committee an English translation of a Russian text book "Theory of waves and wave-making resistance" by A. A. Kostiukov has been prepared by Professor Oppenheimer of the University of Iowa, and edited by Dr. J. N. Newman and Professor L. Landweber.

In its final presentation the Committee has divided the subject into five topics:

I Definition of terms
II Viscous resistance
III Wave resistance
IV Low resistance hull forms
V Restricted water

DEFINITION OF TERMS

The Committee considers that it is essential to the proper understanding of its subject that the important quantities Viscous Resistance and Wave Resistance should be defined in a way that has a clear physical meaning. The most logical basis for doing this is to relate the components of resistance to the method of energy dissipation involved, and the Committee therefore recommends as follows:

Viscous resistance is the component of resistance associated with the expenditure of energy in generating vorticity, vortices and turbulence.

Wave resistance is the component of resistance associated with the expenditure of energy in generating gravity waves.

These two quantities together represent the total dissipation of energy from a normal displacement vessel and they should, therefore, be additive to give the total resistance. At high speeds energy may, however, be dissipated in spray and a third term, Spray Resistance, is required:

Spray resistance is the component of resistance associated with the expenditure of energy in generating spray.

Viscous resistance can be regarded as having two components:

Skin friction resistance, the component of resistance obtained by integrating tangential forces over the hull surface.

Viscous pressure resistance, the component of resistance obtained by integrating pressures due to the thickness of the boundary layer and the wake.

VISCOUS RESISTANCE

The proposed definitions of viscous and wave resistances do not involve any assumptions regarding their independence and a rigorous approach must assume that both are functions of F_n and R_n. Thus the total resistance coefficient C_T for a smooth ship may be represented as follows:

$$C_T = C_V(R_n, F_n) + C_W(F_n, R_n) \tag{A}$$

To perform this sub-division experimentally it is necessary to evaluate the components independently. The viscous resistance can be obtained directly from a wake survey as long as the energy of eventual longitudinal vortices is negligible. A limited number of such surveys have been carried out and the results are illuminating although rather inconsistent, probably due to the use of different techniques of analysis. The Committee recommends that further surveys should be carried out in as many cases as possible, the surveys being so designed that alternative analysis techniques can be applied and compared.

Good approximations to the viscous resistance for ships operating at low F_n are the resistance derived from a submerged double model, or the resistance of a surface model at very low F_n. Provided that turbulence stimulation is effective, these two methods generally give results which are in close agreement but which differ from the results from wake surveys at higher speeds, implying a significant dependence of viscous resistance on wavemaking. Consideration should also be given to a proposal by Horn that an average speed increase may be estimated from measurements of the sinkage of models.

Since the processes for the direct evaluation of viscous resistance are too complex and two time-consuming for day to day use, reliance must be placed on analytical or empirical estimates. These are generally based on the concept of a correlation line and an associated form factor and require the assumption that viscous and wave resistance are independent thus:

$$C_T = (1+k)C_1(R_n) + C_2(F_n) \tag{B}$$

Active research is in progress in an attempt to establish a relationship between k and suitable parameters defining the hull form. The aim of geosim research is to determine whether equation (B) is a sufficiently good engineering approximation to equation (A). Geosim series tend to be limited in range, however, by turbulence stimulation problems at the lower end, and by tank boundary interference at the upper end, and progress is therefore closely linked to progress in the development of blockage corrections.

In relation to the correlation problem itself the Committee strongly recommends that a three dimensional extrapolation should be adopted. Five formulae have been proposed for the evaluation of the form factor, one by Lap, one by Granville, and three by Hughes. In addition considerable work has been done in Japan, and a new basis is being evolved by Hughes. The Committee does not therefore consider that at this stage it can recommend a specific procedure but suggests that tanks should try the various methods and compare the results with the eventual aim of standardisation of a procedure.

Of the two components of viscous resistance, neither viscous pressure resistance nor skin friction resistance can yet be directly evaluated. The total pressure resistance, which has been measured in a few cases, contains both wave resistance and viscous pressure resistance. Skin friction can then be derived by subtracting pressure resistance from total resistance and results obtained in this way suggest a significant relationship between skin friction and wavemaking. Viscous pressure resistance can then be regarded as the difference between skin friction resistance and viscous resistance but, as a small difference between large derived components, accurate determination is unlikely. Direct measurement of skin friction by determining local shear stress would be invaluable and research in this direction should be pursued. The use of Preston tubes for this purpose is being developed and there are indications that hot film probes can be adapted to this work. Both these techniques are critically dependent on calibration of the devices in pipe flow conditions where local friction can be determined from measurements of pressure loss.

Separation is a function of boundary layer flow which is increasing in importance with the increase in fullness of large bulk carriers. Hot film probes have been successfully used to detect separation on model scale. Factors which can cause changes in viscous resistance are of importance for three reasons; firstly as a potential means of reducing viscous resistance, secondly as a cause of irregularities in model experiments, and thirdly as a possible means of controlling scale effects. Five factors have been considered by the Committee:

(a) *Boundary layer control* Boundary layer thickness can be reduced by the application of continuous suction over the afterbody. The quantity of water to be pumped is, however, very large, and the small suction openings are liable to choke. The method has been used in an attempt to reduce the scale effects on propulsion experiments but the suction introduces secondary effects which make analysis difficult.

(b) *Effect of Additives* It has been demonstrated that the addition of dilute solutions of certain long molecule polymers either to the water in a towing tank or by ejection through openings in the sides of a model hull can reduce viscous resistance by up to 60%. Full scale application to torpedoes has been tried but the quantity of solution involved in full scale application to ships appears to be prohibitive. It has been suggested that the effect could be used to reduce scale effects in resistance and propulsion experiments, although it is doubtful if the flow in contaminated water is similar in detail to that in pure water at higher R_n. Considerable research is in progress in an effort to understand how these substances operate and it is currently assumed that the effect is a visco-elastic one, depending mainly on the dimensional and mechanical properties of the molecules.

(c) *Flexible skins* Experiments have not given any clear indication that skin friction can be reduced by the so-called Kramer skin. Theoretical studies suggest that a material with the right elastic and damping properties might reduce turbulent motion, but it is considered unlikely that a material with these properties in the proper proportion could be produced. Further progress in this direction is doubtful.

(d) *Special surface treatment* All materials of comparable roughness have the same skin friction and the only properties to be sought in a new material are the permanence of the surface condition, and its resistance to biological contamination.

(e) *Gas lubrication* Little work has been done in this field. The general indications are that friction would be reduced by a continuous air film and probably slightly increased by a mixture of air bubbles and water.

Three of these factors have a significant bearing on model experiments. A propeller introduces a form of boundary layer control by suction which has been shown to delay or eliminate separation on full afterbodies. Long chain molecules can occur naturally as a bi-product of the decay of algae, and the formation of air bubbles on model hulls can occur in certain circumstances. An enquiry circulated to 55 member organizations yielded 32 replies of which 9 referred to changes in model resistance which were attributed to changes in the water, 3 quoting severe "storms". One of these "storms" resulted from algae and analysis of the water produced direct evidence of the presence of the polymers known to cause such changes. The second was due to uncontrolled bubble formation associated with the method of treatment of the main public water supply, and the third followed refilling of the tank after cleaning. A water board expert reporting on one of these incidents recommended in very strong terms that towing tanks were insufficiently aware of the need to maintain a high standard of water and he recommended the universal adoption of biological filtration.

Practice in regard to turbulence stimulation varies widely. Of the 32 replies received from members, 12 tanks reported having equipment for turbulence detection, generally hot film probes, but none reported a standard practice of checking that stimulation is adequate. Two researches on the efficiency of turbulence stimulators indicate that studs of suitable dimensions are more reliable and effective than trip wires or sand strips.

WAVE RESISTANCE

In spite of considerable effort no significant improvement has been achieved on the Michell integral, which requires the ship to be thin and wave slopes to be small. The application of slender body theory as distinct from thin ship theory has given results which are in worse agreement with experimental results. If it is accepted that practical ships are not thin, but that wave slopes are reasonably small, then it is possible to programme solutions. A direct approach in this direction has been attempted in the US but the computation of the linearised free surface condition has proved to take too long to allow useful application. A further attempt in which the singularity distribution was derived by satisfying the boundary condition at the hull surface and at a rigid water plane, and thence calculating the wave resistance due to this singularity distribution, gave a result which exaggerated the humps and hollows of the resistance curve more than does the Michell solution. On the other hand an approach on these lines has been successfully used as a basis for the derivation of forms of low wave resistance, a problem which is discussed in the next section.

