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Uncertainty Analysis, Example for Resistance Test

1 PURPOSE OF PROCEDURE

The purpose of the procedure is to provide an example for the uncertainty analysis of a model scale towing tank resistance test following the ITTC Procedures 7.5-02-01-01 Rev 00, 'Uncertainty Analysis in EFD, Uncertainty Assessment Methodology' and 7.5-02-01-02 Rev 00, 'Uncertainty Analysis in EFD, Guidelines for Towing Tank Tests.'

2 EXAMPLE FOR RESISTANCE TEST

This procedure provides an example showing an uncertainty assessment for a model scale towing tank resistance test. The bias and precision limits and total uncertainties for single and multiple runs have been estimated for the total resistance coefficient C_T, and residuary resistance coefficient C_R in model scale at one Froude number.

In order to achieve reliable precision limits, it is recommended that 5 sets of tests with 3 speed measurements in each set are performed giving in total 15 test points. In this example the recommended sequence was followed.

Extrapolation to full scale has not been considered in this example. Although it might lead to significant sources of error and uncertainty, it is not essential for the present purpose of demonstrating the methodology.

When performing an uncertainty analysis for a real case, the details need to be adapted according to the equipment used and procedures followed in each respective facility.

2.1 Test Design

By measuring the resistance (R_x), speed (V) and water temperature (t^o), and by measuring or using reference values for the wetted surface (S) and density (ρ) the total resistance coefficient (C_T) can be calculated for a nominal temperature of 15 degrees, according to:

$$C_T^{15\deg} = C_T^{Tm} + (C_F^{15\deg} - C_F^{Tm})(1+k) \tag{2-1}$$

where

$$C_T^{Tm} = \frac{R_x^{Tm}}{0.5\rho V^2 S} \tag{2-2}$$

The residuary resistance coefficient can further be calculated as

$$C_R = C_T^{Tm} - (1+k)C_F^{Tm} = C_T^{15\deg} - (1+k)C_F^{15\deg} \tag{2-3}$$

In Eq. (2-1) the conversion of the resistance coefficients from the measured model temperature (index Tm) to a nominal temperature of 15 degrees is made by the ITTC-1978 prediction method. C_F in Eq. (2-1) is calculated according to the ITTC-1957 frictional correlation line

$$C_F = \frac{0.075}{(\log_{10} Re - 2)^2} \tag{2-4}$$

where *Re* is the Reynolds Number for the respective temperatures.

2.2 Measurement Systems and Procedure

Figure 2.1 shows a block diagram for the resistance test including the individual measurement systems, measurement of individual variables, data reduction and experimental results.

In Section 2.3.1 the bias limits contributing to the total uncertainty will be estimated for the individual measurement systems: hull geometry, speed, resistance and temperature/density/viscosity. The elementary bias limits are for each measurement system estimated for the categories: calibration, data acquisition, data reduction and conceptual bias.

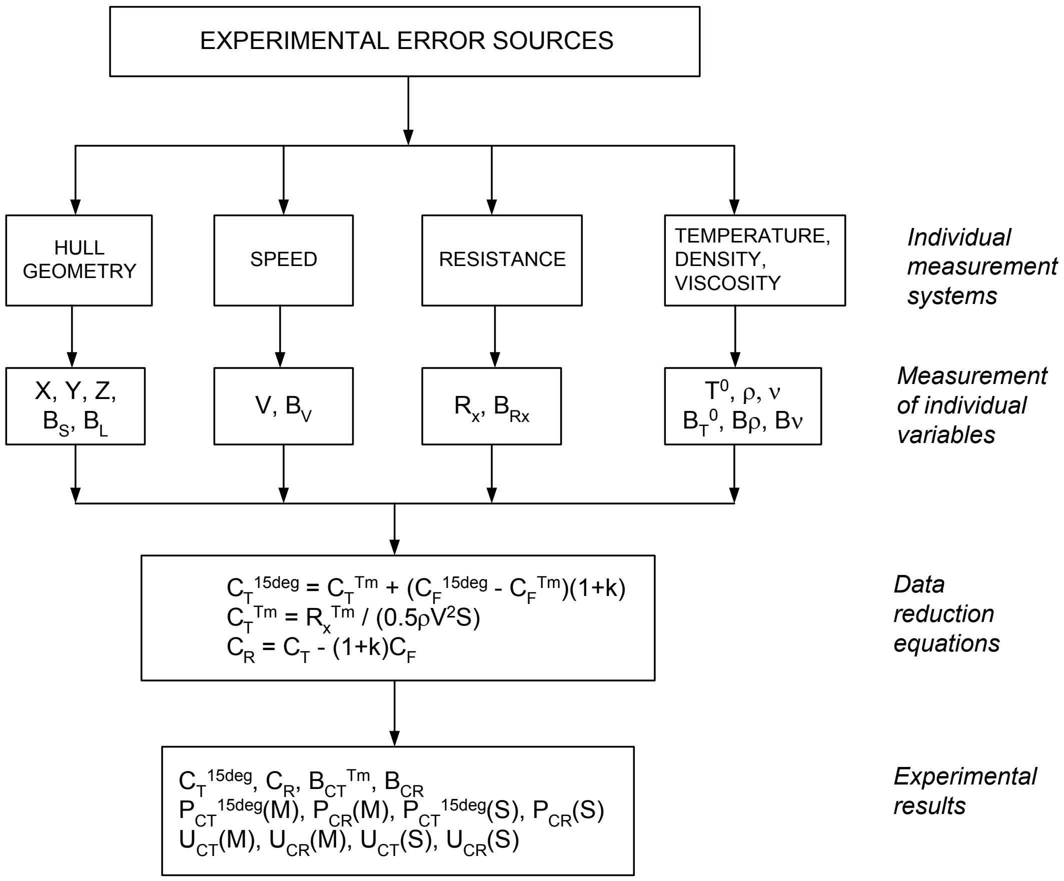

Figure 2.1 Block diagram of test procedure.

