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Numerical Estimation of Roll Damping

1. PURPOSE

This procedure provides a method for roll damping estimation which can be used in the absence of experiment data and can be used for dynamic stability calculations.

2. ESTIMATION METHOD

When considering the motion of a ship in waves most of the hydrodynamic forces acting on a hull can be calculated using a potential theory. However, roll damping is significantly affected by viscous effects. Therefore, a result calculated using a potential theory overestimates the roll amplitude in resonance and is not accurate. It is common practice for the calculation of roll damping to use measured values or estimation methods in order to consider the viscosity effects. In this chapter recommended estimation methods for roll damping are explained.

2.1 Definition of Component Discrete Type Method

In a component discrete type method, the roll damping moment, M_φ, is predicted by summing up the predicted values of a number of components. These components include the wave, lift, frictional, eddy and the appendages contributions (bilge keel, skeg, rudder etc).

$$M_\phi = M_{\phi \mathrm{W}} + M_{\phi \mathrm{L}} + M_{\phi \mathrm{F}} + M_{\phi \mathrm{E}} + M_{\phi \mathrm{APP}} \quad (2.1)$$

The wave and lift components ($M_{\varphi W}$ and $M_{\varphi L}$) are linear components which are proportional to roll angular velocity. The friction, eddy and appendage components ($M_{\varphi F}$, $M_{\varphi E}$ and $M_{\varphi APP}$) are nonlinear components. If the nonlinear components are assumed to be proportional to the square of roll angular velocity, then the equivalent roll damping coefficient in linear form B_{44} can be expressed as follows:

$$B_{44} = B_{44\mathrm{W}} + B_{44\mathrm{L}} + B_{44\mathrm{F}} + B_{44\mathrm{E}} + B_{44\mathrm{APP}} \quad (2.2)$$

where B_{44} is the roll damping coefficient (B_{44} = $B_{\varphi e}$ shown in Eq.(3.5) in section 3.2 which is defined by dividing the roll damping moment M_φ by the roll angular velocity $\omega_E \phi_a$. ϕ_a and ω_E denote the amplitude and circular frequency of the roll motion respectively.

Nonlinear components (e.g. B_{44E}) can be linearized as follows (refer to the section 3.2, ω_E is wave encounter circular frequency):

$$B_{44\mathrm{E}} = \frac{8}{3\pi} M_{\phi \mathrm{E}} \varphi_{\mathrm{a}} \omega_{\mathrm{E}} \quad (2.3)$$

It should be noted that all the coefficients in Eq.(2.1) and (2.2) depend on the roll frequency and the forward speed. $M_{\varphi E}$ (and B_{44E}) and $M_{\varphi APP}$ (and B_{44APP}) sometimes depend on roll amplitude as well as roll frequency because of the *Ke* number effect in the vortex shedding problem. (*Ke* number is Keulegan-Carpenter number expressed as $Ke=U_{max}T/(2L)$. U_{max}: the amplitude of velocity of periodic motion, T: period of motion, L: characteristic length of object).

The roll damping coefficient B_{44} is non-dimensionalized as follows:

$$\hat{B}_{44} = \frac{B_{44}}{\rho \nabla B^2}\sqrt{\frac{B}{2g}} \tag{2.4}$$

The circular frequency of roll motion is also non-dimensionalized as follows:

$$\hat{\omega}_{\mathrm{E}} = \omega_{\mathrm{E}}\sqrt{\frac{B}{2g}} \tag{2.5}$$

where ρ, g, ∇ and B denote the mass density of the fluid, acceleration due to gravity, displacement volume and breadth of the ship's hull respectively (*e.g.* Ikeda *et al*, (1976)). The roll damping coefficient B_{44} can be translated into Bertin's N-coefficient (Bertin, 1874) based form on the assumption that the energy losses over one period are the same (*e.g.* Ikeda *et al*, (1994)):

$$\hat{B}_{44} = \frac{\overline{GM}\varphi_{\mathrm{a}}}{180\, B\hat{\omega}_{\mathrm{E}}} N \tag{2.6}$$

In the following chapter, the sectional roll damping coefficient is sometimes referred to. The sectional roll damping coefficients are expressed with a prime on the right shoulder of a character (e.g. $B'_{44\mathrm{E}}$). For a 3-D ship hull form, the 3-D roll damping coefficient can be obtained by integrating the sectional roll damping coefficient over the ship length. Furthermore, a roll damping coefficient with subscript 0 (e.g. $B'_{44\mathrm{E0}}$) indicates a value at zero forward speed.

2.2 Displacement type mono-hull

2.2.1 Wave making component

The wave making component accounts for between 5% and 30% of the roll damping for a general-cargo type ship. However, the component may have a larger effect for ships with a shallow draught and wide section (Ikeda et al., 1978a).

In the case of zero Froude number, the wave damping can be easily obtained by using the strip method. It is however possible to numerically solve the exact wave problem for a 3-D ship hull form. Using the strip method, the sectional wave damping is calculated from the solution of a sectional wave problem, taking the form:

$$B'_{44\mathrm{W0}} = B'_{22}\left(l_{\mathrm{w}} - \overline{OG}\right)^2 \tag{2.7}$$

where B'_{22} and l_w represent the sectional sway damping coefficient and the moment lever measured from the still water level due to the sway damping force. (For example if the wave damping component is calculated using a strip method based on potential theory, B'_{22} and B'_{42}, which are sectional damping values caused by sway, are obtained from the calculation, and l_w is obtained from B'_{42} divided by B'_{22}.). $\overline{OG}$ represents the distance from the still water level O to the roll axis G with positive being downward.

With non-zero forward ship speed, it is difficult to treat the wave roll damping theoretical1y. However, there are methods that can be used as approximate treatments for predicting the wave damping at forward speed. The first is the method in which the flow field due to roll motion is expressed by oscillating dipoles with horizontal lateral axes. The roll damping is then obtained approximately from the wave-energy loss in the far field. Ikeda *et al*., (1978a) calculated the energy loss in the far field due to a pair of horizontal doublets and compared the results with experiments for models of combined flat plates. From this elementary analysis, they proposed an empirical formula for roll damping of typical ship forms (Ikeda *et al.*, 1978a):

$$\frac{B_{44\mathrm{W}}}{B_{44\mathrm{W}0}}=0.5\begin{bmatrix}(A_2+1)+\\(A_2-1)\tanh(20\Omega-0.3)+\\(2A_1-A_2-1)\times\\ \qquad\exp\left\{-150(\Omega-0.25)^2\right\}\end{bmatrix}\tag{2.8}$$

where:

$$A_1=1+\xi_{\mathrm{d}}^{-1.2}e^{-2\xi_{\mathrm{d}}},\ A_2=0.5+\xi_{\mathrm{d}}^{-1}e^{-2\xi_{\mathrm{d}}}$$

$$\xi_{\mathrm{d}}=\frac{\omega_{\mathrm{E}}^{\ 2}d}{g}\ ,\ \Omega=\frac{V\omega_{\mathrm{E}}}{g}\tag{2.9}$$

$B_{44\mathrm{W}0}$ represents the wave damping at zero forward speed which can be obtained by a strip method. V and d are forward velocity and draught of hull. However, it appears that there are still some difficulties to be considered with this method. There is a limitation in application to certain ship forms, particularly in the case of small draught-beam ratios (Ikeda *et al.*, 1978a).

2.2.2 Hull lift component

Since the lift force acts on the ship hull moving forward with sway motion, it can therefore be concluded that a lift effect occurs for ships during roll motion as well. The prediction formula for this component is as follows (Ikeda *et al.*, 1978a, 1978b):

$$B_{44\mathrm{L}}=\frac{\rho}{2}VLdk_{\mathrm{N}}l_0l_{\mathrm{R}}\left[1-1.4\frac{\overline{OG}}{l_{\mathrm{R}}}+\frac{0.7\overline{OG}}{l_0l_{\mathrm{R}}}\right]\tag{2.10}$$

where

$$l_0=0.3d\quad,\quad l_{\mathrm{R}}=0.5d$$

$$k_{\mathrm{N}}=2\pi\frac{d}{L}+\kappa(4.1\frac{B}{L}-0.045)$$

$$\kappa=\begin{cases}0 & C_{\mathrm{M}}\le 0.92\\ 0.1\quad\text{for} & 0.92<C_{\mathrm{M}}\le 0.97\\ 0.3 & 0.97\le C_{\mathrm{M}}<0.99\end{cases}\tag{2.11}$$

where C_{M} = A_{M}/(B d) (C_{M}: midship section coefficients, A_{M}: area of midship section).

In Eq.(2.10) and (2.11), k_{N} represents the lift slope often used in the field of ship manoeuvring. The lever l_0 is defined in such a way that the quantity $l_0\dot{\varphi}/V$ corresponds to the angle of attack of the lifting body. The other lever l_{R} denotes the distance from the point O (the still water level) to the centre of lift force.

2.2.3 Frictional component

The frictional component accounts for between 8% and 10% of the total roll damping for a 2m long model ship (Ikeda et al., 1976, 1978c). However, this component is influenced by Reynolds number (scale effects), and so the proportion decreases in proportion to ship size and only accounts for between 1% and 3% for full scale ships. Other components of the roll damping do not have such scale effects. Therefore, even if the scale of a ship is varied, the same non-dimensional damping coefficient can be used for the other components excluding the frictional component.

Kato (1958) deduced a semi-empirical formula for the frictional component of the roll damping from experimental results on circular cylinders completely immersed in water. It was found that the frictional damping for rolling cylinders can be expressed in the same form as that given by Blasius (1908) for laminar flow, when the effective Reynolds number is defined as:

$$Re = \frac{0.512\, r^2 \varphi_a^2 \omega_E}{\nu} \tag{2.12}$$

where r is radius of cylinder, ν is kinematic viscosity. The frictional coefficient C_f is defined (Hughes, 1954) as:

$$C_f = 1.328 \left(\frac{3.22\, r_f^2 \varphi_a^2}{T_R \nu} \right)^{-0.5} \tag{2.13}$$

The damping coefficient due to surface friction for laminar flow in the case of zero ship speed can be represented as:

$$B'_{44F0} = \frac{4}{3\pi} \rho S_f r_f^3 \varphi_a \omega_E C_f \tag{2.14}$$

where the value of r_f and S_f for a 3-D ship hull form can be estimated by following regression formulas (Kato, 1958):

$$r_f = \frac{1}{\pi} \left\{ \frac{(0.887 + 0.145 C_B)(1.7d + C_B B) - }{2\overline{OG}} \right\} \tag{2.15}$$

$$S_f = L(1.7d + C_B B) \tag{2.16}$$

This component increases slightly with forward speed, and so a semi-theoretical method to modify the coefficient in order to account for the effect of the forward speed on the friction component was proposed by Tamiya et al, (1972). The combination of Kato and Tamiya's formulae is found to be accurate for practical use and is expressed as:

$$B_{44F} = B_{44F0} \left(1 + 4.1 \frac{V}{\omega_E L} \right) \tag{2.17}$$

where B_{44F0} is the 3-D damping coefficient which can be obtained by integrating the sectional damping coefficient B'_{44F0} in Eq.(2.14) over the ship length.