Considerable development has taken place in the technique and theory of measuring wave resistance. The basis of this is the computation of wave energy from measurements of the surface disturbance. Although the method is approximate, in so far as the energy calculations are based on the application of linearised theory to the real disturbance, the errors are unlikely to be significant except in the wake, where the relationship of wave height to energy is suspect. Nevertheless a number of applications of the approach have been made and some very interesting results have been obtained. In one case of a thin parabolic hull, summation of viscous resistance measured by a wake survey, and wave resistance derived from the surface disturbance has given a good approximation to the total resistance. Two main methods have been developed depending on whether the wave profiles used are measured transversely (transverse cut method) or longitudinally (longitudinal cut method), and a linking theory exists between these two methods. In towing tank work both have an application, the transverse cut because it can be applied directly on a towing carriage, and the longitudinal cut because it is made at fixed points in the water and could be applied not only to models independent of the towing carriage, but also to full scale ships. The Committee considers that the direct measurement of wave resistance in this way is a very powerful technique which should be developed fully.

LOW RESISTANCE HULL FORMS

There are two possible approaches to the problem of developing low resistance hull forms:

(i) to select optimum characteristics from a large store of data, either by inspection or by statistical methods

(ii) to try to optimise the associated hydrodynamic phenomena.

The first of these is adopted by the majority of ship designers but a systematic statistical approach is only of recent development. This has been applied successfully in the case of trawlers and current indications are that significant improvements can be obtained.

The use of wave resistance theory as a guide to optimum design is of also recent origin and appreciable progress has been made. The direct approach to the problem is to minimise the resistance integrals and thence derive a corresponding singularity distribution, but this is limited by the overall limitation of the linearised theory. The alternative is to try to minimise the wave pattern and this has practical advantages, firstly that it can be applied to specific regions, for example the forebody, where boundary layer effects are small, and secondly that results can be verified by visual inspection of the wave system, or by wave pattern measurements.

The development of bulbous bows is closely linked with the theoretical problem of minimising wave resistance, and optimised forms frequently have bulbs. The success of bulb and ram bows in reducing the resistance of large tankers, particularly in the ballast condition, cannot be entirely attributed to wave cancellation since the reduction in resistance is of the order of the total wave resistance, and the effect on viscous resistance therefore requires to be investigated. There is evidence from wind tunnel experiments that trailing vortices are generated from the bilges of very full forms, and it has been suggested that the bulb may reduce such secondary flows.

RESTRICTED WATER

A theoretical approach to the problem of restricted water is subject to the same limitations as the wave resistance calculation. The problem is to derive an augmented velocity distribution and then to use it to calculate a resistance correction. No new developments have taken place in this direction, but the use of available theoretical solutions should be investigated.

The Committee does not consider that blockage corrections are important in the larger tanks for experiments with models of reasonable dimensions. Smaller tanks have a more difficult problem because laminar flow prevents small models being used. Of the 33 replies received in response to the questionnaire, 14 mentioned some use of blockage corrections, 6 used a blockage correction as standard practice either for all work or for specific classes. Where corrections were reported the following methods were used: Hughes (5) Schuster (3) Taniguchi (3) Emerson (1) Landweber/Schlichting (1).

The Committee is unable at this stage to make a specific recommendation but has made arrangements for a comparison of these methods to be carried out. Blockage corrections assume considerable importance in relation to geosim experiments and it is important to devise a test procedure for studying geosim results which is not dependent on any assumptions regarding correlation methods.

When it is necessary to simulate full scale conditions of restricted water additional problems arise both in generalising the data to take account of variations in the geometry of the channel, and in carrying out the experiments and analysis. In particular, additional precautions are necessary to ensure full turbulence, and extra stimulators are frequently required. In extremely restricted conditions the interaction of viscous and wave resistance becomes more serious and the validity of standard extrapolation procedures becomes suspect. As in the case of the basic correlation problem, direct measurements of components of resistance by wake survey and wave pattern measurements are necessary in order to establish a test for approximate methods.

SUMMARY OF RECOMMENDATIONS

(a) Experimental research to determine viscous resistance by wake survey methods should be carried out in as many cases as possible, in order to assist in developing a datum for the comparison of correlation methods. The study of resistance and flow on double models should be extended.

(b) Experimental research to determine wave resistance from the wave pattern should also be carried out in as many cases as possible in order to establish the validity or otherwise of summations of this and wake survey results, because it provides a comparison for calculated wave resistance, because it offers the possibility of full scale determination which would allow the correlation datum to be extended to high R_n, and because it shows immediately the influence of design changes on wave resistance.

(c) Detailed flow studies should be pursued actively because of the possibility of assessing the effects of local hull features such as bulbs, and the effects of separation, and because it may provide information on the possibility of optimising hull forms in relation to skin friction.

(d) Analytical approaches to the evaluation of wave resistance should be pursued actively because of their relation to the design of optimum hull forms.

(e) A three-dimensional correlation method should be adopted as soon as possible but a geneal test for the validity of specific formulations will not exist until more data for (a) and (b) have been obtained and the Committee cannot at this stage recommend a particular formulation.

(f) Blockage corrections are only important for smaller tanks using relatively large models, and in geosim studies. The most widely used correction is that of Hughes. The Committee has arranged for a comparison of formulations to be made and cannot yet make a specific recommendation. The effect of restricted water on the wave resistance derived by linear theory should be checked experimentally in as many cases as possible.

(g) Of the factors which can influence viscous resistance only two are of general importance to towing tanks, additives and air bubble effects, in both cases because of their relation to the reliability of model experiments.

(h) The Committee strongly recommends that all tanks should aim to maintain a high standard of water quality, and that all deviations or storms should be thoroughly investigated by a water expert.

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APPENDIX I

VISCOUS RESISTANCE

by L. Landweber (*Inst. of Hydraulic Research*)

The following note represents a "state-of-the-art" report on certain specific aspects of viscous resistance, and its relation to a fuller understanding of ship resistance. The following topics are discussed in detail:

(1) Correlation Lines and Form Factors.
(2) Measurement Errors.
(3) The Betz-Tulin Method for Obtaining Viscous Drag.
(4) Variation of Viscous Drag with Froude Number.

Correlation Lines and Form Factors

Two new methods of estimating the viscous resistance of a ship form have been proposed by Hughes. In one1) he recommends for the ship-model correlation formula

$$C_V = 0.0620r\,(\log_{10} R_N - 2.18)^{-2}$$

where R_N is the Reynolds number and r is a form factor, to be determined at a sufficiently low Froude number F_N so that the wave resistance coefficient C_W is negligible. This formula is derived as a mean from the analysis of several geosim series. However, because of the wide dispersion of the data about their mean values, and indications of variation of the form factor with Froude number, the case for the above correlation fomula does not seem to be strong.

Dr. Hughes may agree with the above opinion, since he has proposed an alternative correlation formula in a more recent paper[2].

$$\textcircled{C}_V = \frac{\text{viscous drag}}{\nabla^{2/3} v^2} \cdot \text{const.} = x(\log_{10} R_N - 2)^{-2}$$

in which ∇ is the displacement, v the velocity of the model, and x is a form factor, given in terms of a curve and formula as a function of the block coefficient and the ratio $S/\nabla^{2/3}$ where S is the wetted surface area. His analysis shows that, for most models, this frankly empirical procedure gives form factors within ±3 percent of the mean.

Geosim data have also been recently reanalyzed by Professor Tamiya[3] on the basis of the formula

$$C_t - C_{f0} = KC_{f0} + C_w$$

in which the resistance coefficients are based on wetted surface area, and C_{f0} is taken as Schoenherr's flat plate friction coefficient. If the form factor K is a constant, graphs of $C_t - C_{f0}$ versus C_{f0} at constant Froude number would plot as a straight line of slope K and intercept C_w. However, his results show slopes varying with Froude number, and sometimes negative.

It may be recalled from the 1962 report of the Resistance Committee that Yokoo[4] had also found a variation of the form factor with Froude number. Since this variation follows the humps and hollows of the wave resistance, Maruo[5] has suggested that this result is due to scale effect on wave resistance, rather than wave effect on viscous drag.