Using the data reduction Eqs. (2-2) and (2-3) the bias limits are then reduced to $B_{CT}{}^{Tm}$, and B_{CR} respectively. As the adjustments in model temperature from the measured temperature to 15 degrees are very small the bias limits associated with the Eq. (2-1) conversion have not been considered.

The precision limits for the total resistance coefficient at a nominal temperature of 15 degrees $P_{CT}{}^{15deg}$, and residuary resistance coefficient P_{CR} are estimated by an end-to-end method for multiple tests (M) and a single run (S).

Table 2.1 Ship particulars.

Definitions	Symbol	Value (unit)
Length between perp.	L_{PP}	6.500 (m)
Length on waterline	L_{WL}	6.636 (m)
Length overall submerged	L_{OS}	6.822 (m)
Breadth	B	1.100 (m)
Draught even keel	T	0.300 (m)
Wetted surface incl. rudder	S	7.600 (m^2)
Area water plane	A_{WP}	4.862 (m^2)
Displacement	∇	1.223 (m^3)
Block coefficient	$C_B = \nabla / L_{PP} B T$	0.5702 (-)
Water plane coefficient	$C_{WP} = A_{WP} / L_{PP} B$	0.680 (-)
Wetted surface coefficient	$C_S = S / \sqrt{(\nabla L_{PP})}$	2.695 (-)

Table 2.2 Constants.

Definitions	Symbol	Value (unit)
Gravity	g	9.810 (m/s^2)
Density, model basin	ρ	1000 (kg/m^3)
Water temperature (resistance test average)	t^o	15 (degrees)

In Tables 2.1 and 2.2 the ship particulars and constants used in the example are tabulated.

2.3 Uncertainty Analysis

The uncertainty for the total resistance coefficient is given by the root sum square of the uncertainties of the total bias and precision limits

$$\left(U_{C_T}\right)^2 = \left(B_{C_T}\right)^2 + \left(P_{C_T}\right)^2 \tag{2-5}$$

$$\left(U_{C_R}\right)^2 = \left(B_{C_R}\right)^2 + \left(P_{C_R}\right)^2 \tag{2-6}$$

The bias limit associated with the temperature conversion of the measured data, Eq. (2-1), will not be considered in the present example and therefore

$$B_{C_T}^{15\deg} = B_{C_T}^{Tm} \tag{2-7}$$

The bias limit for B_{CT} can therefore be calculated as:

$$\left(B_{CT}\right)^2 = \left(\frac{\partial C_T}{\partial S} B_S\right)^2 + \left(\frac{\partial C_T}{\partial V} B_V\right)^2 + \left(\frac{\partial C_T}{\partial Rx} B_{Rx}\right)^2 + \left(\frac{\partial C_T}{\partial \rho} B\rho\right)^2 \tag{2-8}$$

The bias limit for Eq. (2-3) is

$$\left(B_{CR}\right)^2=\left(\frac{\partial C_R}{\partial C_T}B_{C_T}\right)^2+\left(\frac{\partial C_R}{\partial k}B_k\right)^2+\left(\frac{\partial C_R}{\partial C_F}B_{C_F}\right)^2 \tag{2-9}$$

The precision limits will be determined for C_T^{15deg} and for C_R by an end-to-end method where all the precision errors for speed, resistance and temperature/density/viscosity are included. The precision limits for a single run (S) and for the mean value of multiple test (M) are determined. Regardless as to whether the precision limit is to be determined for single or multiple runs the standard deviation must be determined from multiple tests in order to include random errors such as model misalignment, heel, trim etc. If it is not possible to perform repeat tests the experimenter must estimate a value for the precision error using the best information available at that time. The precision limit for multiple tests is calculated according to

$$P(M)=\frac{K\,SDev}{\sqrt{M}} \tag{2-10}$$

where M = number of runs for which the precision limit is to be established, $SDev$ is the standard deviation established by multiple runs and K=2 according to the methodology.

The precision limit for a single run can be calculated according to

$$P(S)=K\,SDev \tag{2-11}$$

2.3.1 Bias Limit

Under each group of bias errors (geometry, speed, resistance and temperature/density/viscosity) the elementary error sources have been divided into the following categories: calibration; data acquisition; data reduction; and conceptual bias. The categories not applicable for each respective section have been left out.

2.3.1.1 Hull Geometry (Model Length and Wetted Surface Area)

The model is manufactured to be geometrical similar to the drawings or mathematical model describing the hull form. Even though great effort is given to the task of building a model no model manufacturing process is perfect and therefore each model has an error in form and wetted surface. The influence of an error in hull form affects not only the wetted surface but also the measured values by an error in resistance. For example, two hull forms, with the same wetted surface and displacement, give different resistance when towed in water if the geometry is not identical. This error in hull form geometry is very difficult to estimate, and will not be considered here. Only the bias errors in model length and wetted surface area due to model manufacture error are taken into account.

Model length

Data acquisition:
The bias limit in model length (on the waterline) due to manufacturing error in the model geometry can be adopted from the model accu-

racy of ±1 mm in all co-ordinates as given in ITTC Procedure 7.5-01-01-01 Rev 01 'Ship Models.' Hence the bias limit in model length will be B_L=2 mm.

Wetted surface

Data acquisition:

In this example, the error in wetted surface due to manufacturing error in model geometry is estimated using an ad hoc method. By assuming the model error to be ±1 mm in all co-ordinates, as given in ITTC Procedure 7.5-01-01-01 Rev 01, 'Ship Models', the length will increase by 2 mm, beam by 2 mm and draught by 1 mm. If the dimensions are changed while keeping the block coefficient constant, the displacement becomes ∇'= (6.502·1.102·0.301·0.5702)·1000 =1229.8 kg which is an increase of $\nabla'-\nabla$=6.7 kg. Assuming the wetted surface coefficient to be constant, the wetted surface for the larger model becomes $S'=2.696\cdot\sqrt{(\nabla'\cdot L'_{PP})}=7.622$ m^2, which corresponds to an increase of $S'-S$=0.022 m^2 or 0.29% of the nominal wetted surface S.