The applicability of this formula has also been confirmed through Ikeda's analysis (Ikeda et al, 1976) on the 3-D turbulent boundary layer over the hull of an oscillating ellipsoid in roll motion.

2.2.4 Eddy making component

At zero forward speed, the eddy making component for a naked hull is mainly due to the sectional vortices. Fig.2.1 schematically shows the location of the eddies generated around the ship hull during the roll motion (Ikeda *et al.*,(1977a),(1978b)). The number of eddies generated depends on two parameters relating to the hull shape, which are the half breadth-draught ratio H_0 (=$B/2d$) and the area coefficient σ (=A_j/Bd, A_j: the area of the cross section under water).

Ikeda *et al*, (1978c) found from experiments on a number of two-dimensional cylinders with various sections that this component for a naked hull is proportional to the square of both the roll frequency and the roll amplitude. In other words, the coefficient does not depend on *Ke* number, but the hull form only:

$$C_R = \frac{M_{\phi E}}{\frac{1}{2} \rho d^4 L \dot{\varphi} |\dot{\varphi}|} \tag{2.18}$$

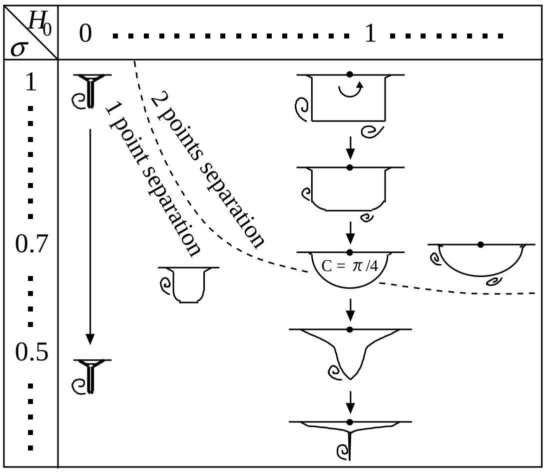

Fig.2.1 Vortices shed from hull. (Ikeda et al., 1977a)

A simple form for the pressure distribution on the hull surface as shown in Fig.2.2 can be used:

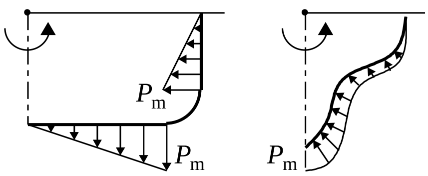

Fig.2.2 Assumed profile of pressure distribution. (Ikeda et al., 1977a).

The magnitude of the pressure coefficient C_p can be taken as a function of the ratio of the maximum relative velocity to the mean velocity on the hull surface $\gamma = V_{max}/V_{mean}$. This can be calculated approximately by using the potential flow theory for a rotating Lewis-form cylinder in an infinite fluid. The C_p-γ curve is thus obtained from the experimental results of the roll damping for 2-D models. The eddy making component at zero forward speed can be expressed by fitting this pressure coefficient C_p with an approximate function of γ, by the following formula (Ikeda *et al*, 1977a, 1978a):

$$B'_{44E0} = \frac{4\rho d^4 \omega_E \varphi_a}{3\pi} C_R \tag{2.19}$$

$$C_R = \left\{ \begin{array}{l} \left(1 - f_1 \dfrac{R}{d}\right)\left(1 - \dfrac{\overline{OG}}{d}\right) + \\ f_2 \left(H_0 - f_1 \dfrac{R}{d}\right)^2 \end{array} \right\} C_p \left(\frac{r_{max}}{d}\right)^2$$

$$C_p = 0.5\left(0.87 e^{-\gamma} - 4 e^{-0.187\gamma} + 3\right)$$

where:

$$f_1 = 0.5\left[1 + \tanh\left\{20(\sigma - 0.7)\right\}\right]$$

$$f_2 = 0.5(1 - \cos \pi\sigma) - 1.5\left\{1 - e^{-5(1-\sigma)}\right\} \sin^2 \pi\sigma$$

and the value of γ is obtained as follows:

$$\gamma = \frac{\sqrt{\pi} f_3 \left(r_{max} + \dfrac{2M}{H}\right) \sqrt{A^2 + B^2}}{2d \left(1 - \dfrac{\overline{OG}}{d}\right) \sqrt{H'_0 \sigma'}} \tag{2.20}$$

$$M = \frac{B}{2(1 + a_1 + a_3)}$$

$$H'_0 = \frac{H_0}{1 - \overline{OG}/d}$$

$$\sigma' = \frac{\sigma - \overline{OG}/d}{1 - \overline{OG}/d}$$

$$H = 1 + a_1^2 + 9a_3^2 + 2a_1(1 - 3a_3)\cos 2\psi - 6a_3 \cos 4\psi$$

$$A_0 = -2a_3 \cos 5\psi + a_1(1-a_3)\cos 3\psi + \left\{(6-3a_1)a_3^2 + (a_1^2 - 3a_1)a_3 + a_1^2\right\}\cos\psi$$

$$B_0 = -2a_3 \sin 5\psi + a_1(1-a_3)\sin 3\psi + \left\{(6+3a_1)a_3^2 + (3a_1 + a_1^2)a_3 + a_1^2\right\}\sin\psi$$

$$r_{\max} = M\sqrt{\left\{(1+a_1)\sin\psi - a_3\sin 3\psi\right\}^2 + \left\{(1-a_1)\cos\psi + a_3\cos 3\psi\right\}^2}$$

where a_1, a_3 are the Lewis-form parameters. ψ represents the Lewis argument on the transformed unit circle. ψ and f_3 are:

$$\psi = \begin{cases} 0 = \psi_1 & (r_{\max}(\psi_1) \geq r_{\max}(\psi_2)) \\ \dfrac{1}{2}\cos^{-1}\dfrac{a_1(1+a_3)}{4a_3} = \psi_2 & \\ & (r_{\max}(\psi_1) < r_{\max}(\psi_2)) \end{cases}$$

$$f_3 = 1 + 4\exp\left\{-1.65\times 10^5 (1-\sigma)^2\right\}$$

For a 3-D ship hull form, the eddy making component is given by integrating B_{E0} over the ship length.

This component decreases rapidly with forward speed and reduces to a non-linear correction for the (linear) lift force on a ship, or wing, with a small angle of attack. From experimental results for ship models a formula for this component at forward speed can be determined empirically as follows (Ikeda *et al*, 1978a, 1978c):

$$B_{44E} = B_{44E0}\frac{(0.04K)^2}{1+(0.04K)^2} \tag{2.21}$$

where K is the reduced frequency ($=\omega L/U$).

The above-mentioned Eq.(2.19) applies to a sharp-cornered box hull with normal breadth-draught ratio, but not to a very shallow draught. Yamashita *et al*, (1980) confirmed that the method gives a good result for a very flat ship when the roll axis is located at the water surface. Standing (1991), however, pointed out that Eq.(2.19) underestimates the roll damping of a barge model. To confirm the contradictions, Ikeda *et al*, (1993) carried out an experimental study on the roll damping of a very flat barge model and proposed a simplified formula for predicting the eddy component of the roll damping of the barge as follows (Ikeda *et al*, 1993):

$$B'_{44E0} = \frac{2}{\pi}\rho L d^4\left(H_0^2 + 1 - \frac{\overline{OG}}{d}\right)\times \left\{H_0^2 + \left(1 - \frac{\overline{OG}}{d}\right)^2\right\}\varphi_a\omega_E \tag{2.22}$$

2.2.5 Appendages component

2.2.5.1 Bilge keel component

The bilge keel component B_{44BK} is divided into four components:

$$B_{44BK} = B_{44BKN0} + B_{44BKH0} + B_{44BKL} + B_{44BKW} \tag{2.23}$$

The normal force component B_{44BKN0} can be deduced from the experimental results of oscillating flat plates (Ikeda *et al*, 1978d, 1979).

The drag coefficient C_D of an oscillating flat plate depends on the *Ke* number. From the measurement of the drag coefficient, C_D, from free roll tests of an ellipsoid with and without bilge keels, the prediction formula for the drag coefficient of the normal force of a pair of the bilge keels can be expressed as follows:

$$C_D = 22.5 \frac{b_{\mathrm{BK}}}{\pi l \varphi_{\mathrm{a}} f} + 2.4 \tag{2.24}$$

where b_{BK} is the breadth of the bilge keel and l is the distance from the roll axis to the tip of the bilge keel. The equivalent linear damping coefficient $B'_{44\mathrm{BKN0}}$ is:

$$B'_{44\mathrm{BKN0}} = \frac{8}{3\pi} \rho l^3 \omega_{\mathrm{E}} \varphi_{\mathrm{a}} b_{\mathrm{BK}} f\, C_D \tag{2.25}$$

where f is a correction factor to take account of the increment of flow velocity at the bilge, determined from the experiments:

$$f = 1 + 0.3 e^{\{-160(1-\sigma)\}} \tag{2.26}$$

From the measurement of the pressure on the hull surface created by the bilge keels, it was found that the coefficient C_p^+ of pressure on the front face of the bilge keels does not depend on the *Ke* number. However, the coefficient C_p of the pressure on the back face of bilge keel and the length of negative-pressure region do depend on the *Ke* number. From these results, the length of the negative-pressure region can be obtained as follows:

$$S_0 / b_{\mathrm{BK}} = 0.3 \frac{\pi l \varphi_{\mathrm{a}} f}{b_{\mathrm{BK}}} + 1.95 \tag{2.27}$$

assuming a pressure distribution on the hull as shown in Fig.2.3.

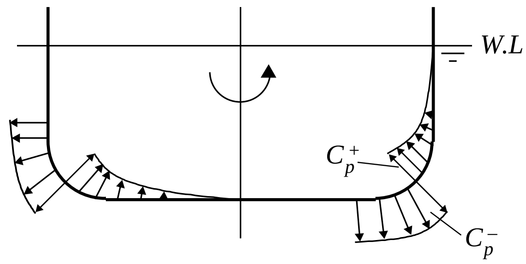

Fig.2.3 Assumed pressure distribution on the hull surface created by bilge keels. (Ikeda et al., 1976)

The roll damping coefficient B'_{BKH0} can be expressed as follows (Ikeda *et al*, 1978d, 1979):

$$B'_{44\mathrm{BKH0}} = \frac{4}{3\pi} \rho l^2 f^2 \omega_{\mathrm{E}} \varphi_{\mathrm{a}} \int_G C_p \cdot l_{\mathrm{p}} dG \tag{2.28}$$

where G is length along the girth and l_{p} is the moment lever.