Another formula for the form factor, based on an approximate but rational theoretical analysis, taking into account the effect of the increased potential-flow velocities on the frictional resistance, the growth of the boundary layer on the viscous pressure resistance, and an empirical formula for airfoil pressure drag, is given by Sasajima and Tanaka[6] in the form

$$K = \sqrt{\frac{\nabla}{L^3}} \left(2.2 C_b + \frac{P}{C_b} \right)$$

where C_b is the block coefficient and the quantity P is given by a curve as a function of the beam-length ratio.

A remarkable paper by Taniguchi[7], which reports on the results with a geosim series of a 45,000 ton tanker, for which the viscous drag was also determined by wake surveys, and the wave drag by surface-profile measurements, will also be mentioned here. He found that the form factor varied from about 1.34 at $F_N = 0.15$ to 1.2 at $F_N = 0.21$, and that in the ballast condition, the wave resistance coefficient is not zero at $F_N = 0.1$ as is assumed in a method of Hughes for determining the form factor.

In studies of form effect, some method of turbulence stimulation is usually applied to assure an early transition to a turbulent boundary layer. Hot-wire studies of the boundary layer of models of two ore carriers, reported by Brard[8], indicate that a helicoidal wire is superior to studs as a turbulence stimulator. Since the surface area over which the laminar boundary layer extends can be determined by means of the hot wire, it was recommended to test without turbulence stimulation, and to determine the resistance with a fully turbulent boundary layer by calculating a correction for the known extent of the laminar boundary layer. An interesting feature of this proposal to allow a larger area of laminar boundary in model tests is that the boundary-layer thickness and the width of the wake would be reduced at the stern, so that the flow in this region would be more nearly similar to that of the ship, and the scale effect on the wave resistance would be reduced.

These various investigations of the concept of a viscous drag given by a correlation line and an associated form factor have not yielded a strong confirmation of the validity of the procedure. Geosim studies indicate that one can select an average form factor for a ship form with which one could probably predict prototype resistance with more realistic "roughness allowances" than with the fixed I.T.T.C. or A.T.T.C. correlation lines. Since the form factors obtained have varied significantly with Froude number, one may well question, as Maruo has done, whether the term $(1+K)C_{f0}$ derived from geosim tests gives the viscous resistance or some mixture of viscous and wave effects.

MEASUREMENT ERRORS

It is well understood that many pitfalls must be avoided to obtain a meaningful value for the resistance of a ship model. Even if the dynamometer and the speed-measuring system are accurate and reliable, the length of run is long enough so that a steady pattern of boundary layer, wake and wave system is established, and the model has been verified to be sufficiently true and smooth, there still remains the uncertainty of the extent of the laminar boundary layer, the error in velocity due to residual currents and surges in the towing tank, the important effect of minute quantities of certain contaminants in the water, and the error due to the tank boundaries.

An overall measure of these errors is given by the results of tests of the I.T.T.C. 15.8-foot standard model. It was stated in the 1962 report of the A.T.T.C. Resistance Committee that a report of tests in four British tanks "presents a discouraging picture of dispersion of data, not only among different tanks, but also in repeated tests at the same tank." Two additional reports on tests of this model indicate a much improved level of consistency. Hughes[9] finds, after 200 tests of the standard model at NPL over a period of four years, that the resistance remains within 1 percent of its mean value. Taniguchi[10] found that his results in the Mitsubishi tank at Nagasaki agreed well with the NPL mean curve, after applying the blockage correction.

$$\Delta v/v = 1.4\,\mathrm{m}(L/B)^{3/4}$$

Comparison with the results by other Japanese tanks shows a maximum discrepancy of about 3 percent. In the Mitsubishi tank the yearly standard deviation varied from 1 to 1.7 percent; no "storm" phenomenon was observed. Nor was there a significant correlation of ΔC_t with the temperature distribution in the water.

A subsequent study by Hughes[11] of the resistance of of a glass plate, about 44 inches long, resulted in values which fluctuated within ±1 percent of the mean at 4 and 5 fps, and 1-1/2 percent at 3 fps, with extreme variations of about twice these values. Since the results tended to be in phase, i.e. all high, medium, or low at the same time, and in qualitative agreement with the tendency of the results with the standard model, it was concluded that the variation was a genuine hydrodynamic one. Use of the glass plate as a daily check for correcting resistance to a standard has been suggested.

A large reduction in the resistance of ship models in the Fort Steyne tank in December 1960 was reported by Barnaby and Dorey[12]. For a standard model the viscous resistance was about 20% less than its usual value. After a thorough investigation it was concluded that the phenomenon was due to contaminants, either a polysaccharide material or certain algae. It was found that the effect could be eliminated by chlorination, or by emptying the tank and introducing fresh water.

The importance of the residual currents in a towing tank was pointed out by Scott[13] on the basis of an analysis of the resistance measurements of the I.T.T.C. standard model in four tanks, the Teddington Nos. 1 and 2, the St. Albans, and the Mitsubishi tank. He concluded that the principal source of both random and systematic error is current drift between runs, even when anti-drift curtains are used, and suggested that an accurate current meter or an improved system for drift damping is required.

The Betz-Tulin Method for Obtaining Viscous Drag

1. An important step in the derivation of the Betz-Tulin formula for the viscous drag of a ship model requires that the potential flow outside the wake be continued analytically into the wake region in such a way that the boundary conditions at the free surface over the wake region be satisfied. It should be borne in mind that the Betz method does not require that the Betz sources be found, nor that there be a unique set of them; all that matters is that it be possible to find a set of such sources.

The problem may be formulated mathematically as follows: It is required to find a potential function $\phi(x,y,z)$ in a region R bounded by a submerged surface S, the assumed border of the wake, and above by the free surface S_0 on which the conditions $\frac{\partial\phi}{\partial n}=0$ and pressure $p=0$ are satisfied. On the submerged surface S both ϕ and $\frac{\partial\phi}{\partial n}$ are assumed to be known. Here n denotes distance increasing in the direction of the outward normals to S and S_0. The Betz source distribution within R would be given by

$$\nabla^2\phi = 4\pi q(x,y,z) \qquad (1)$$

where $q(x,y,z)$ is the volume density of the source distribution.

Application of Green's third formula, modified to suit the present problem, gives the relation

$$\begin{aligned} 4\pi\phi(x,y,z) &+ \iiint_R \left(\frac{1}{r}+H\right)\nabla^2\phi(\xi,\eta,\zeta) \\ &+ \iint_{S_0} \phi\frac{\partial}{\partial n}\left(\frac{1}{r}+H\right)dS_0 \qquad (2) \\ &= \iint_S \left[\left(\frac{1}{r}+H\right)\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}\left(\frac{1}{r}+H\right)\right]dS \end{aligned}$$

where r is the distance between a pair of points of R,

$$r=[(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]^{1/2}$$

and H is a harmonic function such that $\frac{1}{r}+H$ satisfies the free-surface boundary conditions. Then (2) indicates that the potential function $\phi(x,y,z)$ also satisfies the free-surface boundary conditions. By expressing $\nabla^2\phi$ in terms of second differences and replacing the integrals by quadrature formulas, (2) assumes the form of a set of linear equations for an array of values of ϕ on S_0 and in the interior of R, with the right member of (2) giving a set of known values. Thus, although the execution of the indicated numerical solution of (2) may be extremely difficult, the indicated procedure would yield an approximate solution of the stated problem, and hence, by (1), an approximate determination of the Betz source distribution.

2. A refinement of the Betz-Tulin formula for the

viscous drag of a ship model will now be presented. According to the Lagally theorem, the force on a wake source is proportional to the potential flow at the location of the source. Both Betz and Tulin have assumed that, to a sufficient accuracy, this velocity may be taken to be that of the free stream, and have expressed the force on the sources in the form

$$D_{SB} = -\rho \int_{\omega} U(u_1 - u)\,dS \qquad (3)$$

where ρ is the mass density of the fluid, ω the area intersected by a transverse section of the wake, U the free-stream velocity, u the actual longitudinal velocity component, u_1 the corresponding velocity component of the analytically-continued potential flow.