The model is loaded on displacement and therefore an error in hull form with, for example, too large a model are somewhat compensated by the smaller model draught. The increased displacement of 6.7 kg gives, with a water plane area of A_{WP}=4.862 m^2, a decreased draught of 1.38 mm. With a total waterline length of $2\cdot L_{WL}$=13.272 meters the smaller draught decreases the wetted surface by 13.272·0.00138 =0.0183 m^2.

Totally, the bias limit in wetted surface due to the assumed error in hull form will be B_{S1}=0.022-0.0183=0.0037 m^2.

Calibration:

The model weight (including equipment) is measured with a balance and the model is loaded to the nominal weight displacement. The balance used when measuring the model weight is calibrated to ± 1.0 kg. The errors in model and ballast weights are seen in Table 2.3.

Table 2.3 Error in displacement.

Item	**Weights**	**Weights**	
		Individual weights	**Group weights**
Ship model	260 kg	± 1.0 kg	± 1.00 kg
Ballast weights	3x200 kg	± 1.0 kg	$\sqrt{3(1.0)^2}$=± 1.732 kg
	2x150 kg	± 0.75 kg	$\sqrt{2(0.75)^2}$=± 1.061 kg
	6x10 kg	± 0.05 kg	$\sqrt{6(0.05)^2}$=± 0.122 kg
	3x1 kg	± 0.005 kg	$\sqrt{3(0.005)^2}$=± 0.009 kg
Total weight displ.	1223 kg		±2.267 kg

The total uncertainty in weight is given by the root sum square of the accuracy of the group of weights, 2.267 kg.

An increase in model weight of 1 kg gives, with ρ=1000 and a water plane area of 4.862 m^2, an additional draught of 1/4.862=0.206 mm. With a waterline length of 13.272 m this results in an increased wetted surface of 0.000206·13.272=0.00273 m^2 per kg.

For the deviation in displacement of ±2.267 kg, the error in weight displacement equals 2.267/1223 = 0.185%, the error in draught equals 2.267·0.206=0.467 mm and the error in wetted surface equals B_{S2}=2.267·0.00273 =0.0062 m^2.

Finally the error in wetted surface is obtained by the root sum square of the two bias components as $B_S = \sqrt{0.0037^2+0.0062^2}$)=0.0072 m^2 corresponding to 0.10 % of the nominal wetted surface area of 7.6 m^2.

2.3.1.2 Speed

The carriage speed measurement system consists of individual measurement systems for pulse count (c), wheel diameter (D) and 12 bit DA and AD card time base (Δt). The speed is determined by tracking the rotations of one of the wheels with an optical encoder. The encoder is perforated around its circumference with 8000 equally spaced and sized windows. As the wheel rotates, the windows are counted with a pulse counter. The speed circuit has a 100 ms time base which enables an update of the pulse every 10th of a second. A 12-bit DA conversion in the pulse count limits the maximum number of pulses in 100 ms to 4096. The output of the speed circuit is 0-10 V so that 4096 counted in 100 ms corresponds to 10 V output. The output from the encoder is calculated with the equation

$$V = \frac{c\pi D}{8000\Delta t} \tag{2-12}$$

where c is the number of counted pulses in Δt=100 ms and D is the diameter of the carriage wheel (0.381 m).

The bias limit from blockage effects has not been considered.

Pulse count (c)

Calibration:
The optical encoder is factory calibrated with a rated accuracy of ±1 pulse on every update. This value is a bias limit and represents the minimum resolution of the 12-bit AD data acquisition card. Therefore, the bias limit associated with the calibration error will be Bc_1=1 pulse (10V/2^{12}=0.00244 V).

Data acquisition:
In the given data acquisition cycle, the speed data is converted to the PC by two 12-bit conversions. The resolution is resol=10 V/ 2^{12} = 0.00244V / bit. The AD boards are accurate to 1.5 bits or pulses, which was determined by calibrating the boards against a precision voltage source. Therefore, the bias associated with the two conversions is Bc_2= Bc_3=1.5 pulses (0.00366 V).

Data reduction:
The final bias occurs when converting the analogue voltage to a frequency that represents the pulse count over 10 time bases or one second. This is enabled if correlating the given frequency to a corresponding voltage output. The bias limit results from approximating a

calibration (set of data) with a linear regression curve fit. The statistic is called standard error estimate (*SEE*) and is written from Coleman and Steele (1999) as

$$SEE = \sqrt{\frac{\sum_{i=1}^{N}\left(Y_i\text{-}(aX_i+b)\right)^2}{N-2}} \tag{2-13}$$

It is proposed by Coleman and Steele (1999) that a ±2(*SEE*) band about the regression curve will contain approximately 95% of the data points and this band is a confidence interval on the curve fit. The curve fit bias limit is calculated to be 2.5 Hz corresponding to B_{c4}= 0.25 pulse (0.000614 V).

The total bias limit for pulse count will then be

$$B_c = \left(B_{c1}^2 + B_{c2}^2 + B_{c3}^2 + B_{c4}^2\right)^{\frac{1}{2}} = \left(1^2 + 1.5^2 + 1.5^2 + 0.25^2\right)^{\frac{1}{2}} = 2.358\, pulse\,(0.00576V) \tag{2-14}$$

<u>Wheel diameter (*D*)</u>

One of the driving wheels of the carriage is used for the speed measurement. The wheel is measured with constant time intervals to ensure the right calibration constant is used.

Calibration:

The wheel diameter is measured with a high quality Vernier calliper at three locations at the periphery of the wheel which are averaged for a final value of *D*. The wheel diameter is considered accurate to within B_D=0.000115 m.