The coefficient C_p^+ can be taken approximately as 1.2 empirically. From the relation of $C_D = C_p^+ - C_p^-$, the coefficient C_p^- can be obtained as follows:

$$C_p^- = 1.2 - C_D = -22.5 \frac{b_{\mathrm{BK}}}{\pi l \varphi_{\mathrm{a}} f} - 1.2 \tag{2.29}$$

The value of $\int_G C_p \cdot l_{\mathrm{p}} dG$ in Eq.(2.28) can be obtained as follows:

$$\int_G C_p \cdot l_{\mathrm{p}} dG = d^2 \left(-A_0 C_p^- + B_0 C_p^+ \right) \tag{2.30}$$

where:

$$A_0 = (m_3 + m_4) m_8 - m_7^2$$

$$B_0 = \frac{m_2^2}{3(H_0 - 0.215m_1)} + \frac{(1-m_1)^2(2m_3 - m_2)}{6(1-0.215m_1)} + m_1(m_3m_5 + m_4m_6)$$

$$m_1 = R/d$$

$$m_2 = \overline{OG}/d$$

$$m_3 = 1 - m_1 - m_2$$

$$m_4 = H_0 - m_1$$

$$m_5 = \frac{\left\{\begin{array}{l}0.414H_0 + 0.0651m_1^2 - \\ \qquad (0.382H_0 + 0.0106)m_1\end{array}\right\}}{(H_0 - 0.215m_1)(1-0.215m_1)}$$

$$m_6 = \frac{\left\{\begin{array}{l}0.414H_0 + 0.0651m_1^2 - \\ \qquad (0.382 + 0.0106H_0)m_1\end{array}\right\}}{(H_0 - 0.215m_1)(1-0.215m_1)}$$

$$m_7 = \begin{cases} S_0/d - 0.25\pi m_1, & S_0 > 0.25\pi R \\ 0 & , \; S_0 \le 0.25\pi R \end{cases}$$

$$m_8 = \begin{cases} m_7 + 0.414m_1 & , \; S_0 > 0.25\pi R \\ m_7 + 1.414m_1\left(1 - \cos\left(\frac{S_0}{R}\right)\right), & S_0 \le 0.25\pi R \end{cases}$$

where l is distance from roll axis to the tip of bilge keels and R is the bilge radius. These are calculated as follows:

$$l = d\sqrt{\left\{H_0 - \left(1 - \frac{\sqrt{2}}{2}\right)\frac{R}{d}\right\}^2 + \left\{1 - \frac{\overline{OG}}{d} - \left(1 - \frac{\sqrt{2}}{2}\right)\frac{R}{d}\right\}^2} \quad (2.31)$$

$$R = \begin{cases} 2d\sqrt{\dfrac{H_0(\sigma - 1)}{\pi - 4}}, & R < d \;\&\; R < \dfrac{B}{2} \\ d & , H_0 \ge 1 \;\&\; \dfrac{R}{d} > 1 \\ \dfrac{B}{2} & , H_0 \le l \;\&\; \dfrac{R}{d} > H_0 \end{cases} \quad (2.32)$$

To predict the bilge keel component, the prediction method assumes that a cross section consists of a vertical side wall, a horizontal bottom and a bilge radius of a quarter circle for simplicity. The location and angle of the bilge keel are taken to be the middle point of the arc of the quarter circle and perpendicular to the hull surface. It may not be possible to satisfactorily apply these assumptions to the real cross section if it has large differences from a conventional hull with small bilge radius as shown in Fig.2.4 for a high speed slender vessel (Ikeda *et al*, 1994).

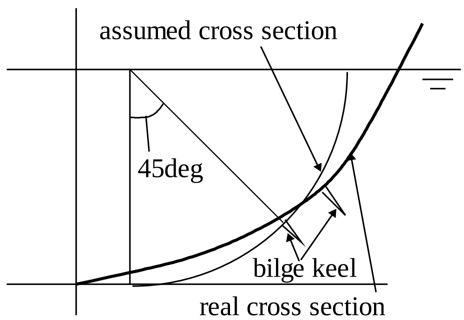

Fig.2.4 Comparison between cross section, fitting position and the angle of bilge keel assumed in prediction method and those of high speed slender vessels. (Ikeda et al, 1994)

These assumptions cause some element of error in the calculation of the moment levers of the normal force of the bilge keels and of the pressure force distributed on the hull surface created by the bilge keel. In such a case, Eq.(2.30) should be calculated directly. The pressure distribution can be taken as shown in Fig.2.3 and the length of negative pressure C_p^- can be defined by using parameter B in Eq.(2.30).

In the estimation method, it is assumed that the effect of forward speed on the bilge keel component is small and can be ignored. However, it is hard to ignore the lift force acting on the bilge keel if a vessel has high forward speed. Since a bilge keel can be regarded as a small aspect ratio wing, Jones's theory can be applied to it where the flow is composed of forward speed $V = Fr\sqrt{gL}$ and the tangential velocity caused by roll motion $u = l_1\dot{\phi} = l_1\phi_a\omega_E$ (where l_1 denotes the distance between the centre of roll axis and the centre of bilge keel) the attack angle and the resultant flow velocity are obtained as $\alpha = \tan^{-1}(u/V)$ and $V_R = \sqrt{V^2 + u^2}$ respectively. On the basis of Jones's theory, the lift force acting on a bilge keel is expressed as (Ikeda *et al*, 1994):

$$L_{\mathrm{BK}} = \frac{\pi\rho\alpha V_R^{\ 2} b_{\mathrm{BK}}^{\ 2}}{2} \tag{2.33}$$

where b_{BK} is the maximum breadth of the bilge keel. The roll damping coefficient due to a pair of bilge keels $B_{44\mathrm{BKL}}$ can be obtained as follows:

$$B_{44\mathrm{BKL}} = \frac{2\, L_{\mathrm{BK}}\, l_1}{\varphi_{\mathrm{a}}\ \omega_{\mathrm{E}}} \tag{2.34}$$

The wave making contribution from the bilge keels at zero forward speed $B_{44\mathrm{BKW0}}$ is expressed as (Bassler *et al*, 2009):

$$\hat{B}_{44\mathrm{BKW0}} \sim C_{\mathrm{BK}}\left(b_{\mathrm{BK}}\right)\exp\left(-\frac{\omega^2}{g} d_{\mathrm{BK}}\left(\varphi\right)\right) \tag{2.35}$$

where the source strength C_{BK} is a function of the bilge keel breadth b_{BK}. In this equation, the bilge keel may be considered as a source, pulsing at frequency ω_e at a depth relative to the free surface, d_{BK} in Fig.2.5, based on the roll amplitude. For simplicity, C_{BK} is assumed to be the ratio of the bilge keel breadth to ship beam. The damping is assumed to be zero for zero roll amplitude. The distance from the free surface to the bilge keel, d_{BK}, is given by:

$$d_{\mathrm{BK}}\left(\varphi\right) = l_{\mathrm{BK}}\left[\left(\frac{\left(2d/B\right)}{\sqrt{1+\left(2d/B\right)^2}}\right)\cos\varphi - \sin\varphi\left(\frac{1}{\sqrt{1+\left(2d/B\right)^2}}\right)\right] \tag{2.36}$$

where d is the draught, B is the beam, and ϕ is the roll angle, Fig.2.5. The effects of forward speed are taken into account by Eq.(2.8).

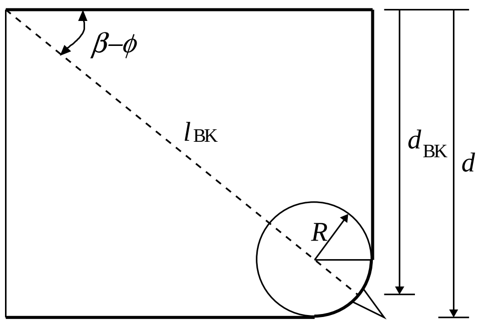

Fig.2.5 Illustration of the bilge keel depth, d_{BK}, as a function of roll angle, ϕ; and distance from the roll axis to the bilge keel, l_{BK}, for the half-midship section of a conventional hull form. (Bassler et al, 2009)

2.2.5.2 Skeg component

The skeg component of the roll damping is obtained by integrating the assumed pressure created by the skeg, as shown in Fig.2.6 over the skeg and the hull surface.

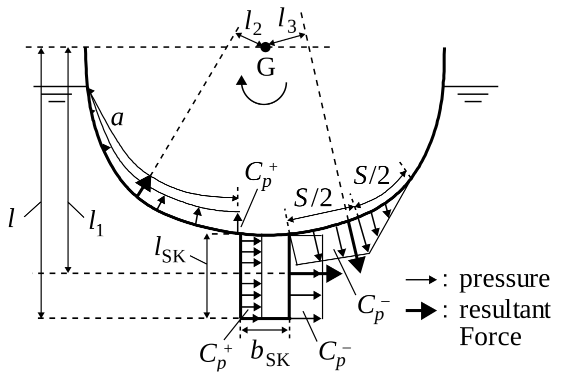

Fig.2.6 Assumed pressure created by a skeg. (Baharuddin et al., 2004)

The skeg component of the roll damping per unit length can be expressed as follows (Baharuddin *et.al*, 2004):

$$B'_{44\mathrm{SK}0} = \frac{4}{3\pi}\varphi_{\mathrm{a}} l^2 \omega_{\mathrm{E}} \rho \left(\begin{array}{l} C_D l_{\mathrm{SK}} l_1 - \\ 0.5 C_p^+ a l_2 + \\ \frac{3}{4} C_p^- S l_3 \end{array} \right) \quad (2.37)$$

$$C_D = (C_p^+ - C_p^-) = C_{D0} e^{\left(\frac{-0.38 b_{\mathrm{SK}}}{l_{\mathrm{SK}}}\right)}$$

$$C_p^+ = 1.2$$

$$C_{D0} = \begin{cases} 2.425 Ke & , \ 0 \le Ke \le 2 \\ -0.3 Ke + 5.45 & , \ Ke > 2 \end{cases}$$

$$Ke = \frac{U_{\max} T_e}{2 l_{\mathrm{SK}}} = \frac{\pi \varphi_{\mathrm{a}} l}{l_{\mathrm{SK}}}$$

$$S = 1.65 Ke^{2/3} \cdot l_{\mathrm{SK}}$$

where C_p^+ , C_p^- and l_2, l_3 denote representative pressure coefficients and their moment levers obtained by integrating the pressure distribution on the hull surface in front of and on the back face of the skeg respectively. l is the distance from the axis of roll rotation to the tip of the skeg. l_{SK} and b_{SK} are the height and thickness of skeg respectively, Ke is the Keulegan-Carpenter number for the skeg, U_{max} is the maximum tangential speed of the edge of the skeg, T_e is the period of roll motion and S is the distribution length of negative pressure on hull surface created by the skeg.