By Gauss' flux theorem, the Betz source distribution q satisfies

$$4\pi \int q\,d\forall = \int_{\omega} (u_1 - u)\,dS \qquad (4)$$

the volume integral extending over the volume of the wake from the body to the control section ω. Applying the Lagally theorem, we obtain for the total force on the sources

$$D_S = -4\pi\rho \int q u_1\,d\forall \qquad (5)$$

Hence, applying the mean-value theorem and (4), we find

$$D_S = -\rho \int_{\omega} \bar{u}_1(u_1 - u)\,dS \qquad (6)$$

where $\bar{u}_1$ is a mean value of u_1 over the region bounded by the control surfaces.

If $\bar{u}_1$ is replaced by the local value u_1, the expression for the viscous drag becomes

$$D_V = \int_{\omega} \Big[p_0 - p + \rho u(U - u) + \frac{1}{2}\rho (U - u_1)(U - 2u + u_1) \Big] dS \qquad (7)$$

instead of the formula of Reference 1),

$$D_V = \int_{\omega} \Big[p_0 - p + \rho u(U - u) + \frac{1}{2}\rho (U - u_1)^2 \Big] dS \qquad (8)$$

VARIATION OF VISCOUS DRAG WITH FROUDE NUMBER

Determinations of viscous drag of ship models by means of wake surveys have been made at several towing tanks since the last I.T.T.C. Conference. For a 10-foot model of a Series 60, 0.60-block model it was found by Wu and Landweber[14] that the viscous drag varies from about 10 percent less than the I.T.T.C. Correlation Line at $F_N = 0.24$ to about 5 percent about it at $F_N = 0.30$. This result has recently been confirmed with the same model by Key[15], using improved equipment and a refined calibration procedure. Thus there appears to be a strong variation of the viscous drag with Froude number for this model, especially for $F_N > 0.24$.

For one of Wigley's parabolic ship forms, wind-tunnel and towing-tank studies of the resistance components have been reported in two papers, one by Shearer and Cross[16], the other by Lackenby[17]. Excluding results from wake traverses at sections closer than a half model length behind the model, it was found that the viscous-drag coefficients from $F_N = 0.27$ to 0.48 are nearly constant. No results were given at lower Froude numbers, except at 1/10 length behind the model, where the method of determining the viscous drag could not be considered reliable. This work should be extended to lower Froude numbers to determine whether the viscous drag varies with Froude number.

For a 4-meter model of Inui's S 201 ($B/L = 0.12$, block coefficient=0.80, prismatic coefficient=0.68), Sharma[18] obtained good agreement between the viscous drags from a wake survey and a double model, but the dispersion of his wake-survey results was too great to determine the effect of Froude number. At the largest Froude number ($F_N = 0.32$), the wake-survey result is 25 percent less than that from the double model.

Taniguchi[7], whose work on a geosim series of a 45,000 ton tanker has already been mentioned, determined the viscous drag from wake surveys with a 7 meter model of this series. Neglecting results from wake measurements at 0.35 L as unreliable, he gives values of the viscous drag at only four Froude numbers, insufficient to define a curve. These viscous-drag values are about 10 percent greater than the results from the geosim form-factor analysis previously described.

The frictional part of the viscous drag has also been investigated since the last Conference. Townsin[19] measured the pressure distribution and the total resistance of a 1/50th scale Victory model at Froude numbers between 0.21 and 0.28, and found that the frictional resistance, obtained by subtracting the pressure drag from the total, is much greater than is usually assumed, varying from 1.34 C_{f0} at $F_N = 0.21$ to 1.56 C_{f0} at $F_N = 0.26$, where C_{f0} refers to Hughes' friction formula for a flat plate. His results also indicated the occurrence of separation at the stern at $F > 0.26$. This suggests that both the frictional and viscous pressure resistance coefficients vary with Froude number.

For the parabolic ship form mentioned previously[16,17], the curve of frictional resistance, also obtained by subtracting the pressure resistance from the total, varies strongly with Froude number, showing humps and hollows in opposite phase to the curve of wave resistance, and does not follow the trend of either the A.T.T.C. or I.T.T.C. extrapolators. On the other hand, the results with a double model in a wind tunnel, obtained in the same manner, are practically coincident with the 1957 I.T.T.C. line.

Studies of the effect of the free surface on flow separation by Chow[20)] at Iowa and Lin[21)] and Webster[22)] at Hydronautics indicate the probable mechanism of the variation of the viscous drag with Froude number. The primary effect of a free surface is to introduce zones of strong adverse piezometric-pressure gradients, where the surface-wave profile along a hull rises from a trough to a crest. This causes the boundary layer to thicken rapidly, the shear stresses to diminish, and the viscous pressure drag to increase, by amounts which depend on the shape of the wave profile, and consequently on the Froude number. The possibility of the occurrence of separation is also increased.

Chow's study of the flow about a vertical strut piercing the free surface showed that separation occurred farthest upstream at the Froude number $F_N=0.25$, and then moved downstream with further increases in the Froude number. It was observed that the presence of a free surface affects the occurrence of separation in two ways:

(a) At Froude numbers less than about 0.40, the adverse piezometric pressure gradient at a free surface exceeds that on the same form when deeply submerged, and hence the occurrence of a zone of separation near a free surface is enhanced.

(b) Because of the curvature of the streamlines at the crests and troughs of a free surface, secondary flows may occur in the boundary layer. This has been suggested as the explanation of isolated zones of separation below the free surface. It appears to be necessary to take these secondary flows into account in any theory for the prediction of separation and viscous drag of shiplike forms at a free surface.

In the Hydronautics study a procedure for computing the line of separation on a ship form, employing linearized gravity-wave potential-flow theory to compute the pressure gradients, and Cooke's method (assuming small deviations between directions of flow inside and outside the boundary layer) for computing the three-dimensional boundary layer, has been applied to the Series 60, 0.60 block form. In remarkable agreement with Chow's observations on an entirely different form, it was also found that the earliest separation occurred at $F_N=0.25$. Observations at the Davidson Laboratory, however, failed to show any separation, as one might expect for so fine a form. This indicates that the calculation procedure used is as yet unsuited for making absolute predictions, although the agreement with Chow's observations on a fuller form suggests that the predicted trend is probably correct.

Townsin[23)] has also found that the extent of the separation zone varies with Froude number. Hot-film surface probes were used to detect separation on a 100-inch Taylor Model of 0.875 block coefficient, an 8-foot "Victory" ship model, and an 8-foot BSRA model of 0.80 block coefficient. The Taylor model showed much more separation at $F_N=0.122$ than at $F_N=0.183$. No separation was detected on the Victory model. The BSRA model showed a larger separation zone at $F_N=0.119$ than at $F_N=0.219$. Although these results indicate that the viscous drag varies with Froude number, the trend of the extent of the separation zone is opposite to that observed by Chow and predicted by Webster.

It appears, from the aforementioned research, that both the frictional resistance and the viscous drag of a ship model are functions which vary sensitively with the ship form and the Froude number. The refinement of methods of obtaining the viscous drag, and its routine use by towing tanks, seems to be a necessary step in the development of a more rational procedure for predicting the resistance of a ship from model tests.

APPENDIX II

REPORT ON WAVE RESISTANCE

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1. Since the design of ships always involves compromise between various desired characteristics the theory of wave resistance should, from the point of practical application, solve the following two main problems:

a) the determination of wave resistance of a given hullform,

b) reduced wave resistance development of hull forms for which the reduced wave resistance is incorporated in the compromise of the desired hull characteristics.

These two main problems have produced different trends in research although an adequate theory is able to handle them both.

In order to restrict the synopsis we consider only ship moving uniformly and rectilinearly at or under a smooth surface of deep water.

2. Looking back through the years a big step forward in this field of naval hydrodynamics was taken when Kelvin published his linearized theory of ship waves. True, in this theory the most drastic approximation of all was made, i.e. the entire moving hull was assumed to be collapsed into a moving point singularity thus leaving one parameter, namely its strength to describe the ship. At first sight one would perhaps not expect such a theory to produce much of practical and theoretical value. However, that is not so because Kelvin's theory yields, even when it is treated approximately by the means of stationary phase, important aspects of the wave pattern created by the ship, aspects which are in good accord with observations. For example, the existence of two different wave trains, the confinement of the disturbance within a sector with an angle fixed independent of the speed and the speed wave-length relation are all correctly given by this theory. It is quite remarkable that such a drastic approximation gives these results; the only phenomenon left out is the wave interference.