<u>Time base (Δt)</u>

The time base of the speed circuitry is related to the clock speed of its oscillator module.

Calibration:

The oscillator module is factory calibrated and its rated accuracy is 1.025 10^{-5} seconds on every update giving $B_{\Delta t}$= 1.025 10^{-5} seconds.

The data reduction equation is derived from Eq. (2-12) and can be written

$$B_V = \left(\left(\frac{\partial V}{\partial c}B_c\right)^2 + \left(\frac{\partial V}{\partial D}B_D\right)^2 + \left(\frac{\partial V}{\partial \Delta t}B_{\Delta t}\right)^2\right)^{\frac{1}{2}} \tag{2-15}$$

Using the nominal values of c=1138.4, D=0.381 m and Δt=0.1 s for the mean speed of V=1.7033 m/s the partial derivatives can be calculated as

$$\frac{\partial V}{\partial c} = \frac{\pi D}{8000\Delta t} = 0.00150 \tag{2-16}$$

$$\frac{\partial V}{\partial D} = \frac{c\pi}{8000\Delta t} = 4.4705 \tag{2-17}$$

$$\frac{\partial V}{\partial \Delta t} = \frac{c\pi D}{8000}\left(-\frac{1}{\Delta t^2}\right) = -17.0327 \tag{2-18}$$

The total bias limit can then be calculated according to Eq. (2-15) as

$$B_V = \begin{pmatrix} (0.00150\cdot 2.358)^2 + (4.4705\cdot 0.000115)^2 \\ + (17.0327\cdot 1.025\ 10^{-5})^2 \end{pmatrix}^{\frac{1}{2}} = 0.00357 \tag{2-19}$$

The total bias limit for the speed is B_V=0.00357 m/s corresponding to 0.21% of the nominal speed of 1.7033 m/s.

The bias limit for the speed could alternatively be determined end-to-end, by calibrating against a known distance and a measured transit time.

2.3.1.3 Resistance

The horizontal x-force is to be measured for the model when towed through the water.

Calibration:

The resistance transducer is calibrated with weights. The weights are the standard for the load cell calibration and are a source of error, which depends on the quality of the standard. The weights have a certificate that certifies their calibration to a certain class. The tolerance for the individual weights used is certified to be ± 0.005%. The calibration is performed from 0 to 8 kg with an increment of 0.5 kg. The bias error arising from the tolerance of the calibration weights, B_{Rx1}, is calculated as the accuracy of the weights, times the resistance measured according to Eq. (2-20).

$$B_{Rx1} = \text{accuracy of weights} \cdot \text{Rx} = 0.00005 \cdot 41.791 = 0.00209\ \text{N} \qquad (2\text{-}20)$$

Data acquisition:

The data from the calibration tabulated in Table 2.4 shows the mass/volt relation. From these values the *SEE* can be calculated with Eq. (2-13) to *SEE*=0.0853 resulting in a bias for the curve fit to be B_{Rx2}=0.1706 N.

The third error is manifest in the load cell misalignment, i.e., difference in orientation between calibration and test condition. This bias limit is estimated to be ±0.25 degrees and will effect the measured resistance as

$$B_{Rx3} = R_x - \left(\cos 0.25°\ Rx\right) = 41.791\,(1 - \cos 0.25^o\,) = 0.00040\,\text{N} \qquad (2\text{-}21)$$

Resistance data is acquired by an AD converter, which normally has an error of 1 bit out of AD accuracy of 12 bits. AD conversion bias error in voltage shall be given by AD converter error in bit multiplied by AD range (-10 volts to 10 volts) divided by AD accuracy. This voltage can be translated into Newton by using the slope value of calibration.

$$B_{Rx4} = \frac{1 \cdot 20}{2^{12}} 12.582 = 0.0614\,\text{N} \qquad (2\text{-}22)$$

Table 2.4 Resistance transducer calibration.

Output (Volt)	**Mass (kg)**	**Force (N)**
4.930	0.000	0.000
4.556	0.500	4.905
4.157	1.000	9.810
3.767	1.500	14.715
3.373	2.000	19.620
2.972	2.500	24.525
2.595	3.000	29.430
2.200	3.500	34.335
1.820	4.000	39.240
1.430	4.500	44.145
1.040	5.000	49.050
0.644	5.500	53.955
0.262	6.000	58.860
-0.121	6.500	63.765
-0.530	7.000	68.670
-0.919	7.500	73.575
-1.303	8.000	78.480

R=62.089-Volt·12.582

Data reduction:

The transducer is fitted in the middle of a special rod, which connects the model to the carriage and tows the model. During the resistance tests the running trim and sinkage of the model result in an inclination of the towing force compared to the calibration which is expressed as a bias limit B_{Rx5}. The mean running trim fore and aft are measured to be ΔTf=4.22 mm and ΔTa=8.34 mm. If the towing force is applied in Lpp/2 the sinkage + trim in the towing point ΔTtp can be calculated as $\Delta Ttp=(\Delta Tf+\Delta Ta)/2$=6.28 mm. The rod used for towing the model is 500 mm long and therefore the inclination of the towing force will be arcsin(6.28/500)=0.72 degrees compared to the calm water level. The bias limit can then be computed as

$$B_{Rx5}=R_x\text{-}\left(\cos 0.72^o\ R_x\right)=$$
$$41.791\left(1-\cos 0.72^o\right)=0.0033\,\mathrm{N} \qquad \text{(2-23)}$$

This error can be corrected for during the measurements if the angle in the rod is measured. If the transducer is mounted directly to the carriage and is constructed to take loads only in the x-direction this error will be eliminated.

The total bias limit in resistance is obtained by the root sum square of the four bias components considered B_{Rx} = $\sqrt{(0.00209^2+0.1706^2+0.00040^2+0.0614^2+ +0.0033^2)}$ = 0.1814 N corresponding to 0.43 % of the mean resistance of 41.791 N.