2.3 Hard chine type hull

Generally the roll damping acting on a cross section can be divided into a frictional component, a wave making component, an eddy making component, a bilge-keel component and a skeg component. Bilge keel and skeg components are caused by separated vortices. However, it is more convenient practically to treat them as independent components, without including them in the eddy making component. Although the friction component may be around 10% of the roll damping from measured model data (;model length under approximate 4m, refer to IMO MSC.1/ Circ.1200 ANNEX, Page 7, 4.3.2), it is only up to approximately 3% for a full scale vessel. This means therefore, that the friction component can be effectively ignored. The wave making component can again be treated using the theoretical calculation based on potential theory as defined previously for displacement hulls. Therefore it is recommended to also apply these calculation methods to hard chine type hulls.

2.3.1 Eddy making component

The eddy making component of a hard chine type hull is mainly caused by the separated vortices from the chine. The sectional pressure distribution on hull caused by this separated vortex is approximated by a simple formulation and the roll damping is calculated by integrating it along the hull surface.

The length and the value of the pressure distribution are decided upon based on the measured pressure and the measured roll damping. Initially the estimation method is used for the case where the rise of floor is 0. The pressure distribution is assumed to like that shown in Fig.2.7.

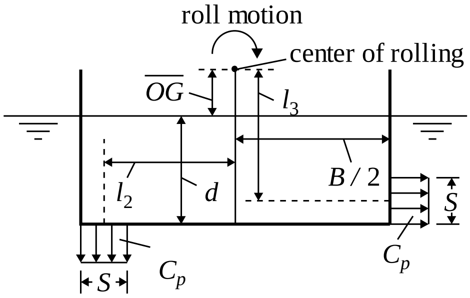

Fig.2.7 Assumed pressure distribution created by separated flow from hard chine. (Ikeda et al, 1990)

The sectional roll damping coefficient is calculated from the following:

$$B'_{44\mathrm{E0}} = \frac{4}{3\pi}\rho\varphi_{\mathrm{a}}\omega_{\mathrm{E}}C_p\,S\,(l_2+l_3)l^2 \quad (2.38)$$

where, l_2 and l_3 are the moment levers shown in Fig.2.7, and l is the distance from the axis of roll rotation to the chine (Ikeda et al, 1990).

The length of the negative pressure S and its pressure coefficient C_p are expressed as the function $H_0{*}\left(=B/\left(2d-2\overline{OG}\right)\right)$. These are obtained from the following equations based on measured data:

$$S = (0.3H_0{*} - 0.1775 + \frac{0.0775}{H_0{*}^2})d \quad (2.39)$$

$$C_p = \exp(k_1 H_0{*} + k_2) \quad (2.40)$$

where:

$$k_1 = -\exp\begin{pmatrix} -0.114H_0^{\,2} + \\ 0.584H_0 - 0.558 \end{pmatrix} \quad (2.41)$$

$$k_2 = -0.38H_0^{\,2} + 2.264H_0 + 0.748$$

When there is a rise of floor, the moment lever not only changes, but the length of the negative pressure distribution and its pressure coefficient also change. However, the effect of the rise of floor on the size of a separated vortex is not well understood. Therefore, the effect of rise of floor is taken into consideration by modifying the coefficient as a function of the rise of floor. S and C_p are multiplied by the following empirical modification coefficient (Ikeda *et al*, 1990):

$$f_1(\alpha) = \exp(-2.145\beta) \quad (2.42)$$

$$f_2(\alpha) = \exp(-1.718\beta) \quad (2.43)$$

Using the above method, the eddy making component of a cross section can be estimated. The depth of the chine d_c, the half breadth to draught ratio H_0 ($=B/2d$) of a cross section, draught d, rise of floor β, and vertical distance from water surface to the centre of gravity (axis of roll rotation) $\overline{OG}$ (downward positive) are required for the estimation.

2.3.2 Skeg component

The estimation method for the skeg component has been proposed by Tanaka *et al*, (1985). Using the estimation method, the shape of the approximated pressure distribution is shown in Fig.2.8.

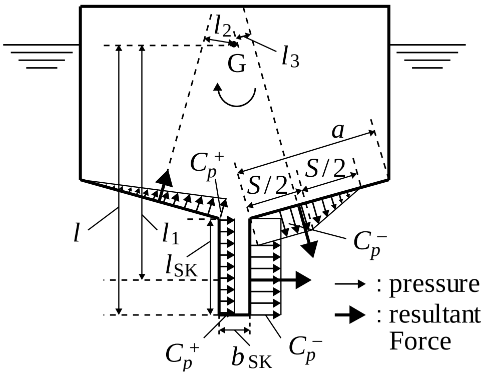

Fig.2.8 Assumed pressure distribution created by skeg. (Tanaka et al., 1985)

From the integration of the pressure distribution, the roll damping coefficient for the cross section is expressed by the following:

$$B'_{44\mathrm{SK}0} = \frac{8}{3\pi}\rho\,\varphi_{\mathrm{a}} l^2 \omega_{\mathrm{E}} \left\{ \begin{array}{l} C_D l_{\mathrm{SK}} l_1 - \\ 0.5 C_p^+ a l_2 + \\ \frac{3}{4} C_p^- S l_3 \end{array} \right\} \quad (2.44)$$

$$C_D = C_p^+ - C_p^-$$

$$C_p^- = -3.8$$

$$C_p^+ = 1.2$$

$$S = 1.65 Ke^{\frac{2}{3}} l_{\mathrm{SK}}$$

$$Ke = U_{\max} \frac{T_e}{2 l_{\mathrm{SK}}}$$

Here, $U_{\max}$ is the maximum tangential speed at the centre of skeg, T_e is roll period, l_{SK}, b_{SK} are the height and thickness of skeg, and l is the distance from the axis of roll rotation to the tip of the skeg. In this estimation method, the skeg is assumed to be a flat plate and the pressure coefficient is assumed to be constant based on the measured results from an oscillated flat plate with a flat plate skeg (Tanaka *et al*, 1985). However, an Asian coastal fishing boat may have a wide breadth due to the stability requirements for the boat and due to the strength of the skeg required in service (Ikeda *et al*, 1990). In this case, not only should the measured results from a flat plate be considered, but also the measured results of the drag coefficients from oscillating square cylinders (Ikeda *et al*, 1990), in order to decide upon a suitable drag coefficient. It is expressed by the following (Ikeda *et al*, 1990):

$$C_D \left(= C_p^+ - C_p^-\right) = C_{D0} \exp\left(-0.38 \frac{b_{\mathrm{SK}}}{l_{\mathrm{SK}}}\right)$$

$$C_{D0} = \begin{cases} 2.425 Ke & (0 \le Ke \le 2) \\ -0.3 Ke + 5.45 & (2 < Ke) \end{cases}$$

$$C_p^+ = 1.2 \quad (2.45)$$

2.4 Multi-hull

Katayama *et al*. (2008) experimentally investigated the characteristics of roll damping of two types of multi-hull vessels: a high speed catamaran; and a trimaran. They proposed a method of estimating the roll damping for these types of craft.

2.4.1 Wave making component

The wave making component $B_{44\mathrm{W}}$ is generated by the almost vertical motion of the demihull. For this component, the wave interaction between the hulls is considered significant, as also indicated by Ohkusu, (1970). However, for simplicity, this component can be estimated by using the heave potential damping of the demihull B_{33}. It should be noted however, that the B_{33} term does not include the wave interaction effects between the hulls. A strip method, including the end term effects, is used for the calculation of B_{33} (Katayama *et al.* 2008):

$$\begin{aligned} B'_{44W}\dot{\varphi} &= B'_{44W}\omega_E\varphi_a \\ &= 2b_{demi}\ B'_{33}\ b_{demi}\omega_E\varphi_a \\ &= 2b_{demi}{}^2\ B'_{33}\dot{\varphi} \end{aligned} \tag{2.46}$$

where b_{demi} is the distance of the centre of demihull from the vessel's centre line.

2.4.2 Lift component

A method for the estimation of the lift component of a multi-hull vessel can be constructed based on Eq.(2.10). Based on the relative location of each hull in the multi-hull craft, l_R, l_0 and $\overline{O'G}$ are defined as shown in Fig.2.9.

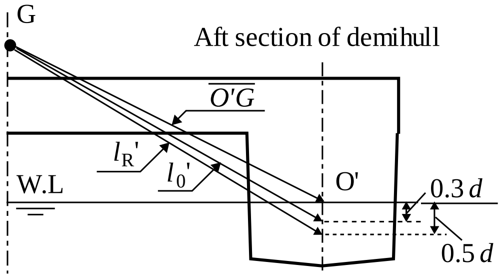

Fig.2.9 Coordinate system to calculate l'$_0$ and l'$_R$ and $\overline{O'G}$. (Katayama et al., 2008)

This allows the lift component to be described as follows (Katayama *et al.* 2008):

$$B_{44L} = \frac{1}{2}\rho A_{HL} V k_N l'_0 l'_R \left(1 - 1.4\frac{\overline{O'G}}{l'_R} + 0.7\frac{\overline{O'G}^2}{l'_0\, l'_R} \right) \tag{2.47}$$

$$k_N = \frac{2\pi d}{L_{PP}}$$

where A_{HL} is the lateral area of the demihulls or side hulls under water line and L_{PP} is the length between perpendiculars.

2.4.3 Frictional component

For multi-hull vessels, the frictional component is created by the vertical motion of the demihull or side hull. This component is assumed to be smaller than the other components. Based on the estimation method proposed in the previous chapters, the friction component for the demihull or side hull can be estimated as follows (Katayama *et al.* 2008):

$$B'_{44F} = \frac{8}{3\pi}\rho A_{HL}\varphi_a\omega_E b_{demi}{}^3 C_f \left(1 + 4.1\frac{V}{\omega_E L_{PP}}\right) \tag{2.48}$$

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}}} \qquad Re = \frac{4\varphi_a b_{demi} d}{T_e \nu}$$

where A_{HL} is the lateral area of the demihulls or side hulls under water line, and b_{demi} is the distance of the centre of the demihull from the centre line, ν is kinematic viscosity. The effects of forward speed can be taken into account with Eq.(2.17).

2.4.4 Eddy making component

Significant vortex shedding has been observed from flow visualization around multi-hull vessels whilst rolling. It was observed that one vortex was shed from each demihull of the catamaran and from each side hull of the trimaran. The location of the vortex shedding was found to be at the keel or the outside bilge of demihull/side hull. This is shown in Fig.2.10. (Katayama *et al*. 2008).