Kelvin's theory was superseded by the more advanced linearized theory by Michell. This theory was however overlooked and forgotten for many years. In his theory Michell assumed that the entire hull of the ship was collapsed into a slender vertical disc—a so-called thin ship. Thus, apart from assuming the well known linearized conditions, the boundary condition at the wetted surface of the ship was satisfied on the vertical centre line plane of the form rather than on its wetted surface.

Later Havelock developed his linearized theory for a source moving under the free surface and suggested generating bodies by locating a continuous distribution of sources and sinks or directed dipoles on the vertical centre line plane of the body or on a surface on or inside its wetted surface. This theory is in many ways more descriptive than Michell's approach and hence appeals more to engineers. By placing the continuous source and sink distribution over the vertical centre line plane of the body and by determining the source and sink strengths for a thin ship and neglecting the disturbance of the free surface, i.e. assuming unbounded water or extremely low Froude numbers, it will be found that not only are Michell's and Havelock's wave resistance formulae identical but their velocity potentials are completely identical too.

3. The search for a better approximation to the determination of the source strengths in the linearized problem has continued in later years. Assuming a source distribution on the wetted surface the determination of the source strengths involves the solution of a Fredholm integral equation of the second kind over the wetted surface. Hess and Smith have indicated how this problem can be solved when assuming a finite number of sources over the wetted surface of the ship and neglecting the disturbance of the free surface. Thus their method has, from the view point of the wave resistance theory, some grave limitations but is, on the other hand, valid for arbitrary bodies.

It is understood that a computing program is under way in connection with the solution of the Fredholm integral equation for the determination of the source strengths in which the disturbance of the free surface is taken account of.

If the distribution is over the vertical centre line plane the determination of the source strengths, when the free surface is taken into account, involves the solution of a Fredholm integral equation of the first kind.

4. In the modern derivation of Michell thin-ship approximation of the velocity potential it is common to begin with the exact boundary conditions for the problem, assuming irrotational flow of an inviscid fluid. The various unknown functions are then assumed to be analytic functions of some parameter describing the thinness of the ship. After various manipulations the original boundary-value problem is replaced by a new linearized one whose solution is taken to be an approximation to the desired one. However, there is some uncertainty as to the proper restriction for this approximation to be a good one. Renewed research into Michell thin-ship theory has made it clear that the resistance computed by this theory is the most important term in the more exact wave resistance expression, assuming the beam/length ratio to be small but with no restriction on the beam/draught ratio or the slopes of the wetted surface.

5. In Michell's theory the thin ship is, in effect, held fixed in space with the water streaming past. Attempts have been made to generalize Michell's theory in allowing the ship to be a floating rigid body performing small oscillations relative to a motion of a translation with constant velocity. It then turns out that the rolling, yawing, and swaying oscillations are all damped while the surging, pitching and heaving oscillations are not damped to the first order. These results are what one would expect and comes about because of the way in which the slenderness parameter is defined for the gen-

eralized Michell theory, i.e. from the assumption that the ship is a thin disc, the plane of symmetry of which is the vertical centre line plane of the ship. In the absence of friction, the only mechanism available for damping is the creating of surface waves which carry off energy to infinity. For a Michell ship an oscillation of infinitesimal amplitude of first order in pitching, heaving and surging would create surface waves having amplitudes of second order at least, while the other possible oscillations of a Michell ship, having amplitudes of first order, can cause surface waves of first order. Since for actual ships, however, the pitching and heaving modes are damped to about the same extent as the other modes one is again reminded of the slogan that we should be concerned with what type of theory fits the ship and not what type of ship fits the theory. One obvious method of overcoming the difficulty is to retain the theory and simply add damping terms with coefficients to be fixed empirically. An other and more satisfactory possibility is to explore other ways of linearization, for example by assuming the draught/length ratio to be small. The analysis in the latter case is however still incomplete and suffers from drawbacks similar to the thin ship, but with the vertical and transverse modes reversed.

6. There has been a good deal of research into the application of the linearized slender body theory, which have been well established in aerodynamics, to the prediction of wave resistance and ship motions in general. This is a linearized theory based on the assumption that the hull is slender with respect to both beam and draught. Hence the hull is regarded as being collapsed into a line segment and the effect of the hull is generally replaced by appropriate distributions of singularities along this line segment. Alternatively, use is made of Michell thin-ship theory allowing the draught to become small, i.e. by expanding Michell's integral for small draughts. It is optimistically suggested by some authors that the slender body theory will eventually lead to a rational and successful theory for predicting ship motion in waves. It should be noted however that the wave resistances predicted by the different linearized slender body theories are not in agreement with each other over the range of Froude numbers.

7. The problem of finding a hull form which presents the least resistance under conditions imposed by practical requirements has always been a goal for the naval architects. Indeed the vast efforts at the experimental towing tanks aims at fulfillment of this goal. On the other hand, mathematical methods have been tried to the determination of the ship form of minimum resistance under simplifying conditions.

Separating the total resistance of a ship into a component due to viscosity and a component due to wave-making both these components will depend, among other things, on the ship's form. The relation between the viscous resistance and the ship form is however not sufficiently understood to make an analysis feasible. Under extremely simplifying conditions the wave resistance is given by the celebrated Michell's integral involving either the functions which define the shape of the hull or, as shown by Havelock, the functions which define the distribution over the vertical centre line plane of sources and sinks or directed dipoles by which the hull is assumed to be generated. The problem to minimizing Michell's integral is a mathematical problem in the calculus of variations and has stimulated the interest of hydrodynamicists for a number of years. In any practical applications of the theoretical results which may be obtained one is forced to accept the assumption that the linearized theory of wave motion is an accurate enough approximation to the actual fluid motion about the hull and that the exclusion of viscosity will have no serious effect.

The problem of minimum wave resistance can be tackled only when some side conditions are imposed. This is due to the fact that without any restrictions there is a solution which creates no waves. In fact, we know of a class of singularities which does not create waves in the linearized case. Even with the side condition of fixed displacement and draught the minimum wave resistance problem has no solution. Thus the minimum problem which has been tackled has usually been less general in scope, having been simplified to find the curve of sectional area for a form whose transverse sections are given by a prescribed equation. The simplest case is the elementary wall-sided ship of infinite draught. The condition of constant displacement is now interpreted as constant water plane area. It has been found however that the problem has really no solution in the strict sense since the end conditions are violated. A similar situation appears in the case of finite draught because of the logarithmetic singularity in the kernel of integral equation which has to be solved. In these cases Michell's integral has been interpreted as involving the functions defining the shape of the hull.

The situation is however somewhat different when Michell's integral is interpreted as involving functions which define the distribution of dipoles over the vertical centre line plane. In this case the problem of minimizing the wave resistance of infinite deep struts under a single side condition of constant sectional area has a solution. The same is true in the case of elementary ships of finite draught. The minimizing problem with the side conditions of constant displacement, draught and beam, i.e. constant block coefficient is of great practical interest but has no solution. If, instead of constant beam, constant moment of inertia of the water plane with respect to the transverse axis is assumed a unique solution exists. The minimizing problem with the side conditions of constant displacement and constant wetted surface has also been investigated. The solution of this problem is, approximately, equivalent to the ship form for which the sum of wave resistance and skin friction is a minimum.

8. Attempts have been made to find a slender ship form of minimum wave resistance but the solution involves only ship forms of a restricted class and is by no means the best among all admissible forms. Alternatively, the aysmptotic form of the minimum solution of Michell's integral for vanishing draught has been investigated but the results are not yet quite conclusive.

9. The use of bulbous bows in order to obtain reduced wave resistance for ships has become very popular with ship owners in later years. The waves left far behind any body is, in the linearized case, an integral summation of elementary sine and cosine waves. These elementary waves travel in all directions θ satisfying $-\pi/2 \leqq \theta \leqq \pi/2$, where θ is the angle between the normal to a straight line through the ridge of each elementary wave and the direction of motion of the body. The amplitude function of each elementary wave is a function of θ. Analysed in this manner it is not difficult to determine which waves are due to the bow, which are due to the stern and so on for an elementary form.