2.3.1.4 <u>Temperature/Density/Viscosity</u>

<u>Temperature</u>

Calibration:

The thermometer is calibrated by the manufacturer with a guaranteed accuracy of ±0.30 degrees within the interval -5 to +50 degrees. The bias error limit associated with temperature measurement is B_{t^o}=0.3 degrees corresponding to 2 % of the nominal temperature of 15 degrees.

<u>Density</u>

Calibration:

The density-temperature relationship (table) according to the ITTC Procedure 7.5-02-01-03 Rev 00 'Density and Viscosity of Water' for g=9.81 can be expressed as:

$$\rho=1000.1 + 0.0552\cdot t^o - 0.0077\cdot t^{o2} + 0.00004\cdot t^{o3} \qquad \text{(2-24)}$$

$$\left|\frac{\partial\rho}{\partial t}\right|=\left|0.0552-0.0154t^\circ+0.000120t^{\circ 2}\right| \qquad \text{(2-25)}$$

Using Eq. (2-25) with t^o=15 degrees and B_{t^o}=0.3 degrees the bias $B_{\rho l}$ can be calculated according to:

$$B_{\rho l}=\left|\frac{\partial\rho}{\partial t^\circ}\right|B_{t^\circ}=0.1488\cdot 0.3=0.04464\,\mathrm{kg/m^3} \qquad \text{(2-26)}$$

Data reduction:

The error introduced when converting the temperature to a density (table lookup) can be calculated as two times the *SEE* of the curve fit to the density/temperature values for the whole temperature range. Comparing the tabulated values with the calculated values (Eq. 2-24) the bias error $B_{\rho 2}$ can be calculated as $B_{\rho 2}$=0.070 kg/m^3.

Conceptual:

The nominal density according to the ITTC-78 method is ρ =1000. Using this method introduces a bias limit as the difference between ρ (15 degrees) = 999.34 and ρ = 1000 such as $B_{\rho 3}$ = 1000.0-999.345 = 0.655 kg/m^3 corresponding to 0.0655% of the density.

The bias for ρ can then be calculated according to:

$$
\begin{aligned}
B_\rho &= \sqrt{(B_{\rho 1})^2 + (B_{\rho 2})^2 + (B_{\rho 3})^2} \\
&= \sqrt{(0.1488 \cdot 0.3)^2 + 0.070^2 + 0.655^2} \\
&= 0.660\,\text{kg/m}^3
\end{aligned}
\qquad (2\text{-}27)
$$

The bias limit for density is thus B_ρ=0.660 kg/m^3 corresponding to 0.066 % of ρ = 1000. If using the density value determined by the temperature, the bias limit $B_{\rho 3}$ will be eliminated.

<u>Viscosity</u>

Calibration:

The viscosity-temperature relationship for fresh water adopted by ITTC Procedure 7.5-02-01-03; Rev 0, 'Density and Viscosity of Water' can be calculated as

$$
\begin{aligned}
\nu &= ((0.000585(t^\circ - 12.0) - 0.03361) \\
&\quad (t^\circ - 12.0) + 1.2350)10^{-6} \\
&= (0.000585 t^{\circ 2} - 0.04765 t^\circ + 1.72256)10^{-6}
\end{aligned}
\qquad (2\text{-}28)
$$

Partial derivative of Eq. (2-28) is

$$\frac{\partial \nu}{\partial t^\circ} = (0.00117 t^\circ - 0.04765)10^{-6} \qquad (2\text{-}29)$$

Using Eq. (2-29) with t°=15 degrees and B_{t°=0.3 degrees the bias $B_{\nu 1}$ can be calculated according to:

$$B_{\nu 1} = \left|\frac{\partial \nu}{\partial t^\circ}\right| Bt = 0.0301 10^{-6} \cdot 0.3 = 0.0090 10^{-6}\ \text{m}^2/\text{s} \qquad (2\text{-}30)$$

Data reduction:

For a nominal temperature of 15.0 degrees this formula results in ν=1.13944 10^{-6} m^2/s. Meanwhile the fresh water kinematic viscosity according to the table in ITTC Procedure 7.5-02-01-03, Rev 00, for 15.0 degrees is equal to ν=1.13902 10^{-6} m^2/s. Using this method introduces a bias error due to the difference between ν(15.0) = 1.139435 10^{-6} m^2/s and ν=1.139020 10^{-6} m^2/s such as $B_{\nu 2}$= -4.15 10^{-10} m^2/s.

With these results the total bias limit can be calculated as

$$B_\nu = \sqrt{(B_{\nu 1})^2 + (B_{\nu 2})^2} \qquad (2\text{-}31)$$

The bias limit associated with fresh water viscosity due to temperature measurement and viscosity calculation method is thus B_ν= 9.04 10^{-9} m²/s corresponding to 0.793 % of the kinematic viscosity.

2.3.1.5 Skin Frictional Resistance Coefficient

The skin frictional resistance coefficient is calculated through the ITTC-1957 skin friction line

$$C_F = \frac{0.075}{(\log_{10}\frac{VL}{\nu} - 2)^2} \tag{2-32}$$

Bias errors in skin friction calculation may be traced back to errors in model length, speed and viscosity. Bias limit associated with C_F can be a found as

$$\left(B_{C_F}\right)^2 = \left(\frac{\partial C_F}{\partial V}B_V\right)^2 + \left(\frac{\partial C_F}{\partial L}B_L\right)^2 + \left(\frac{\partial C_F}{\partial \nu}B_\nu\right)^2 \tag{2-33}$$

partial derivatives of Eq. (2-33) by model speed, model length and viscosity are

$$\frac{\partial C_F}{\partial V} = 0.075\left(-\frac{2}{(\log\frac{VL}{\nu} - 2)^3}\right)\left(\frac{1}{V\ln 10}\right) \tag{2-34}$$

$$\frac{\partial C_F}{\partial L} = 0.075\left(-\frac{2}{(\log\frac{VL}{\nu} - 2)^3}\right)\left(\frac{1}{L\ln 10}\right) \tag{2-35}$$

$$\frac{\partial C_F}{\partial \nu} = 0.075\left(-\frac{2}{(\log\frac{VL}{\nu} - 2)^3}\right)\left(-\frac{1}{\nu\ln 10}\right) \tag{2-36}$$

By substituting B_V=0.0036 m/s, B_L=0.002 m, B_ν=-9.04 10^{-9} m^2/s, bias limits associated with C_F in model scale is B_{CF}=4.258 10^{-6} corresponding to 0.142 % of the nominal value of C_F= 2.990 10^{-3}.