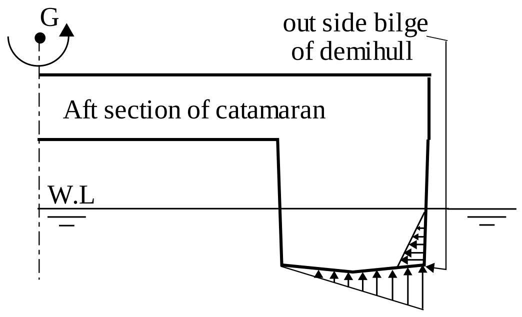

Fig.2.10 Assumed vortex shedding point and pressure distribution of aft section of catamaran. (Katayama et al., 2008)

The scale of the eddy may be similar to that for barge vessels. Therefore, these damping forces can be estimated by integrating the pressure created by eddy-making phenomena over the hull surface. The pressure coefficient at the point of vortex shedding can be assumed to be 1.2 and the profile of pressure distribution is assumed as shown in Fig.2.10. In addition, the effects of forward speed are taken into account by Eq.(2.21).

2.5 Additional damping for a planing hull

Typical planing craft have a shallow draught compared to their breadth, with an immersed lateral area that is usually very small. Even if the vessel runs at a very high speed, the horizontal lift component is small. Conversely, the water plane area is very large and the vertical lift force acting on the bottom of the craft is also large. As a result, this may play an important role in the roll damping. It is therefore necessary to take into account the component due to this effect. Assuming that a craft has small amplitude periodic roll motion about the center of gravity, a point y on a cross section shown in Fig.2.11, has a vertical velocity $u_z(y)$ [m/sec.] defined as:

$$u_z(y) = \dot{\varphi}\, y \tag{2.49}$$

where $\dot{\phi}$ [rad./sec.] denotes roll angular velocity and y [m] is transverse distance between the centre of gravity and point y.

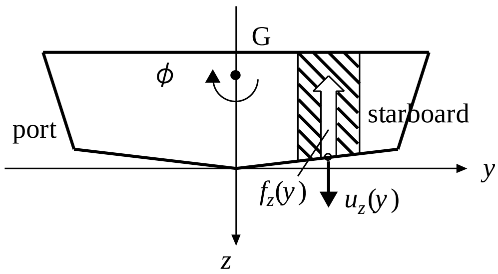

Fig.2.11 Cross section of a ship. (Ikeda et al., 2000)

When the craft has forward speed V [m/sec.], the buttock section including point y, experiences an angle of attack $\alpha(y)$ [rad] for the relative flow as shown in Fig.2.12.

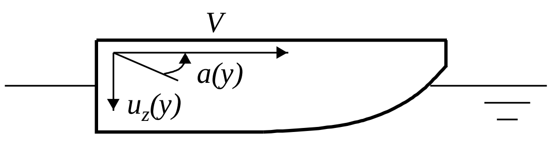

Fig.2.12 Buttock section of a craft. (Ikeda et al., 2000)

The angle $\alpha(y)$ can be calculated as follows:

$$\alpha(y) = \tan^{-1}\frac{u_z(y)}{V} = \tan^{-1}\frac{\dot{\varphi}y}{V} \cong \frac{\dot{\varphi}y}{V} \tag{2.50}$$

Assuming that the running trim angle is θ_1 [rad.], the vertical lift force acting on the craft is expressed as the virtual trim angle $\theta(y)$ [rad.] with the relative flow described as:

$$\theta(y) = \theta_1 + \alpha(y) \cong \theta_1 + \frac{\dot{\varphi} y}{V} \quad (2.51)$$

For planing craft, the magnitude of the hydrodynamic lift force significantly depends on the trim angle. The vertical lift force $f_z(y)$ [kgf/m] (positive upwards) acting on the buttock line including point y, with attack angle $\alpha(y)$ [rad.], is calculated as follows:

$$f_z(y) = \frac{1}{2}\rho B_{\mathrm{w.l}} V^2 k_{\mathrm{L}}(\theta_1)\alpha(y) \quad (2.52)$$

where ρ [kgf sec.^{2}/m^4] denotes the density of the fluid, $B_{\mathrm{w.l}}$ denotes the water line breadth and $k_{\mathrm{L}}(\theta_1)$ [1/rad.] is the lift slope. This is the non-dimensional vertical lift coefficient C_L differentiated by trim angle as follows:

$$k_{\mathrm{L}}(\theta_1) = \frac{\partial C_L}{\partial \theta} \quad (2.53)$$

On the basis of the quasi-steady assumption, $f_z(y)$ [kgf/m] is assumed to be the mean value of the hydrodynamic lift force L [kgf] acting on the planing hull in steady running condition:

$$f_z(y) = \frac{L}{B_{\mathrm{w.l}}} = \frac{1}{2}\rho B_{\mathrm{w.l}} V^2 C_L \quad (2.54)$$

where the lever arm for the roll moment about the center of gravity is y [m]. The roll moment is then given by:

$$\begin{aligned} M_{\varphi} &= \int_{-\frac{B_{\mathrm{w.l}}}{2}}^{\frac{B_{\mathrm{w.l}}}{2}} f_z(y)\cdot y\,dy \\ &= \frac{1}{24}\rho B_{\mathrm{w.l}}{}^4 V k_{\mathrm{L}}(\theta_1)\dot{\varphi} = B_{\mathrm{VL}}\dot{\varphi} \end{aligned} \quad (2.55)$$

This method of predicting the vertical lift component for planing craft is combined with the prediction method for a hard chine hull as an additional component $B_{44\mathrm{VL}}$ (Ikeda *et al*, 2000).

2.6 Additional damping for flooded ship

Flood water dynamics is similar to the effects of anti-rolling tank. The tank is classified according to its shape, such as a U-tube type or open-surface type. The ship motion including the effects of the tank has been theoretically established for each type (*e.g.* Watanabe, (1930 & 1943), Tamiya, (1958), Lewison, (1976)). However, in order to calculate the resultant ship motion, experiments such as forced oscillation tests are required to obtain some characteristics of the tank.

Based on experimental results by Katayama *et al*, (2009), and Ikeda *et al*, (2008) a proposed estimation formula for the roll damping component created by flooded water was obtained. It should be noted that the prediction formula only applies to smaller roll angles, but can be applied to cases without a mean heel angle.

$$\begin{aligned} B_{44\mathrm{IW}} = {} & A\left(\frac{h}{B_{comp}}, \varphi_a, \frac{\overline{OG}}{B}\right)\times \\ & C\left(\omega_{\mathrm{E}}, \frac{h}{B_{comp}}\right)^{B\left(\frac{h}{B_{comp}},\varphi_a\right)} \times \\ & \exp\left\{-C\left(\omega_{\mathrm{E}}, \frac{h}{B_{comp}}\right)^{B\left(\frac{h}{B_{comp}},\varphi_a\right)}\right\}\times \\ & l_{\mathrm{comp.}}\rho B_{\mathrm{comp.}}^5 \sqrt{\frac{2g}{B_{\mathrm{comp.}}}} \end{aligned} \quad (2.56)$$

$$A(\frac{h}{B_{comp}},\varphi_a,\frac{\overline{OG}}{B})=\frac{1.8\frac{h}{B_{comp}}-1.9882\varphi_a+0.429}{1.2\frac{\overline{OG}}{B}+1}$$

$$B(\frac{h}{B_{comp}},\varphi_a)=40.842\frac{h}{B_{comp}}-10.502\varphi_a+2.1$$

$$C(\omega_{\mathrm{E}},\frac{h}{B_{comp}})=\frac{1}{\pi}\sqrt{\frac{B}{g}}\cdot\left(\frac{\omega_{\mathrm{E}}}{\sqrt{h/B_{comp}}}\right)=\frac{\omega_{\mathrm{E}}}{\omega_{\mathrm{IW}}}$$

$$\omega_{\mathrm{IW}}=\frac{\pi}{B_{comp}}\sqrt{gh}$$

where h is water depth. l_{comp} and B_{comp} are the length and the breadth of flooding compartment. ρ and g are the density of fluid and acceleration of gravity respectively. ω_{E} is roll frequency, ϕ_{a} is roll amplitude, ω_{IW} is the natural frequency of the water in a tank.

3. ESTIMATION OF ROLL DAMPING COEFFICIENTS

Many ways of representing roll damping coefficients have been expressed, depending on whether the roll damping is expressed as a linear or nonlinear form. In this section, some of the expressions most commonly used are introduced, and the relations among them are reviewed and they are transformed into terms of linearized damping coefficients.

3.1 Nonlinear damping coefficients

The equations of ship motion are expressed in six-degrees-of freedom. Roll motion has coupling terms of sway and yaw motions, even if the form is a linear motion equation under small motion amplitude and symmetrical hull assumptions. In this section, in order to discuss the problem of nonlinear roll damping, however, the equation of the roll motion of a ship is expressed as the following simple single-degree-of-freedom form:

$$I_\phi\ddot{\varphi}+B_\phi(\dot{\varphi})+C_\phi\varphi=M_\phi(\omega_{\mathrm{E}}t) \qquad (3.1)$$

Here, if the roll motion is assumed to be a steady periodic oscillation, ϕ in Eq.(3.1) is expressed with its amplitude ϕ_{a} and its circular frequency ω_{E}. I_φ is the virtual mass moment of inertia along a longitudinal axis through the center of gravity and C_φ is the coefficient of restoring moment. Furthermore, M_φ is the exciting moment due to waves or external forces acting on the ship, and t is the time. Finally, B_φ denotes the nonlinear roll damping moment.

The damping moment B_φ can be expressed as a series expansion of $\dot{\phi}$ and $|\dot{\phi}|$ in the form:

$$B_\phi=B_{\phi1}\dot{\varphi}+B_{\phi2}\dot{\varphi}|\dot{\varphi}|+B_{\phi3}\dot{\varphi}^3+\cdots \qquad (3.2)$$

which is a nonlinear representation. The coefficients $B_{\varphi1}$, $B_{\varphi2}$, in Eq.(3.2) are considered constants during a steady periodic oscillation concerned. For the case of large amplitude roll motion, where the bilge keel may be above water surface at the moment of maximum roll angle, $B_{\varphi1}$, $B_{\varphi2}$ in Eq.(3.2) are proposed as a piecewise function of roll angle by Bassler *et al*, (2010). It should be noted that these coefficients may be not same values for a different steady periodic oscillation, in other words, they may depend on the amplitude ϕ_a and the frequency ω_e of steady periodic oscillation.