If the strength of the source distribution is a cosine function in the horizontal direction and, for example, a constant in the vertical direction, i.e. the ship has approximately U-shaped sections, the two wave systems far off of the ship can be represented by an integral summation of elementary sine waves focused at the bow and stern respectively all with positive amplitude functions. It is well known that the waves created by a point doublet, (i.e. a sphere) can be represented by an integral summation of elementary sine waves focused in the free surface vertically over the centre of the doublet, all with a negative amplitude function. This suggests that by a suitable choice of both the strength and the vertical position of doublet at the bow it will be possible to obtain a considerable reduction of the bow waves for this type of model at a chosen speed. It is however not possible to obtain a waveless bow by this device. Indeed no finite ship can have zero wave resistance.

Similarly a suitable doublet at the stern would reduce the stern waves at a chosen speed when assuming inviscid fluid and when the effect of propellers and other attachments are neglected. However, the influence of the viscosity and the wake near the stern is so important that the stern problem should be considered separately in connection with practical application.

The bow waves are purely positive sine waves if the strength of the source distribution is an even power series and are purely cosine waves if an odd power series is used. It has been found that these positive sine bow waves can be completely eliminated by a concentrated doublet distribution (i.e. a cylinder) of suitable strength along the forward end of the distribution and extending vertically to infinite depth. As the infinite deep doublet distribution can not be realized in any practical application, it must be truncated at a finite depth. The truncation invalidates the perfect cancellation of the waves generated by each systems. However the deeply submerged part does not influence too much the surface waves.

Although a doublet is a good device for cancelling positive sine bow waves it has no application in connection with cosine bow waves. A singularity of one step higher order than a doublet is a quadrupole. The waves created by a quadrupole consists of cosine elementary waves focused in the free surface vertically over the centre of the quadrupole, all with a negative amplitude function.

The cosine bow waves formed by a source distribution which is an odd power series can be completely eliminated by a quadrupole distribution of suitable strength along the forward end of the distribution and extending vertically to infinite depth. A quadrupole itself in a uniform stream does not produce a closed body, but it may, when combined with a doublet line. This suggests that these quadrupoles can be used to improve the form of the bulb and hence its effect when both cosine and sine waves are present.

Negative cosine bow waves can also be eliminated by a source distribution of suitable strength along the forward end of the distribution and extending vertically to infinite depth. Of course, a sink distribution at the afterbody of the ship has to be employed in this case in order to have a closed body.

In general, the linearized bow waves consists of elementary waves which have different characteristics in each direction of advance. Therefore it is not possible to match in all direction of advance the three main characteristics of elementary waves, i.e. focus point of wave, phase and amplitude of elementary waves from bulb's with those from a general ship bow so that all waves are cancelled everywhere. Indeed, the ship shapes and hence the source distribution for ships with bulb has to be chosen with care because:

a) positive sine bow waves require a doublet bulb,
b) negative cosine bow waves require a source bulb,
c) strong positive cosine waves with weak sine bow waves require a doublet bulb combined with either a source (sink) or a quadrupole bulb.

In this type of research the source distribution has, in general, been taken as given and the hull form (stream lines of a double model) determined under the assumption that the disturbance at the free surface is negligible. This hull form is then used in the model experiments.

For a model assumed to create only positive sine bow and stern waves a forward shift of the wave phase has been observed. This necessitated fitting a bulb which projected quite a bit forward of the bow in order to obtain a favourable wave resistance instead of locating the centre of the bulb at the stem, which is the position the linearized first order theory suggested.

Various explanations for this shift have been put forward, the most recent being based on higher order wave theory. In the expression of Green's function in the classical wave theory a particular term, which is supposed to be small, is neglected in the first order theory. For a particular simple source distribution this term was included in Green's function and it was found then that this led to a forward shift of the wave phase compared with the results from the first order wave theory. This suggests that the discrepancy between experiments and the first order wave theory as regards the wave phase may be due to this term. It indicates also why the use of ram bow is favoured in many cases.

A sizable bulb can also be used for the purpose of guiding the water flow toward the flat bottom from the beginning rather than letting the water spill over the bilges to reach the bottom at a later stage. Used in this manner a bulb prevents the formation of eddies and hence reduces the form drag. If in addition the combination hull and bulb produce low wave resistance the total resistance is reduced. This has been put forward as an explanation why a large bulb on tanker models can result in a great reduction in total resistance at low Froude numbers.

10. The method of steep descent has lately been put forward as a possible method for the purpose of modifying given hull forms in order to obtain reduced wave resistance.

Note is taken of the close relationship between the Euler-Lagrange equations of the classical calculus of variations and the gradient of a function in a finite-dimensional vector space. Indeed, the Euler-Lagrange equations may be regarded analogously as the gradient of a functional in function space. This suggests that the wave resistance as well as the hull form should be described in terms of a finite number of the same variables. Considering each of these variables to represent a coordinate in a finite-dimensional vector space, the gradient of the wave resistance in that vector space is determined. If these defining variables are changed in a direction as nearly parallel to this gradient vector as the necessary side conditions permit we decrease the wave resistance as rapidly as possible for a given amount of motion through the vector space. In this manner we produced an improved hull form. Following the trajectory far enough we may approach a stationary value. This description leads to the employment of a method of steep descent for the calculation of improved hull froms.

11. The Froude scaling method of analyzing ship model resistance data presuppose that the total resistance experienced by a ship may be decomposed into two parts, i.e. the viscous component which depends on the Reynolds number, and the residual part which depends on the Froude number. Although this was an ingenious suggestion made years ago for solving a troublesome scaling problem, grave difficulties have been encountered in the practical application of this scaling law. One of these difficulties is the interaction between ship waves and boundary layer. Although this problem has been recognized for years no one has actually looked deeper into its nature until recently.

The total vectorial force experienced by a body moving in a real fluid consists of two terms, namely the vectorial pressure resistance drag which consists of the surface integral (taken over the wetted surface) of the normal fluid stress, and the vectorial skin friction drag which consists of the surface integral (taken over the wetted surface) of the fluid shear stress. For flows of infinite extent, the pressure drag, which vanishes in inviscid flow, is due to the pressure distribution being modified from the potential flow by the presence of the boundary layer, and is usually in this case called the form drag. In the presence of a free surface, however, gravity and capillary surface waves now exists to further modify the pressure distribution. In this case the pressure drag includes not only the form drag of the viscous origin but also the resistance which is intrinsically due to wave-making. The concept of wave resistance arises only when the far-field is brought into the picture. Otherwise both viscous form drag and wave resistance are contained in the surface integral of the normal fluid stress. A feature of the surface wave and boundary layer interaction is that it occurs as a whole entirety. The existence of the boundary layer effects the external potential flow and its wave system; the waves in turn influence the pressure distribution in the boundary layer and hence also skin friction. It follows that the shape and structure of the boundary layer in the presence of waves must be determined with the interaction taken into account.

A complete calculation of the boundary layer for a given body which moves in or beneath a free surface and produces gravity waves may, at present, be too complicated. Since we are mainly interested in determining with adequate accuracy overall characteristics, e.g. skin friction of momentum thickness, rather than details of the flow the momentum-integral equation for two-dimensional incompressible boundary layers is made use of together with certain more recent developments in the boundary layer theory. With the boundary layer determined in this manner the skin friction is deduced as well as an equivalent ship form, this being equal to the original form plus the displacement thickness. The wave resistance, which is determined in the usual manner but for the equivalent ship form rather than for the original form, consists now of the following terms:

a) The wave resistance experienced by the ship when moving in an inviscid fluid.

b) The wave resistance experienced by a form corresponding to the boundary layer represented by its displacement thickness and moving in an inviscid fluid.

c) The body-boundary layer interaction.

One should not expect too much of such a research at present because too many simplifying assumptions have been made. The important part is however that a start has been made towards an investigation of the real problem which concerns three-dimensional flow of a turbulent boundary layer near the condition of separation.

12. Although we are concerned with the three-dimensional ship wave problem the research carried out in connection with two-dimensional waves should not be overlooked.

The question of determining the plane potential flow past a circular cylinder beneath a free surface under gravity is an old problem which recently, however, has undergone renewed investigations.