2.3.1.6 Form Factor

The recommended method for the experimental evaluation of the form-factor is that proposed by Prohaska. If the wave-resistance component in a low speed region (say 0.1 < Fr <0.2) is assumed to be a function of Fr^4, the straight-line plot of C_T/C_F versus Fr^4/C_{F} will intersect the ordinate (Fr =0) at (1+k), enabling the form factor to be determined.

hence

$$(1+k) = \frac{C_{\mathrm{T}}}{C_{\mathrm{F}}} \text{ at low Froude numbers} \tag{2-37}$$

In the case of a bulbous bow near the water surface these assumptions may not be valid and care should be taken in the interpretation of the results.

The bias limit $B_{(1+k)}$ can be determined from the data reduction Eq. (2-37). The determination of the precision limit requires about 15 set of tests for several speeds. As there was no example data available, the uncertainty in form factor has for the time being and for indicative purposes been assumed to be 0.02, equal to 10% of k or 1.66% of 1+k.

2.3.1.7 Total Bias Limit- Total Resistance Coefficient

In order to calculate the total bias and precision limits the partial derivatives have to be calculated using input values of Rx=41.791 N, g=9.81 m/s^2, ρ=1000 kg/m^3, S=7.60 m^2 and V=1.7033 m/s.

$$\frac{\partial C_T}{\partial S} = \frac{R_x}{0.5\rho V^2}\left(-\frac{1}{S^2}\right) = -4.988\ 10^{-4} \quad (2\text{-}38)$$

$$\frac{\partial C_T}{\partial V} = \frac{R_x}{0.5\rho S}\left(-\frac{2}{V^3}\right) = -0.00445 \quad (2\text{-}39)$$

$$\frac{\partial C_T}{\partial Rx} = \frac{1}{0.5\rho V^2 S} = 9.07\ 10^{-5} \quad (2\text{-}40)$$

$$\frac{\partial C_T}{\partial \rho} = \frac{R_x}{0.5V^2 S}\left(-\frac{1}{\rho^2}\right) = -3.791\ 10^{-6} \quad (2\text{-}41)$$

The total bias limit can then be calculated according to Eq. (2-8) as
$B_{C_T} = 2.3296\ 10^{-5}$
corresponding to 0.615% of the total resistance coefficient C_T=3.791 10^{-3}.

2.3.1.8 Total Bias Limit- Residuary Resistance Coefficient

Residuary resistance can be obtained from Eq. (2-3) as

$$C_R = C_T - (1+k)C_F \quad (2\text{-}42)$$

The bias limit of residuary resistance coefficient can be calculated according to

$$\left(B_{C_R}\right)^2 = \left(\frac{\partial C_R}{\partial C_T} B_{C_T}\right)^2 + \left(\frac{\partial C_R}{\partial k} B_k\right)^2 + \left(\frac{\partial C_R}{\partial C_F} B_{C_F}\right)^2 \quad (2\text{-}43)$$

partial derivatives of Eq.(2-42):

$$\frac{\partial C_R}{\partial C_T} = 1 \quad (2\text{-}44)$$

$$\frac{\partial C_R}{\partial k} = -C_F = -0.00299 \quad (2\text{-}45)$$

$$\frac{\partial C_R}{\partial C_F} = -(1+k) = -1.2 \quad (2\text{-}46)$$

by using Eq. (2-43):

$$B_{C_R} = \sqrt{\left(1\cdot 2.3311\ 10^{-5}\right)^2 + \left(-0.00299\cdot 0.02\right)^2 + \left(-1.200\cdot 4.258\ 10^{-6}\right)^2} = 6.438\ 10^{-5} \quad (2\text{-}47)$$

The total bias limit associated with residuary resistance coefficient is 6.438 10^{-5} corresponding to 31.72 % of the nominal value of C_R=0.203 10^{-3}.

2.3.2 Precision Limit

In order to establish the precision limits, the standard deviation for a number of tests, with the model removed and reinstalled between each set of measurements, must be determined. In this example 5 sets of testing (A-E) with 3 speed measurements in each set have been performed giving totally 15 test points. This is the best way to include random errors in the set-up such as model misalignment, trim, heel etc.

As resistance is highly dependent on viscosity, the resistance values measured have to be corrected to the same temperature. For a single towing tank the resistance values can preferably be corrected to the mean temperature of the tests in order not to make too large a correction. If the results are to be compared to results from other facilities all the resistance values must be corrected to the same temperature. In the present case the total resistance coefficient for the measured resistance and speed are corrected to the temperature of 15 degrees centigrade, according to the ITTC-78 method, by the following:

Table 2.5 Standard deviation of C_T and C_R.