Dividing Eq.(3.1) with Eq.(3.2) by I_φ, another expression per unit mass moment of inertia can be obtained:

$$\ddot{\varphi}+2\alpha\dot{\varphi}+\beta|\dot{\varphi}|\dot{\varphi}+\gamma\dot{\varphi}^3+\omega_\phi^2\varphi=m_\varphi(\omega_E t) \quad (3.3)$$

where:

$$\begin{aligned}&2\alpha=\frac{B_{\phi 1}}{I_\phi},\quad \beta=\frac{B_{\phi 2}}{I_\phi},\quad \gamma=\frac{B_{\phi 3}}{I_\phi},\\ &\omega_\phi=\sqrt{\frac{C_\phi}{I_\phi}}=\frac{2\pi}{T_\phi},\quad m_\phi=\frac{M_\phi}{I_\phi}\end{aligned} \quad (3.4)$$

In Eq.(3.4) the quantities ω_φ and T_φ represent the natural frequency and the natural period of roll, respectively.

3.2 Equivalent linear damping coefficients

Since it is difficult to analyze strictly the nonlinear equation stated in the preceding section, the nonlinear damping is usually replaced by a certain kind of linearized damping as follows:

$$B_\phi(\dot{\varphi})=B_{\varphi e}\dot{\varphi} \quad (3.5)$$

The coefficient $B_{\varphi e}$ denotes the equivalent linear damping coefficient. Although the value of $B_{\varphi e}$ depends in general on the amplitude and the frequency, because the damping is usually nonlinear, it can be assumed that $B_{\varphi e}$ is constant during the specific motion concerned.

There are several ways to express the coefficient $B_{\varphi e}$ in terms of the nonlinear damping coefficients $B_{\varphi 1}$, $B_{\varphi 2}$ and so on. The most general way is to assume that the energy loss due to damping during a half cycle of roll is the same when nonlinear, and linear damping are used (Tasai, 1965). If the motion is simple harmonic at circular frequency ω_E, then $B_{\varphi e}$ can be expressed as:

$$B_{\phi e}=B_{\phi 1}+\frac{8}{3\pi}\omega_E\varphi_a B_{\phi 2}+\frac{3}{4}\omega_E{}^2\varphi_a^2 B_{\phi 3} \quad (3.6)$$

For more general periodic motion, Eq.(3.6) can be derived by equating the first terms of the Fourier expansions of Eqs.(3.5) and (3.2) (Takaki *et al*, 1973).

Corresponding to Eq.(3.3), an equivalent linear damping coefficient can be defined, α_e= $B_{\varphi e}/2I_\varphi$ per unit mass moment of inertia:

$$\alpha_e=\alpha+\frac{4}{3\pi}\omega_E\varphi_a\beta+\frac{3}{8}\omega_E^2\varphi_a^2\gamma \quad (3.7)$$

In the case of irregular roll motion, there is another approach to the linearization of the roll damping expression. Following the work of Kaplan, (1966), Vassilopoulos, (1971) and others, it can be assumed that the difference of the damping moment between its linearized and nonlinear forms can be minimized in the sense of the least squares method. Neglecting the term $B_{\varphi 3}$ for simplicity the discrepancy δ in the form can be defined:

$$\delta=B_{\phi 1}\dot{\varphi}+B_{\phi 2}|\dot{\varphi}|-B_{\phi e}\dot{\varphi} \quad (3.8)$$

Then, E{δ^2} can be minimized, the expectation value of the square of δ during the irregular roll motion, assuming that the undulation of the roll angular velocity $\dot{\phi}$ is subject to a Gaussian process and that the coefficients $B_{\varphi e}$, $B_{\varphi 1}$ and $B_{\varphi 2}$ remain constant:

$$\frac{\partial E\{\delta^2\}}{\partial B_{\phi e}} = -2\left(B_{\phi 1} - B_{\phi e}\right)E\{\delta^2\} - 2B_{\phi 2}E\{\dot{\varphi}^2|\dot{\varphi}|\} = 0 \tag{3.9}$$

and then:

$$B_{\phi e} = B_{\phi 1} + \sqrt{\frac{8}{\pi}}\sigma_{\dot{\phi}} B_{\phi 2} \tag{3.10}$$

where the factor $\sigma_{\dot{\phi}}$ represents the variance of the angular velocity $\dot{\phi}$ (JSRA, 1977). Furthermore, as an unusual way of linearization, the nonlinear expression can be equated to the linear one at the instant when the roll angular velocity takes its maximum value during steady oscillation:

$$B_{\phi e} = B_{\phi 1} + \omega_{\mathrm{E}}\varphi_{\mathrm{a}} B_{\phi 2} \tag{3.11}$$

This form seems to correspond to a collocation method in a curve-fitting problem, whereas Eq.(3.6) corresponds to the Galerkin approach. Since there is a difference of approximately 15% between the second terms of the right hand sides of Eqs.(3.6) and (3.11), the latter form may not be valid for the analysis of roll motion. However, it may be used as a simple way of analyzing numerical or experimental forced-oscillation test data to obtain the values of these coefficients quickly from the time history of the roll moment.

3.3 Decay coefficients

a free-roll test, the ship is rolled to a chosen angle and then released. The subsequent motion is obtained. Denoted by ϕ_n, the absolute value of roll angle at the time of the n-th extreme value, the so-called decay curve expresses the decrease of ϕ_m as a function of mean roll angle. Following Froude and Baker (Froude, (1874), Idle *et al*, (1912)), the decay curve is fitted using a third-degree polynomial:

$$\Delta\varphi = a\varphi_m + b\varphi_m^2 + c\varphi_m^3 \tag{3.12}$$

where:

$$\Delta\varphi = \varphi_{n-1} - \varphi_n$$
$$\varphi_m = [\varphi_{n-1} + \varphi_n]/2$$

The angles in degrees are usually used in this process.

The coefficients *a*, *b* and *c* are called decay coefficients. The relation between these coefficients and the damping coefficients can be derived by integrating Eq.(3.1) without the external-force term over the time period of a half roll cycle and then equating the energy loss due to damping to the work done by the restoring moment. The result can be expressed in the form:

$$\Delta\varphi = \frac{\pi}{2}\frac{\omega_\phi}{C_\phi}\varphi_m \times \left(B_{\phi 1} + \frac{8}{3\pi}\omega_\phi\varphi_m B_{\phi 2} + \frac{3}{4}\omega_\phi^2\varphi_m^2 B_{\phi 3}\right) \tag{3.13}$$

Comparing Eq.(3.13) with Eq.(3.12) term by term, the following relations can be obtained:

$$a = \frac{\pi}{2}\frac{\omega_\phi}{C_\phi}B_{\phi 1} = \frac{\pi}{2}\frac{2\alpha}{\omega_\phi} = \frac{\pi}{2}\kappa_\alpha$$

$$b\frac{180}{\pi} = \frac{4}{3}\frac{\omega_\phi^2}{C_\phi}B_{\phi 2} = \frac{4}{3}\beta \tag{3.14}$$

$$c\left(\frac{180}{\pi}\right)^2 = \frac{3\pi}{8}\frac{\omega_\phi^3}{C_\phi}B_{\phi 3} = \frac{3\pi}{8}\omega_\phi\gamma$$

It should be noted that the condition for the validity of Eq.(3.14) is that the coefficients $B_{\varphi 1}$, $B_{\varphi 2}, \cdots$ and $a, b, \cdots$ should be independent of the roll amplitude. As the section 2.2.5.1 explained, the effect of bilge keels appears mainly in the term $B_{\varphi 2}$ and, further, the value of $B_{\varphi 2}$ varies with roll amplitude. In such a case, Eq.(3.14) will not remain valid. Only the part of B_2 which is independent of the amplitude is related to the coefficient b. The other part of $B_{\varphi 2}$ that is inversely proportional to the amplitude will apparently be transferred to the coefficient a, and the part proportional to the amplitude will appear in c. In place of a term-by-term comparison, therefore, it will probably be reasonable to define an equivalent extinction coefficient a_e and to compare it with the equivalent linear damping coefficient $B_{\varphi e}$ as in the form:

$$a_e = a + b\varphi_m + c\varphi_m{}^2 = \frac{\pi}{2}\frac{\omega_\phi}{C_\phi}B_{\phi e} \tag{3.15}$$

Bertin's expression by Motora, (1964), can be written in the form:

$$\Delta\varphi = \frac{\pi N}{180}\varphi_m^2 \tag{3.16}$$

The coefficient N can be taken as a kind of equivalent nonlinear expression and it has been called an "N-coefficient". As seen from Eq.(3.12):

$$N = a\frac{180}{\pi\varphi_m} + b + c\frac{\pi\varphi_m}{180} \tag{3.17}$$

The value of N depends strongly on the mean roll angle ϕ_m so that its expression is always associated with the ϕ_m value, being denoted as N_{10}, N_{20} and so on, where N_{10} is the value of N when mean roll angle is 10 degrees, *etc*.

4. PARAMETERS

4.1 Parameters to be taken into account

The main parameters that need to be considered when dealing with roll damping are presented below.

<u>Hull Form including Appendages (bilge keel, skeg and rudder etc)</u>

- Body plan or 3D-data of hull
- Principal particulars of hull (Length, Breadth and Draught)
- Dimensions of appendages (length, width, thickness and position)

<u>Loading Condition of Ship</u>

- Weight or draught of ship
- Height of the centre of gravity: $\overline{KG}$
- Roll natural period T_φ

<u>Rolling Condition</u>

- Roll period T_R or wave period T_w
- Wave direction χ
- Forward speed V or Froude number Fr
- Roll amplitude ϕ_a

5. NOMENCLATURE

Symbol	Explanation	Section
y	transverse position on cross section	2.5
A_0	$A_0 = -2a_3 \cos 5\psi + a_1(1-a_3)\cos 3\psi + +\left\{(6-3a_1)a_3^{\,2} + (a_1^{\,2}-3a_1)a_3 + a_1^{\,2}\right\}\cos\psi$	2.2 2.2.4
A_0	$A_0 = (m_3 + m_4)m_8 - m_7^{\,2}$	2.2 2.2.5.1
A_1	$A_1 = 1 + \xi_d^{\,-1.2} e^{-2\xi_d}$	2.2 2.2.1
A_2	$A_2 = 0.5 + \xi_d^{\,-1} e^{-2\xi_d}$	2.2 2.2.1
A_M	midship section area	2.2 2.2.2
A_{HL}	lateral area of the demihulls or side hulls under water line	2.4 2.4.2 2.4 2.4.3
A_j	area of cross section under water line	2.2 2.2.4
a	length acting on C_{fp} sectional girth length from keel to hard chine or water line	2.2 2.2.5.2 2.3 2.3.2
a, b, c	decay coefficient (obtained from free-roll test)	3.3
a_1, a_3	Lewis-form parameter	2.2 2.2.4
a_e	equivalent extinction coefficient	3.3
B	breadth of hull	2.1 2.2 2.2.2 2.2 2.2.3 2.2 2.2.5.1
B_0	$B_0 = -2a_3 \sin 5\psi + a_1(1-a_3)\sin 3\psi + \left\{(6+3a_1)a_3^{\,2} + (3a_1 + a_1^{\,2})a_3 + a_1^{\,2}\right\}\sin\psi$	2.2 2.2.4
B_0	$B_0 = \dfrac{m_2^{\,2}}{3(H_0 - 0.215m_1)} + \dfrac{(1-m_1)^2(2m_3 - m_2)}{6(1-0.215m_1)} + m_1(m_3 m_5 + m_4 m_6)$	2.2 2.2.5.1
B_{33}	linear coefficient of heave damping	2.4 2.4.1
B_{44}	equivalent linear coefficient of total roll damping	2.1
B_{44AP}	equivalent linear coefficient of apendage component of roll damping	2.1
B_{44BK}	equivalent linear coefficient of bilge-keel component of roll damping	2.2 2.2.5.1
B_{44BKL}	equivalent linear coefficient of bilge-keel lift component of roll damping	2.2 2.2.5.1
B_{44BKW}	equivalent linear coefficient of bilge-keel wave making component of roll damping	2.2 2.2.5.1
B_{44E}	equivalent linear coefficient of eddy making component of roll damping	2.1 2.2 2.2.4
B_{44F}	equivalent linear coefficient of friction component of roll damping	2.1 2.2 2.2.3
B_{44IW}	equivalent linear coefficient of flooded water component of roll damping	2.6
B_{44L}	equivalent linear coefficient of lift component of roll damping	2.1 2.2 2.2.2 2.4 2.4.2
B_{44VL}	equivalent linear coefficient of vertical lift component of roll damping	2.5