The first approximation is to replace the cylinder by the dipole potential, modified so as to satisfy the linearized free-surface condition. However, no closed body is generated by the first approximation. In particular the front and rear stagnation points are on different streamlines. Moreover it appears likely that at no finite order of approximation is a closed body generated. This finding may have a bearing on the related linearized problem of determining ship-like bodies from given source distributions.

As to the second approximation, circumstances are known in which the effect of the second approximation to the Bernoulli equation at the free surface is more important than to include modifications to the flow due to the fact that the singularity distribution which generates the body in an infinite fluid no longer does so exactly in a fluid with a linearized free surface. This finding also may have a bearing on the related ship wave problem where consideration has been given to whether in numerical terms some improvement in accuracy might be achieved by going back to the exact boundary condition on the actual wetted surface.

Second-order effects are present even with a linear free surface condition. Large as the corresponding corrections to the first-order linearized expressions for the wave-induced forces on the cylinder are, the non-linear second order corrections are considerably larger. This clearly indicates the importance of being quite clear under what circumstances the linearization process has a rational justification. It can not be said that we have reached this stage yet in the theory of ship waves.

APPENDIX III

SHIPS OF MINIMUM WAVE RESISTANCE

by G. WEINBLUM (*Univ. Hamburg*)

1. The development of forms of *least resistance* is a problem of utmost practical importance; it represents a central problem in ship-building science, in analytical ship hydrodynamics, as well as in model work. This statement applies particularly to *wave* resistance because of its strong variability with the ship form. Theoretical research on ships of minimum wave resistance is advancing now on a broad front and is beginning to inspire experimental investigations. Notwithstanding meritorious attempts the application of this research to design is still in a rudimentary state because (1) theoretical results are based on the *ideal fluid* concept, and (2) even under this idealized assumption linearized solutions have only been obtained for restricted classes of hull forms. As an example of (2), we mention that no reasonable expression exists so far for the determination

of the wave resistance of ships with a transom stern.

It is our purpose to summarize briefly what so far has been achieved, and to indicate what should be done in the future to meet urgent needs of naval architecture. Theoretical work must finally lead to the determination of ships of minimum *total* resistance or rather of optimum *performance* including propulsion effects*, later even considering seaway conditions. From the point of view of technical application the theoretical methods so far available rarely represent more than a heuristic approach. It must be emphasized, however, that finding forms of low resistance rather than "exact" optimum forms represents the technical problem, thus mitigating the rigor of analytic work.

2. Assuming ideal fluid, linearized theory, and simplified ship forms (or singularity distributions), the optimization is based on

i) finding the minimum of certain resistance integrals or

ii) on the extinction of the wave pattern (especially of bow waves).

The first approach is theoretically the straightforward and appropriate procedure. The second, however, has some advantages in so far as it can be applied to regions where the ideal fluid concept is valid to a reasonable approximation, i.e. to the bow, or, more generally, to the region of the forebody.

To 2 i): Minimum solutions or, more generally, solutions yielding low values have been

a) investigated primarily for Michell's integral as the most important and classical wave resistance formula.

b) Similar investigations refer to the wave resistance of wholly submerged bodies of revolution and

c) to *Pienoids* (bodies generated by singularities distributed over a skeleton surface).

Further integrals which can, and should be handled in a similar way are those given by

d) *Havelock* for systems of continuous and discrete distributions (but not for those distributed over a *fixed* surface!)

e) *Hogner's* integral for pressure systems,

f) Maruo's expression for semi-submerged ships (can be subsumed under d),

g) pertinent resistance integrals for motion on shallow (finite depth) water.

The mathematics involved have caused a lot of concern. It has been shown that in many cases no solutions exist of the underlying problem of the calculus of variation. By generalizing the class of functions admitted, and proper interpretation of results, most difficulties can be avoided. These painstaking investigations have led e.g. to the interesting results: keeping the displacement constant changes of hull forms (distributions) are possible such that the wave resistance is not influenced by these changes. This indicates the fact that equivalent resistance properties can be attained by widely different hulls. On the other hand other calculations show that, to slight changes of form under certain conditions, large variations in resistance may correspond.

Although the "exact" method of calculus of variations has yielded valuable results, the direct method (Ritz' method) appears to be more fruitful from a practical point of view. The influence of vertical distribution of displacement, in addition to the dominant longitudinal distribution, on the minimum resistance has recently been clarified. The wave drag becomes almost negligible for theoretical optimum forms below a Froude number of say 0.27 and similarly for optimum full forms (high prismatic forms) at usual cargo ship speeds. The importance of bulbous forms is supported by this analysis.

To 2 ii): The same result concerning the bulb is obtained by the inspection of the wave pattern, and methods designed for wave extinction. Because of its evidence and advantages from the point of view of physics investigations in this field appear to be promising (started by Inui, his school and followers).

Notwithstanding the physical shortcomings of the ideal fluid concept efforts should be concentrated to study the following problems.

1. Development of hydrodynamic (singularity) models for all important families of ship forms (e.g. with transom stern) and determination of pertinent velocity potentials under simplified conditions.

We mention as a solution of more general interest (not applicable immediately to ships) as found recently by Bessho for optimum dipole distributions over lines or planes normal to the direction of translation.

2. The determination of body forms generated by singularities in presence of a free surface (Tuck, Kajitani) and

2a. the inverse problem—determination of generating singularities for a given body moving close to a free surface.

3. Second order theory (in first line for the Michell theory) including critical survey of non orthodox attempts to improve the linear theory e.g. such as made by Guilloton.

Results of second order investigations should greatly further endeavours to find optimum forms.

3. The mentioned steps in developing second order

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* Unfortunately, a paper by Sisov on the optimization of thrust deductions so far has not been available.

ideal fluid theory, important in itself, are perhaps still more decisive as a necessary prerequisite for deriving optimum forms with respect to wave resistance and later to total resistance in the real fluid.

The theoretical and experimental work carried out within the last years admits a favorable outlook (Inui and his school, Wahausen and his school, Eggers, Sharma, Shearer, Hogben, Gadd, Ward, etc. for the determination of wave resistance, as well as T. Y. Wu's fundamental study on the influence of waves on frictional resistance). It should be emphasised that even our knowledge of the viscous drag of ship-like bodies moving in unbounded fluid ("double model technique") is absolutely inadequate. These investigations should be extended on a broad front including the problems presented by the influence of the frictional wake and of separation on wave generation. There is no qeustion that the determination of optimum (favorable) forms of *total* resistance and the possibility to convert model data to the full size ship will be decisively promoted by this research. Already at present the impact of analytical methods of optimization on model technique is extremely beneficial.

APPENDIX IV

EFFECTS OF SHALLOW AND RESTRICTED WATER

by A. J. W. LAP (*Netherlands Ship Model Basin*)

In principle most of the problems dealt with in the report apply also the resistance of models or ships in shallow and restricted water. In general it can be said that the greater the restriction in dimensions of the channel, the more intense the phenomena and unfortunately also the greater the difficulties in both conducting model tests and judging and extrapolating their results. A few items, however, need maybe some further consideration.

1. INTERACTION OF WAVE AND VISCOUS RESISTANCE

As the distance between ship form and boundaries of the channel becomes smaller, the interference of viscous and wave resistance becomes much more serious. Especially in the case of long and comparatively slow models it may well occur that the gap between ship bottom and tank bottom is completely filled up by boundary layer flow, in which case the question must be put, in how far the general equation for the total resistance may be split up as:

$$C_t = C_v(R_n, F_n) + C_w(R_n, F_n)$$

For very small water depths this will probably not be the case and a solution must be found to determine how far under these circumstances the viscous resistance depends on Froude number and how far the serious disturbances in the potential flow due to viscosity and their effect on viscous resistance may be taken into account as a form correction factor.

Part of these interaction problems can in principle be solved by conducting geosim tests in very shallow water in order to check whether or not lines of constant Froude number are still parallel, when plotted to a base of R_n or $f(R_n)$. Much useful information can also be obtained by determining the viscous resistance by means of a wake traverse method or by pressure measurements on the hull. The interference effects further form an additional difficulty in calculating the wave resistance in shallow and restricted waters in a non-ideal fluid. An effective general solution suitable for practical application does not yet exist.