Series /run	Measured values				Nominal speed /temp	
					Eq.(2-1)	Eq.(2-3)
	***Rx* (N)**	***V* (m/s)**	**Temp (deg)**	**$C_T\cdot$ 1000**	**$C_T\cdot$ 1000**	**$C_R\cdot$ 1000**
A1	41.713	1.702	16.0	3.789	3.806	0.217
A2	41.352	1.702	16.0	3.757	3.773	0.185
A3	41.564	1.702	16.0	3.776	3.792	0.204
B1	41.365	1.703	15.9	3.753	3.768	0.180
B2	41.763	1.705	15.9	3.781	3.795	0.208
B3	41.742	1.705	15.9	3.779	3.793	0.206
C1	41.744	1.702	16.0	3.792	3.808	0.220
C2	42.007	1.705	16.0	3.803	3.819	0.232
C3	41.938	1.703	16.0	3.805	3.822	0.234
D1	41.482	1.703	14.9	3.764	3.762	0.175
D2	41.646	1.705	14.9	3.770	3.768	0.181
D3	41.556	1.703	14.9	3.771	3.769	0.181
E1	41.577	1.703	16.1	3.773	3.790	0.203
E2	41.577	1.703	16.1	3.773	3.790	0.203
E3	41.736	1.703	16.1	3.787	3.806	0.217
MEAN					3.791	0.203
SDev					0.0192	0.0192

The residual resistance C_R, which is considered temperature independent, is calculated by

$$C_{\mathrm{R}} = C_{\mathrm{T}}^{Tm} - C_{\mathrm{F}}^{Tm}(1+k) \tag{2-48}$$

where index T_m= measured temperature (compare also Eq. (2-3)).

C_T for 15 degrees is then calculated from:

$$C_T^{15deg} = C_R + C_F^{15deg}(1+k) \tag{2-49}$$

By combining equation Eq. (2-48) and Eq. (2-49) C_T can be calculated as in Eq. (2-1).

In the above table the total resistance coefficient is calculated for each run, using the measured resistance and speed. This corrects the measured resistance to the nominal speed by the assumption that the resistance is proportional to V^2. For small deviations in speed this assumption is considered accurate.

The mean value over 15 runs for C_T^{15deg} (corrected to nominal speed and temperature) is calculated as $\overline{C_T} = 3.791\ 10^{-3}$ as shown in table 2.5. With Eq. (2-2), using the nominal values for speed, density and wetted surface, the corrected, mean resistance can be recalculated to $\overline{Rx} = 41.791\,\mathrm{N}$.

The precision limit for the mean value of 15 runs is calculated as

$$P_{\overline{C_T}} = \frac{K\ SDev_{\overline{C_T}}}{\sqrt{M}} = \frac{2 \cdot 0.0192\ 10^{-3}}{\sqrt{15}} = 0.00989\ 10^{-3} \tag{2-50}$$

according to Eq. (2-10) and corresponding to 0.26% of C_T. For a single run the precision limit is calculated as

$$P_{C_T} = K\ SDev_{\overline{C_T}} = 2 \cdot 0.0192\ 10^{-3} = 0.0383\ 10^{-3} \tag{2-51}$$

according to Eq. (2-11) and corresponding to 1.01 % of C_T.

The residual resistance coefficient can also be calculated as shown in table 2.5. The precision limit for the mean value of 15 runs is calculated as

$$P_{\overline{C_R}} = \frac{K\ SDev_{\overline{C_R}}}{\sqrt{M}} = \frac{2 \cdot 0.0192\ 10^{-3}}{\sqrt{15}} = 0.00989\ 10^{-3} \tag{2-52}$$

according to Eq. (2-10) and corresponding to 4.87% of C_R. For a single run the precision limit is calculated as

$$P_{C_R} = K\ SDev_{\overline{C_R}} = 2 \cdot 0.0192\ 10^{-3} = 0.0383\ 10^{-3} \tag{2-53}$$

according to Eq. (2-11) and corresponding to 18.88 % of C_R.

2.3.3 Total Uncertainties

Combining the precision limits for multiple and single tests with the bias limits the total uncertainty can be calculated according to Eq. (2-5) and Eq. (2-6).

The total uncertainty for C_T for the mean value of 15 runs will then be

$$\left(U_{C_T}\right) = \left(\left(B_{C_T}\right)^2 + \left(P_{C_T}\right)^2\right)^{\frac{1}{2}} =$$
$$\left(0.02331^2 + 0.00989^2\right)^{\frac{1}{2}}\ 10^{-3} = 0.02532\ 10^{-3}$$

(2-54)

which is corresponding to 0.67% of C_T.

Correspondingly the total uncertainty for a single run can be calculated as

$$\left(U_{C_T}\right)=\left(\left(B_{C_T}\right)^2+\left(P_{C_T}\right)^2\right)^{\frac{1}{2}}=$$
$$\left(0.02331^2+0.0383^2\right)^{\frac{1}{2}}\ 10^{-3}=0.04483\ 10^{-3} \quad (2\text{-}55)$$

which is 1.18% of C_T.

The total uncertainty for C_R for the mean value of 15 runs can similarly be calculated as

$$\left(U_{C_R}\right)=\left(\left(B_{C_R}\right)^2+\left(P_{C_R}\right)^2\right)^{\frac{1}{2}}=$$
$$\left(0.06438^2+0.00989^2\right)^{\frac{1}{2}}\ 10^{-3}=0.06514\ 10^{-3} \quad (2\text{-}56)$$

which is corresponding to 32.09% of C_R.

Correspondingly the total uncertainty for a single run can be calculated as

$$\left(U_{C_R}\right)=\left(\left(B_{C_R}\right)^2+\left(P_{C_R}\right)^2\right)^{\frac{1}{2}}=$$
$$\left(0.06438^2+0.0383^2\right)^{\frac{1}{2}}\ 10^{-3}=0.07493\ 10^{-3} \quad (2\text{-}57)$$

which is 36.91% of C_R.

As can be seen from the values above the uncertainty will decrease if it is calculated for the **mean value** of 15 tests compared to the single run value. This is also displayed in Figure 2.2 where the bias is constant regardless of the number of tests while the precision and total uncertainty are decreasing with increasing number of repetitions.