B_{44W}	equivalent linear coefficient of wave making component of roll damping	2.1 2.2 2.2.1
B_{440}	[subscript 0] indicates the value without forward speed equivalent linear coefficient of total roll damping without forward speed	2.1
B_{44BKH0}	equivalent linear coefficient of bilge-keel's hull pressure component of roll damping without forward speed	2.2 2.2.5.1
B_{44BKN0}	equivalent linear coefficient of bilge-keel's normal force component of roll damping without forward speed	2.2 2.2.5.1
B_{44E0}	equivalent linear coefficient of eddy making component of roll damping without forward speed	2.2 2.2.4
B_{44F0}	equivalent linear coefficient of frictional component of roll damping without forward speed	2.2 2.2.3
B_{44W0}	equivalent linear coefficient of wave making component of roll damping without forward speed	2.2 2.2.1
B'_{22}	[prime '] indicates sectional value sectional equivalent linear coefficient of sway damping	2.2 2.2.1
B'_{33}	sectional linear coefficient of heave damping	2.4 2.4.1
B'_{42}	sectional equivalent linear coupling coefficient of roll damping by swaying	2.2 2.2.1
B'_{44}	sectional linear coefficient of total roll damping	2.1
B'_{44F}	sectional equivalent linear coefficient of frictional component of roll damping	2.4 2.4.3
B'_{44W}	sectional equivalent linear coefficient of wave making component of roll damping	2.4 2.4.1
B'_{44BKH0}	sectional equivalent linear coefficient of bilge-keel's hull pressure component of roll damping without forward speed	2.2 2.2.5.1
B'_{44BKN0}	sectional equivalent linear coefficient of bilge-keel's normal force component of roll damping without forward speed	2.2 2.2.5.1
B'_{44E0}	sectional equivalent linear coefficient of eddy making component of roll damping without forward speed	2.2 2.2.4 2.3 2.3.1
B'_{44F0}	sectional equivalent linear coefficient of frictional component of roll damping without forward speed	2.2 2.2.3
B'_{44SK0}	sectional equivalent linear coefficient of skeg component of roll damping without forward speed	2.2 2.2.5.2 2.3 2.3.2
B'_{44W0}	sectional equivalent linear coefficient of wave making component of roll damping without forward speed	2.2 2.2.1
$\hat{B}_{44}$	[^] indicates non-dimensional value non-dimensional equivalent linear coefficient of total roll damping	2.1
$\hat{B}_{44BKW0}$	non-dimensional equivalent linear coefficient of bilge-keel component of roll damping without forward speed	2.2 2.2.5.1
B_{comp}	breadth of flooding component	2.6
$B_{w.l}$	water line breadth	2.5
B_φ $B_\phi(\dot{\varphi})$	nonlinear coefficient of roll damping	3.1 3.2
$B_{\varphi1}$ $B_{\varphi2}$ $B_{\varphi3}$	coefficients of nonlinear representation of roll damping	3.2 3.3 3.1
$B_{\varphi e}$	equivalent linear coefficient of roll damping	3.2
b_{BK}	breadth of bilge-keel	2.2 2.2.5.1 2.2 2.2.5.1
b_{SK}	thickness of skeg	2.2 2.2.5.2 2.3 2.3.2

b_{demi}	distance from the centre line to the centre of demihull	2.4 2.4.1 2.4 2.4.3
C_B	Block coefficient $C_B = \nabla/(L\,B\,d)$	2.2 2.2.3
$C_{BK}(b_{BK})$	source strength C_{BK} (a function of b_{BK})	2.2 2.2.5.1
C_D	drag coefficient of something	2.2 2.2.5.1 2.2 2.2.5.2 2.3 2.3.2
C_{D0}	drag coefficient of skeg or flat plate without thickness	2.2 2.2.5.2 2.3 2.3.2
C_f	Frictional resistance coefficient	2.2 2.2.3 2.4 2.4.3
C_L	vertical lift coefficient	2.5
C_M	midship section coefficients $C_M = A_M/(B\,d)$	2.2 2.2.2
C_p	pressure coefficient	2.2 2.2.4 2.2 2.2.5.1 2.3 2.3.1
C_p^-	negative pressure coefficient behind of bilge keel	2.2 2.2.5.1
C_p^-	pressure coefficient behind skeg	2.2 2.2.5.2 2.3 2.3.2
C_p^+	positive pressure coefficient front of bilge keel	2.2 2.2.5.1
C_p^+	pressure coefficient front of the skeg	2.2 2.2.5.2 2.3 2.3.2
C_R	drag coefficient proportional to velocity on surface of rotating cylinder	2.2 2.2.4
C_φ	coefficient of roll restoring moment	3.1 3.3
d	draught of hull	2.2 2.2.1 2.2 2.2.2 2.2 2.2.3 2.2 2.2.4 2.4 2.4.2 2.4 2.4.3
$d_{BK}(\phi)$	depth of the position attached bilge-keel on hull	2.2 2.2.5.1
d_c	depth of chine	2.3 2.3.1
$E\{\delta^2\}$	expectation value	3.2
f	correction factor to take account of the increment of flow velocity at bilge	2.2 2.2.5.1
f_1	$f_1 = 0.5\left[1+\tanh\{20(\sigma-0.7)\}\right]$	2.2 2.2.4
f_2	$f_2 = 0.5(1-\cos\pi\sigma)-1.5\left\{1-e^{-5(1-\sigma)}\right\}\sin^2\pi\sigma$	2.2 2.2.4
f_3	$f_3 = 1+4\exp\left\{-1.65\times10^5(1-\sigma)^2\right\}$	2.2 2.2.4
$f_1(\alpha)$	modification coefficient as a function of the rise of floor (S)	2.3 2.3.1
$f_2(\alpha)$	modification coefficient as a function of the rise of floor (C_p)	2.3 2.3.1
$f_z(y)$	vertical lift force acting on the buttock line including point $A(y)$, with attack angle $\alpha(y)$ [rad.]	2.5
G	the center of gravity	2.2 2.2.1
G	girth length	2.2 2.2.5.1
$\overline{GM}$	Distance of centre of gravity to the metacentre	2.1

g	**Gravity acceleration**	**2.1** **2.2 2.2.1** **2.2 2.2.5.1**
H	$H = 1 + a_1^2 + 9a_3^2 + 2a_1(1-3a_3)\cos 2\psi - 6a_3 \cos 4\psi$	**2.2 2.2.4**
H_0	**half breath draught ratio** $H_0 = B/(2d)$	**2.2 2.2.4** **2.2 2.2.5.1** **2.3 2.3.1**
H_0^*	$H_0^* = \dfrac{B}{2(d - \overline{OG})}$	**2.3 2.3.1**
H'_0	$H'_0 = \dfrac{H_0}{1 - \overline{OG}/d}$	**2.2 2.2.4**
h	**Water depth**	**2.6**
I_φ	**the virtual mass moment of inertia along a longitudinal axis through the centre of gravity**	**3.1**
K	**reduced frequency** $K = \omega L/U$	**2.2 2.2.4**
Ke	**Keulegan-Carpenter number**	**2.1** **2.2 2.2.5.1** **2.2 2.2.5.2** **2.3 2.3.2**
k_1	$k_1 = -\exp(-0.114H_0^2 + 0.584H_0 - 0.558)$	**2.3 2.3.1**
k_2	$k_2 = -0.38H_0^2 + 2.264H_0 + 0.748$	**2.3 2.3.1**
k_L, $k_L(\theta_l)$	**lift slope of vertical lift (for planing hull)**	**2.5**
k_N	**lift slope of horizontal lift (ship in maneuvering)**	**2.2 2.2.2** **2.4 2.4.2**
L	**characteristic length of object (length of ship hull)**	**2.1** **2.2 2.2.2** **2.2 2.2.3** **2.2 2.2.4**
L	**hydrodynamic lift force acting on planing hull**	**2.5**
L_{BK}	**lift force acting on a bilge keel**	**2.2 2.2.5.1**
L_{PP}	**Length between perpendiculars**	**2.4 2.4.3**
l	**distance from the centre of gravity or roll to the tip of skeg or the tip of bilge-keel or chine**	**2.2 2.2.5.1** **2.2 2.2.5.2** **2.3 2.3.1** **2.3 2.3.2**
l_0	**lever defined that the quantity** $l_0\dot{\varphi}/U$ **corresponds to the angle of attack of the lifting body**	**2.2 2.2.2**
l_0'	**distance from the center of gravity to the point of 0.5*d* on center line of demihull**	**2.4 2.4.2**
l_1	**distance from the centre of gravity or roll to the centre of skeg or bilge-keel**	**2.2 2.2.5.1** **2.2 2.2.5.2** **2.3 2.3.2**
l_2	**moment lever integrated pressure along hull surface front of skeg or baseline**	**2.2 2.2.5.2** **2.3 2.3.2** **2.3 2.3.1**