2. FORM EFFECT CORRECTION TO CORRELATION LINE

For deep water it is generally agreed that form effect corrections must be applied to the correlation line, although there are differences of opinion about type and magnitude of these corrections. In particular there is uncertainty about the behaviour of the viscous pressure resistance and as a consequence the question must be put, whether the correct correlation line is represented by the line of zero Froude number of a model family. This problem, not even solved for deep water, becomes of extreme importance for the extrapolation of test results for shallow and restricted water, since the differences between the line $F_n=0$ and the basic correlation line may quite well become four or five times as large as they are in deep water. Consequently the form effect correction to the correlation line, if determined from single model tests on base of the assumption that the low speed part of the C_t curve coincides with the required correlation line, may become extremely large. This results in a much steeper extrapolation and a much lower smooth ship resistance prediction. On the other hand a roughness correction has to be applied to the frictional resistance, that is maybe much greater than that for the same ship in deep water. This is also due to the form effect.

3. SHALLOW WATER AND CANAL EFFECT ON THE COMPONENTS OF RESISTANCE

One of the most important problems in the evaluation of tests in shallow and restricted waters is the conversion of the results obtained into those for other water depths and widths. This is necessary because not always the full size channel can be reproduced to scale by the available basin. It is also necessary sometimes in order to be able to limit the number of tests.

As far as resistance is concerned, several closely related conversion methods were developed, all based on two basic assumptions:

1) the total resistance of a ship form can be split up into viscous and wave resistance, the first of which can be calculated for any required condition.

2) Schlichting's assumption, that the wave resistance per ton is the same in first approximation when the generated waves have the same length.

It is known that these methods give unsatisfactory results in many cases, which is not so surprising, since:

a) they do not take into account any form effect in viscous resistance, as a result of which considerable errors are maybe made in splitting up the total resistance into its components.

b) it was recently suggested that Schlichting's assumption is an oversimplification. By assuming that the resistance per ton is only a function of λ/L* Schlichting introduced the parameter $\frac{D}{\frac{1}{2}\rho V^2\Omega}$*, which is not a constant.

A better assumption is therefore maybe that the specific resistance C_t is dependent on λ/L only.

The physical meaning of Schlichting's oversimplification is that he neglected the wave height or, in other words, that he assumed the wave height automatically to be equal as soon as the wave length was equal.

Quite recently it was shown that on base of three-dimensional extrapolation and the modified Schlichting assumption good correlation could be obtained between results of model tests at various water depths as long as comparatively slender ship forms are considered. For bluff bodies, such as barge tows, that are big relative to the channel, the method still yields unsatisfactory results. There is a possibility that this is caused by laminar flow effects or by incorrect treatment of the viscous pressure resistance, which may be relatively great for this type of bodies.

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* λ=length of generated waves L=ship length
V=ship speed Ω=wetted surface.

APPENDIX V

RESULTS OF QUESTIONNAIRE ON BLOCKAGE CORRECTIONS, TANK STORMS, AND USE OF TURBULENCE DETECTION TECHNIQUES

In August 1964 a letter was circulated to 55 member organizations and individuals. The text was as follows:

"The Resistance Committee was asked by the 10th I.T.T.C. to make recommendations regarding certain aspects of towing tank work, and to do this needs information on current practice and experience in these matters. It would therefore be most helpful if you could supply any relevant information on the questions listed below.

1. *Blockage*: Is it your standard practice to use a blockage correction, or have you had experience of using such corrections? If so, has the method been described in published work; if not, are you willing to offer information for circulation to other members?

2. *Water Contamination*: Have you ever experienced a change of resistance directly associated with a change in the chemical or biological state of the water, and if so, was a cause detected or an analysis of the water made?

3. *Turbulence Detection*: Have you a standard technique for turbulence detection or for the measurement of turbulence level, and if so, is this commercially available, or has it been described in published work?

If you have unpublished experience in any of these fields and are willing to let the Committee have information, it would be useful if you could name the officer in your laboratory who may be contacted for further details. All information should be sent to the Secretary

Table 1. Use of blockage corrections

Country	Organisation	Method & Application
Austria	Schiffbautechnische Versuchsanstalt, Vienna	Schuster method for models over 20 ft long.
Canada	National Research Council, Ottawa	Hughes method for large, full models.
Fed. Rep. of Germany	Versuchsanstalt fur Wasserbau und Schiffbau, Berlin	Standard blockage correction by Schuster method.
Japan	S. P. Div. Transport. Technical Research Inst., Tokyo	Taniguchi method used in No. 2 Tank only.
"	Mitsubishi S. & E. Ltd., Nagasaki	Taniguchi method used in scientific work only.
Sweden	Staten Skeppsprovningsanstalt, Gothenberg	Schuster method included in correlation factor.
United Kingdom	Saunders Roe Div., Westland Aircraft Ltd., Cowes	Standard blockage correction by modified Hughes method.
"	John Brown Ltd., Clydebank	Hughes method has been used for special problems.
"	NPL Ship Division, Feltham	Hughes method used for special problems.
"	University of Newcastle	Emerson method used for tank comparison.
U.S.A.	University of Michigan	Standard correction based in part on Hughes.
"	Webb Institute, New York	Landweber/Schlichting method has been used for special problems.

Table 2. Towing tank storms

Country	Organization	Information
France	Bassin d'Essais des Carenes, Paris	Storms have occurred e.g. in the Spring and after refilling. Water now chlorinated.
Japan	Trans. Tech. Res. Inst., Tokyo	Occasional variations in resistance.
U.K.	Saunders Roe Div., Westland Ltd., Cowes	Occasional variations in resistance.
"	John Brown Ltd., Clydebank	Severe storm reported, associated with formation of gas bubbles. (See Ref. 30)
"	Vickers Ltd., St. Albans	Occasional variations in resistance. Water now treated with sodium hypochlorite and algicide.
"	Admiralty Experiment Works, Haslar	Occasional storms, and random variations.
"	Messrs. Thornycrofts Ltd., Fort Steyne	Severe storm reported, associated with algal growth Cured by chlorination. (See Ref. 31)

Table 3. Use of turbulence detection techniques

Country	Organisation	Method & Application
Canada	National Reseach Council, Ottawa	Occasional use of hot film probe.
Fed. Rep. of Germany	Versuchsanstalt fur Wasserbau und Schiffbau, Berlin	Occasional use of colour probes and hot film probes.
France	Bassin d'Essais des Carenes, Paris	Hot film probe used in research work.
Japan	S. P. Div. Trans. Tech. Res. Inst., Tokyo	Benzoic acid film used when required.
Norway	Skipsmodeltanken, Trondheim	Hot film probe used in research work.
Sweden	Staten Skeppsprovningsanstalt, Gothenberg	Soluble film method sometimes used.
U.K.	Saunders Roe Div., Westland Ltd., Cowes	Hot film or inkstream used for special applications.
"	Vickers Ltd., St. Albans	Hydroquinone diacetate, or hot film sometimes used.
"	NPL Ship Division, Feltham	Hot film probe used to check stimulation on full models, and in research work.
"	Admiralty Experiment Works, Haslar	Hot film probe sometimes used.
"	University of Newcastle	Hot film probes used.
U.S.A.	Webb Institute, New York	Hot film probes sometimes used.

of the Resistance Committee.

Your co-operation in this work will be greatly appreciated."

32 replies were received and in the following tables reference is made to those which contained information of a positive nature. The Committee would like to acknowledge the assistance given by member organizations in this matter.

APPENDIX VI

CONTRIBUTIONS TO THE 11TH I.T.T.C.

During the course of its deliberations the Committee reviewed and discussed a very large amount of published and unpublished matter. In addition to this a great deal of valuable new material has already been presented in the form of formal contributions to the 11th I.T.T.C. These were not available when the Committee prepared its report, but they do much to advance our knowledge in certain of the fields covered in the Report.

In Viscous Resistance there are a number of reports of streamflow experiments on models of full form tankers. Although the indications of these are not specific they tend to support the view that longitudinal vortices may form a significant element of the flow round these full hulls. There are also important contributions on the various factors influencing the accuracy of towing tank measurements, and in particular on the effect of contaminants and additives. The latter confirm the strong influence that both synthetic and natural contaminants can have on friction resistance, and also show that this influence can be deliberately controlled with some precision. A valuable paper on roughness effects has also been received.

In Wave Resistance advances are reported in the theoretical calculation of wave resistance, in the derivation of ship forms of low wave resistance and in the calculation of wave resistance from measurements of surface disturbance.