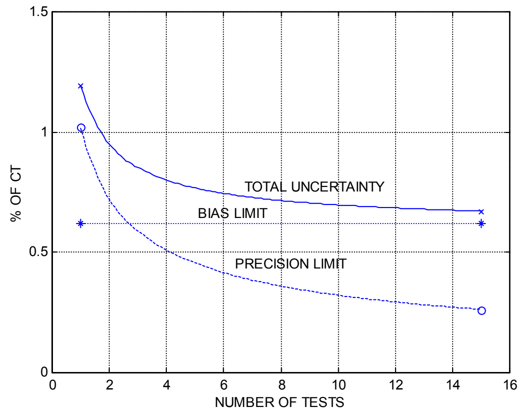

Figure 2.2 Bias, precision and total uncertainty.

Expressed in relative numbers the bias for C_T represents only 27% percent of the total uncertainty for a single run but as much as 85% of the total uncertainty for the mean value of 15 tests. The bias for C_R represents 74% of the total uncertainty for a single run and 98% of the total uncertainty for the mean value of 15 tests.

By comparing the bias and precision limits and the uncertainties, the relative contribution of each term can be calculated. This makes it possible to determine where an upgrade in the measurement system has the largest effect.

The bias and precision limits and the uncertainties for the total resistance coefficient are summarised in Table 2.6 where the relative contribution of each term is calculated. This makes it possible to determine where an upgrade in the measurement system has the largest effect. If considering the total resistance coefficient in this example, the most effective would therefore be to improve the speed and resistance measurement systems as they respectively contribute too 47% and 50% of the total bias limit. The uncertainty in speed consists of 98% of the uncertainty in pulse count Bc. This uncertainty consists of over 80% of the bias limits B_{c2} and B_{c3}. The bias limit in resistance consists of almost 100% of the uncertainty in acquisition, R_{x2} and R_{x4}. It is therefore most important to:

1. Upgrade the resistance measurement system by changing the resistance transducer to a transducer with better linearity (Reduction of error B_{Rx2}).
2. Upgrade the data acquisition cycle in the speed measurement system (Reduction of error B_{c2} and B_{c3}).

Table 2.6 Error contributions to total uncertainty.

Term	Value	Percentage values	
Model geometry (m^2)	7.600		
B_{S1} (m^2)	3.666E-03	25.97	% of B_S^2
B_{S2} (m^2)	6.189E-03	74.03	% of B_S^2
B_S (m^2)	7.193E-03	0.09	% of S
Model speed (m/s)	1.703		
B_{c1} (bit)	1.000	17.98	% of B_c^2
B_{c2} (bit)	1.500	40.45	% of B_c^2
B_{c3} (bit)	1.500	40.45	% of B_c^2
B_{c4} (bit)	0.250	1.12	% of B_c^2
B_c (bit)	2.358	0.21	% of c=1138
B_D (m)	1.150E-04	0.03	% of D=0.381
$B_{\Delta t}$ (s)	1.025E-05	0.01	% of Δt =0.1 s
$\theta^V_c B_c$ (m/s)	3.529E-03	97.69	% of B_V^2
$\theta^V_D B_D$ (m/s)	5.141E-04	2.07	% of B_V^2
$\theta^V_{\Delta t} B_{\Delta t}$ (m/s)	-1.746E-04	0.24	% of B_V^2
B_V (m/s)	3.570E-03	0.21	% of V
Model resistance (N)	41.791		
B_{Rx1} (N)	2.090E-03	0.01	% of B_{Rx}^2
B_{Rx2} (N)	1.706E-01	88.48	% of B_{Rx}^2
B_{Rx3} (N)	3.978E-04	0.00	% of B_{Rx}^2
B_{Rx4} (N)	-6.143E-02	11.47	% of B_{Rx}^2
B_{Rx5} (N)	3.296E-03	0.03	% of B_{Rx}^2
B_{Rx} (N)	1.814E-01	0.43	% of Rx
Model Density (kg/m^3)	1000.000		
Temperature (deg)	15.000		
B_T (deg)	0.300	2.00	% of 15 deg
$B_{\rho1}$ (kg/m3)	-4.464E-02	0.46	% of ρ^2
$B_{\rho2}$ (kg/m3)	7.002E-02	1.12	% of ρ^2
$B_{\rho3}$ (kg/m3)	6.553E-01	98.42	% of ρ^2
B_ρ (kg/m3)	6.605E-01	0.07	% of ρ
Total Resistance Coefficient	3.791E-03		
$\theta^{CT}_S B_S$	-3.588E-06	2.37	% of B_{CT}^2
$\theta^{CT}_V B_V$	-1.589E-05	46.56	% of B_{CT}^2
$\theta^{CT}_{Rx} B_{Rx}$	1.646E-05	49.92	% of B_{CT}^2
$\theta^{CT}_\rho B_\rho$	-2.504E-06	1.16	% of B_{CT}^2
B_{CT}	2.329E-05	0.61	% of C_T
P_{CT} (S)	3.829E-05	1.01	% of C_T
P_{CT} (M)	9.886E-06	0.26	% of C_T
U_{CT} (S)	4.482E-05	1.18	% of C_T
U_{CT} (M)	2.530E-05	0.67	% of C_T
Residual Resist. Coefficient	2.030E-04		
$\theta^{CR}_{CT} B_{CT}$	2.329E-05	13.09	% of B_{CR}^2
$\theta^{CR}_k B_k$	-5.980E-05	86.28	% of B_{CR}^2
$\theta^{CR}_{CF} B_{CF}$	-5.109E-06	4.81	% of B_{CR}^2
B_{CR}	6.438E-05	31.71	% of C_R
P_{CR} (S)	3.832E-05	18.88	% of C_R
P_{CR} (M)	9.895E-06	4.87	% of C_R
U_{CR} (S)	7.492E-05	36.91	% of C_R
U_{CR} (M)	6.513E-05	32.09	% of C_R

where $\theta_i^r = \dfrac{\partial r}{\partial i}$

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