l_3	moment lever integrated pressure along hull surface behind skeg or baseline	2.2 2.2.5.2 2.3 2.3.2 2.3 2.3.1
l_{comp}	length of flooding component	2.6
l_p	moment lever between the centre of gravity or roll and the centre of integrated pressure along hull	2.2 2.2.5.1
l_{BK}	distance from the centre of gravity or roll to the position attached bilge-keel on hull	2.2 2.2.5.1
l_R	distance from still water level to the centre of lift	2.2 2.2.2
l_R’	distance between the center of gravity and the cross point of 0.7*d* water line and the center line of a demihull	2.4 2.4.2
l_{SK}	height of skeg	2.2 2.2.5.2 2.3 2.3.2
l_w	moment lever measured from the still water level due to the sway damping force	2.2 2.2.1
M	$M = \dfrac{B}{2(1+a_1+a_3)}$	2.2 2.2.4
M_φ	roll damping moment	2.1 2.5 3.1
$M_{\varphi APP}$	appendage component of roll damping	2.1
$M_{\varphi E}$	eddy making component of roll damping	2.1 2.2 2.2.4
$M_{\varphi F}$	frictional component of roll damping	2.1
$M_{\varphi L}$	lift component of roll damping	2.1
$M_{\varphi W}$	wave making component of roll damping	2.1
m_1	$m_1 = R/d$	2.2 2.2.5.1
m_2	$m_2 = \overline{OG}/d$	2.2 2.2.5.1
m_3	$m_3 = 1 - m_1 - m_2$	2.2 2.2.5.1
m_4	$m_4 = H_0 - m_1$	2.2 2.2.5.1
m_5	$m_5 = \dfrac{\left\{0.414H_0 + 0.0651m_1^{\ 2} - (0.382H_0 + 0.0106)m_1\right\}}{(H_0 - 0.215m_1)(1-0.215m_1)}$	2.2 2.2.5.1
m_6	$m_6 = \dfrac{\left\{0.414H_0 + 0.0651m_1^{\ 2} - (0.382 + 0.0106H_0)m_1\right\}}{(H_0 - 0.215m_1)(1-0.215m_1)}$	2.2 2.2.5.1
m_7	$m_7 = \begin{cases} S_0/d - 0.25\pi m_1 & ,S_0 > 0.25\pi R \\ 0 & ,S_0 \le 0.25\pi R \end{cases}$	2.2 2.2.5.1
m_8	$m_8 = \begin{cases} m_7 + 0.414m_1 & ,S_0 > 0.25\pi R \\ m_7 + 0.414m_1\left(1-\cos\left(\dfrac{S_0}{R}\right)\right) & ,S_0 \le 0.25\pi R \end{cases}$	2.2 2.2.5.1
m_φ	$m_\varphi = \dfrac{M_\varphi}{A_\varphi}$	3.1
N	Bertin’s *N*-coefficient	2.1 3.3

N_{10}	**Bertin's *N*-coefficient at ϕ = 10 degrees**	3.3
N_{20}	**Bertin's *N*-coefficient at ϕ = 20 degrees**	3.3
O	**origin of the fixed coordinate system on ship (the point on still water level)**	2.2 2.2.1
O'	**origin of the fixed coordinate system on demihull (the point on still water level)**	2.4 2.4.2
$\overline{OG}$	**distance from O to G with positive being download**	2.2 2.2.1 2.2 2.2.2 2.2 2.2.3 2.2 2.2.4
$\overline{O'G}$	**distance from O' to G**	2.4 2.4.2
P_m	**pressure on hull caused by vortex shedding**	2.2 2.2.4
R	**bilge radius**	2.2 2.2.4 2.2 2.2.5.1
Re	**Reynolds number**	2.2 2.2.3 2.4 2.4.3
r	**radius of cylinder**	2.2 2.2.3
r_f	$r_{\mathrm{f}} = 1/\pi \times (0.887 + 0.145 C_B)(1.7d + C_B B) - 2\overline{OG}$	2.2 2.2.3
r_{max}	$r_{\max} = M\sqrt{\begin{array}{l}\{(1+a_1)\sin\psi - a_3\sin 3\psi\}^2 \\ +\{(1-a_1)\cos\psi + a_3\cos 3\psi\}^2\end{array}}$	2.2 2.2.4
S	**length of pressure distribution on cross section**	2.2 2.2.5.2 2.3 2.3.1 2.3 2.3.2
S_0	**length of negative-pressure region**	2.2 2.2.5.1
S_f	$S_{\mathrm{f}} = L(1.7d + C_B B)$	2.2 2.2.3
T	**period of motion**	2.1
T_R	**roll period**	2.2 2.2.3
T_e	**wave encounter period (roll period in waves)**	2.2 2.2.5.2 2.3 2.3.2 2.4 2.4.3
$T_{\square}$	**natural roll period**	3.1
U_{max}	**amplitude of motion velocity or maximum speed of something**	2.1 2.2 2.2.5.2 2.3 2.3.2
u	**maximum speed of the tip of bilge-keel**	2.2 2.2.5.1
$u_z(y)$	**vertical velocity at a point $A(y)$**	2.5
V	**forward velocity** $V = Fr\sqrt{gL}$	2.2 2.2.1 2.2 2.2.2 2.2 2.2.3 2.2 2.2.4 2.2 2.2.5.1 2.4 2.4.2 2.4 2.4.3 2.5
V_R	**relative flow velocity** $V_R^2 = U^2 + u^2$	2.2 2.2.5.1
V_{max}	**maximum relative velocity on the hull surface**	2.2 2.2.4
V_{mean}	**mean velocity on the hull surface**	2.2 2.2.4
y	**transverse distance between the centre of gravity and point $A(y)$**	2.5
y	**lever arm for the roll moment**	2.5

α	Attack angle $\alpha = \tan^{-1}(u/U)$	2.2 2.2.5.1
β	rise of floor (deadrise angle)	2.3 2.3.1
α, β, γ	extinction coefficients $\alpha = \frac{B_1}{2I_\phi}$ $\beta = \frac{B_2}{I_\phi}$ $\gamma = \frac{B_3}{I_\phi}$	3.1 3.3
α_e	equivalent linear extinction coefficient	3.2
$\alpha(y)$	experiences an angle of attack	2.5
δ	discrepancy	3.2
ϕ	roll displacement	3.1
ϕ_a	roll amplitude	2.1 2.2 2.2.3 2.2 2.2.4 2.2 2.2.5.1 2.2 2.2.5.2 2.3 2.3.1 2.3 2.3.2 2.4 2.4.1 2.4 2.4.3 2.6 3.1 3.2
ϕ_m	mean roll angle	3.3
ϕ_n	absolute value of roll angle at the time of the n-th extreme value in free-roll test	3.3
$\dot{\phi}$	roll angular velocity $\varphi_m = [\varphi_{n-1} + \varphi_n]/2$	2.2 2.2.2 2.2 2.2.4 2.2 2.2.5.1 2.4 2.4.1 2.5 3.1
$\ddot{\phi}$	roll angular acceleration	3.1
$\Delta\phi$	$\Delta\phi = \phi_{n-1} - \phi_n$	3.3
γ	ratio of maximum velocity to mean velocity on hull surface $\gamma = V_{max} / V_{mean}$	2.2 2.2.4
κ	modification factor of midship section coefficient	2.2 2.2.2
κ_α	$\kappa_\alpha = \frac{2\alpha}{\omega_\varphi}$	3.3
ν	kinematic viscosity	2.2 2.2.3 2.4 2.4.3
$\theta(y)$	virtual trim angle	2.5
θ_1	running trim angle	2.5
ξ_d	$\xi_d = \omega_e^2 d / g$	2.2 2.2.1

ρ	mass density of fluid	2.1 2.2 2.2.2 2.2 2.2.3 2.2 2.2.4 2.2 2.2.5 2.2 2.2.5.2 2.3 2.3.1 2.3 2.3.2 2.4 2.4.2 2.4 2.4.3 2.5
σ	area coefficient $\sigma = A_j / (Bd)$	2.2 2.2.4 2.2 2.2.5.1
$\sigma_{\dot{\phi}}$	variance of roll angular velocity	3.2
σ'	$\sigma' = \dfrac{\sigma - \overline{OG}/d}{1 - \overline{OG}/d}$	2.2 2.2.4
ψ	Lewis argument on the transformed unit circle	2.2 2.2.4
ψ_1	$0 = \psi_1 \qquad (r_{\max}(\psi_1) \geq r_{\max}(\psi_2))$	2.2 2.2.4
ψ_2	$\dfrac{1}{2}\cos^{-1}\dfrac{a_1(1+a_3)}{4a_3} = \psi_2 \qquad (r_{\max}(\psi_1) < r_{\max}(\psi_2))$	2.2 2.2.4
Ω	$\Omega = U\omega_E / g$	2.2 2.2.1
ω_E	wave encounter circular frequency (roll circular frequency in waves)	2.1 2.2 2.2.1 2.2 2.2.3 2.2 2.2.4 2.2 2.2.5.1 2.2 2.2.5.2 2.3 2.3.1 2.3 2.3.2 2.4 2.4.1 2.4 2.4.3 2.6 3.1 3.2
$\hat{\omega}_E$	non-dimensional wave encounter circular frequency (non-dimensional roll circular frequency in waves)	2.1
ω_{IW}	natural circular frequency of water in a tank $\omega_{IW} = \dfrac{\pi}{B_{comp}}\sqrt{gh}$	2.6
ω_φ	roll natural circular frequency $\omega_\phi = \sqrt{\dfrac{C_\phi}{A_\phi}} = \dfrac{2\pi}{T_\phi}$	3.1
∇	displacement volume	2.1

6. VALIDATION

6.1 Uncertainty Analysis

None

6.2 Bench Mark Model Test Data

6.2.1 Wave making component and Lift component

Refer to Ikeda *et al.*, (1978a) or (1978c)

6.2.2 Frictional component

None

6.2.3 Eddy making component

Refer to Ikeda et al.,(1977a) or (1978b).

6.2.4 Appendages component

a) Bilge keel component

Refer to Ikeda et al., (1976), (1977b) or (1979)

b) Skeg component

Refer to Baharuddin et al., (2004)

6.2.5 Hard chine hull

Refer to Ikeda et al.,(1990) or Tanaka et al., (1985)

6.2.6 Multi-hull

Refer to Katayama et al, (2008).

6.2.7 Planing hull

Refer to Ikeda et al., (2000)

6.2.8 Frigate

Refer to Etebari et al.,(2008), Bassler et al., (2007), Grant et al., (2007), Atsa-vapranee et al., (2007) or (2008).

6.2.9 Water on deck or water in tank

Refer to Katayama et al, (2009).

6.3 Bench Mark Data of Full Scale Ship

Refer to Atsavapranee *et al.*, (2008). Flow visualization around bilge keel and free decay test results are indicated.

6.4 Measurement of Roll Damping

6.4.1 Free Decay Test

Refer to IMO MSC.1/ Circ.1200 AN-NEX, Page 11, 4.6.1.1 Execution of roll decay tests.

6.4.2 Forced Roll Test

6.4.2.1 Fully Captured tests

Refer to Ikeda et al., (1976), (1977a), (1978a), (1990), (1994), (2000), Katayama et al., (2008) or (2009), Bassler et al., (2007).

6.4.2.2 Partly Captured tests

Refer to Hashimoto et al., (2009).

7. REFERENCES

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